DOUGLAS H. CLEMENTS

COMPUTERS IN ELEMENTARY MATHEMATICS EDUCATION

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To teachers dedicated to a new vision
of mathematics education,

To Mike Battista, for untold hours
of arguments leading to insights,

and especially,

To Holly and Ryan, who kept me
from going too fast—
often in the wrong direction

PREFACE

This book is addressed to those concerned with teaching mathematics to elementary school students. Given that computers are increasingly available in the classroom, there is a need to consider how these tools *can* be used and how they *should* be used to help students develop sound mathematical understandings. This book provides guidelines for such use, as well as numerous illustrations and suggestions for specific programs and activities.

To begin with the "big picture," Part I, Foundations of a New Vision, poses three critical questions: What are the present problems of elementary mathematics education? How do students learn mathematics? How best might computers aid in overcoming the problems and helping students learn? The answers to these questions can be surprising.

Part II, The Computer as Tutor, Tool, Tutee, introduces three roles the computer can play in the classroom. It can act as a sophisticated *teaching machine*, instructing students and monitoring their progress. It can serve as a *tool* for graphing or calculating. It can be *programmed* by students as they solve problems and explore mathematical ideas. For each of these roles, strengths, weaknesses, general guidelines, and specific suggestions are provided.

Part III, Computers and the Evolving Curriculum, provides detailed suggestions for using computers to teach the major topics in elementary mathematics education. Sample computer applications are examined in depth, and practical ideas for teaching are developed. Extensive lists of available computer programs are provided at the end of each of these chapters.

Part IV, Focusing the Vision, addresses four important questions regarding the implementation of computers into the classroom: When should we use computers? How can we integrate computer use into classroom routines? How can we integrate computer-enhanced mathematics with other subjects? How can we use computers to contribute to the mathematics education of students with special needs?

Thus, the focus of this book is not on computers per se, but on teaching mathematics *with* computers. Appropriate use of computers can help both teachers and students. This successful use, however, is not easily achieved. The goal of this book is to provide information to teachers who are willing to accept this challenge for the sake of their students. Such informed, interested teachers will pave the way for others and will gather immeasurable benefits for their students—and for themselves—along the way.

Douglas H. Clements

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CHILDREN, MATHEMATICS, AND COMPUTERS

You are undoubtedly familiar with computers, at least to some extent. You've seen computer games and computer programs that give children practice with arithmetic facts. Whether or not you've actually worked with computers, the basic picture seems clear. You also have had years of experience learning, and possibly teaching, mathematics, and have developed certain impressions. This book offers a glimpse of new views—possibly visions—of how children can experience mathematics with computers.

I'd like to share visions of computers in mathematics education that may take a different perspective from those you've already seen. These visions may help you use computers to teach better mathematics and to teach mathematics better. But why *do* we need a new view of teaching mathematics, with or without computers? Doesn't present-day teaching represent the "best teaching"?

PROBLEMS IN ELEMENTARY MATHEMATICS EDUCATION

Results of the second and third National Assessments of Educational Progress (NAEP) in mathematics indicate major deficiencies in students' learning of mathematics. According to Carpenter et al. (1980), "students' performance showed a lack of understanding of basic concepts and processes in many content areas," and "it appeared that most students had not learned basic problem-solving skills, and attempted instead to mechanically apply some mathematical calculation to whatever numbers were given in a problem" (p. 28). Thus, it appears that the dominant focus of school mathematics instruction in the last decade has been on computational skills (which students are learning fairly well), but that the development of problem-solving skills and conceptual understandings has been inadequate.

Indeed, the NAEP provides empirical support for the National Council of Teachers of Mathematics' (NCTM, 1980) recommendation that teachers of mathematics provide opportunities for their students to be actively involved in learning, experimenting with, exploring, and communicating about mathematics as part of an environment that encourages problem solving.

This sounds grandly impressive. But what does this have to do with the "basics"? Let's ask a more *basic* question: How do children learn mathematics?

THEORIES OF LEARNING MATHEMATICS

Why Theories?

Many teachers seem to believe that the only reason that "theory" appears in books and courses is to satisfy some incomprehensible whim of the professor. It is true that theories in mathematics education are far from complete and that drawing direct implications for classroom practice is not always easy. It is equally true, however, that every "fact" gets its importance from being described from within the framework of some theory.

Let's look at a nonmathematical example. You have undoubtedly heard parents espouse the modern equivalent of "spare the rod and spoil the child." What theory lies behind this belief? Something like the following: Children share a certain natural tendency such that, without punishment (preferably physical punishment), their inherent selfishness and desire to break rules would grow without bounds. You have also undoubtedly heard a different opinion (usually from the parent of some young visitor who is tearing apart large portions of your house): "I don't want to stifle his curiosity and creativity, and besides, boys will be boys." Two theories are probably operative here: first, a misinterpretation of Freud's ideas about repressions; second, an impression that there is a natural tendency for males to misbehave vigorously, and that there is not much anyone can do about it. Understanding a bit about the theories that underlie their prescriptions would help these parents apply the theories more consistently and effectively. It also would help them apply them more correctly! One of these notions is invalid, the other (Freudian psychology) woefully misapplied.

In the preceding section we referred to two different approaches to mathematics education. One stated that learning is primarily a constructive process in which students take responsibility for building their knowledge; the other, that children have to be taught basic skills. Both are based on beliefs about mathematics and theories about how children learn. What are these theories? Is one more valid than the other? Are they incompatible? In this section we begin to answer these questions. The rest of the book will build on this beginning, based on the premise that:

> Without practice, theory is a flower not smelled or seen, a library whose dust-covered books are not read.
>
> Without theory, practice is a mere bag of tricks, a trivial compendium that, in mathematics education especially, may hurt more than it helps.

The Two Voices of Piaget

Piaget did not directly study teaching or classroom learning. Nevertheless, his investigations of the process of learning and the nature of knowing have profound implications for education. He stated two major ideas.

The first voice: The child actively constructs knowledge. Piaget believed that knowledge is not a state children are in, but rather a process in which they are engaged. Children know about balls and bouncing because they have played with balls and other objects that bounce. Thus, children construct their own knowledge. They learn by inventing.

But what of mathematics? Maybe we can accept that children learn about bouncing balls on their own, but can children really invent *mathematics* on their own? One interesting research study provides an answer. Groen and Resnick (1977) measured how long preschoolers took to solve simple addition problems. They found, as they expected, that it took longer for these children to solve 5 + 3 than it took them to solve 4 + 2, because the children would count 4 fingers (or other objects), then count 2 fingers, and then count all 6. Naturally, counting 5, then 3, then all 8 took more time. As they measured the children's responses over time, however, an amazing and unexpected thing happened. The times no longer fit the same pattern. Children began solving 2 + 8 faster than 4 + 5. The researchers examined the response times and discovered what was happening: The children were now starting with the larger of the two addends (e.g., 8) and counting up (or "counting on") from that number (e.g., "eight . . . nine, ten"). This took less time (even less time than did "five . . . six, seven, eight, nine"). *During the research study, these 4-year-old children had invented, on their own, a new and sophisticated method of solving addition problems—a method they had never been taught!*

This study is not the only one showing such results. We now know that students of all ages invent their own solutions for solving mathematical problems. Unfortunately, they are not usually rewarded for such inventions (most of which are never even recognized by the teacher). They are often told to "do it the right way."

Piaget believed that children must be engaged in direct action with the content of the curriculum. He also believed that this occurred too infrequently in school.

> If the aim of intellectual training is to form the intelligence rather than to stock the memory, and to produce intellectual explorers rather than mere erudition, then traditional education is manifestly guilty of a grave deficiency. (Piaget, 1970, p. 51)

Piaget suggested that educators provide children with things and ideas to manipulate that will make them conscious of problems and will encourage them to find answers for themselves. Because real comprehension involves reinvention by the child, the teacher should be less the giver of lessons and more the organizer of engaging, problematic situations.

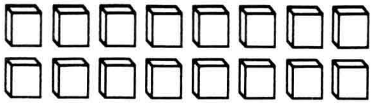
Is the need for this type of education as urgent today as when Piaget wrote? The NAEP results indicated that students "perceive their role in the mathematics classroom to be primarily passive. . . . They feel they have little opportunity to interact with their classmates about the mathematics being studied, to work on exploratory activities, or to work with manipulatives" (Carpenter et al., 1980, p. 36). Remember, too, that problem solving was their weakest area. It is no wonder that the National Council of Teachers of Mathematics recommends that students be actively involved in learning, experimenting with, exploring, and communicating about mathematics.

The second voice: Development proceeds through stages. A stage is a period of time in which a child's thinking reflects a particular mental structure. Piaget's periods follow an invariant sequence, with each period building on and incorporating the previous one. The *sensorimotor period* starts at birth and ends at about 2 years of age. Infants' learn through sensory and motor activity. They begin with reflex actions and gradually integrate them into exploratory and experimental actions.

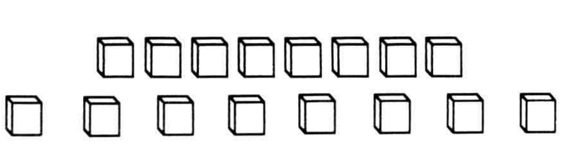
The second period, from approximately age 2 to 7, is the *preoperational stage*. Children learn to use images and language. These symbol systems begin to free thought from concrete action. However, there are limitations to thought during this period. The symbols cannot be manipulated to produce completely logical thought sequences. Preoperational children's thought is:

> *Centered*. They find it difficult to take another's point of view. They also "center on," or consider, only one aspect of a situation and ignore other aspects.
>
> *Irreversible*. They cannot move back and forth between situations, relating before to after.

For example, you might show a child two rows of blocks, matched one to one:



While he or she is looking, spread out one row and ask: Do the rows have the same number of blocks, or does one row have more?



The child probably will center on the length of the row, ignoring the density (how close the blocks are to one another), and state that the second row contains a greater number of blocks. The child does not see that one could reverse the action by moving the second row back into one-to-one correspondence with the first. Children do not conserve number; they do not believe that the number of objects in a group remains the same (is conserved) when the spatial arrangement of the objects is changed.

From about age 7 to 12, during most of the elementary school years, the child is in the period of *concrete operations*. Children can think logically, applying such operations as classification, ordering, reversibility, and conservation (of number, and also of length and area). They can decenter, taking others' perspectives and taking into consideration several aspects of a situation. They can reverse their thinking; for example, they might understand that any addition can be "undone" by a subtraction operation.

But these students have not yet achieved the final period of *formal operational thought*. When they do, they will be able to deal with abstractions and hypotheses that have no direct connection to the real world. They will be able to think about ideas, about thoughts themselves. They will grasp such abstract notions as proportion. They will address problems systematically, scientifically.

Many theorists and teachers believe with Piaget that students actively learn mathematics. They manipulate objects and ideas in a continuous process of building up their own understandings. Not everyone, as we shall see in Chapter 4, believes that students' thought is absolutely "bound" by a given developmental period (actually, Piaget would have agreed to an extent). However, knowledge of these stages of cognitive growth is invaluable to teachers for understanding the thinking processes (and "errors") of their students.

Richard Skemp: One Name (Mathematics), Two Subjects

In the words of Richard Skemp, there are "two effectively different subjects being taught under the same name, 'mathematics'" (1976, p. 22). One subject, *instrumental mathematics*, consists of a limited number of "rules without reasons." The other, *relational mathematics*, is "knowing both what to do and why." It involves building up conceptual structures from which a learner can produce an unlimited number of rules to fit an unlimited set of situations. As Skemp continues, "what constitutes mathematics is not the subject matter, but a particular kind of knowledge about it" (p. 26). From this perspective, the current elementary mathematics curriculum is deficient because it neglects relational understanding. There is too much emphasis on instrumental understanding—formal symbolism and naming—and not enough on analysis, synthesis, and problem solving—on *meaning*.

To Skemp, mathematics is a system of concepts that becomes organized at increasingly higher levels of abstractions. To learn these concepts, students need examples, such as meaningful applications of arithmetic operations (e.g., subtraction). Such concepts, once learned, serve as meaningful examples for higher-level concepts. Rote or instrumental learning actually blocks later learning, because students do not build the necessary mental structures that support higher-level concepts.

Two Kinds of Worthwhile Mathematical Thinking

Skemp's arguments seem plausible; children should develop thinking processes. But don't they need practice for mastery? An answer to this question has come from research in psychology. There are two different types of worthwhile mathematical thinking. One, *automatic thinking*, involves fast, effortless performance. If certain skills and facts are not learned well, too much of children's thinking (cognitive processing capacity) is used up, and there is not enough left for higher-level problem solving. For example, children ultimately have to be able to count forward and backward from any number, without having to "think about it" too much.

The other kind of thinking is *reflective thinking*. Here, children are consciously aware of the problem and the solution processes they use to solve it. Although it is still true that children need lower-level skills and knowledge to become "experts," they also need to develop reflective thinking at each stage of their development of mathematical knowledge. Therefore, from the earliest years they need to be challenged to solve problems based on the skills and knowledge they currently possess. This helps them organize all their knowledge into strong, useful frameworks upon which future learning can be built.

Wait. Why is "automatic" thinking good, but "rote" or "instrumental" mathematics bad? Aren't these really two names for the same thing? No. Automatic thinking is necessary, but it can be distinguished from rote recitation of facts or mechanical processing of numbers. Students who have automatized a process they have learned with understanding can—at any point—pause and explain what they are doing and why. Students who have learned by rote cannot; they are on a meaningless treadmill of mechanical manipulations.

In somewhat simplified terms, then, we actually want students to act

FIGURE 1-1 Jon Secaur contemplates a problem-solving program with his students. (Photo by Gary Harwood.)