

WORLD SCIENTIFIC SERIES ON
NONLINEAR SCIENCE



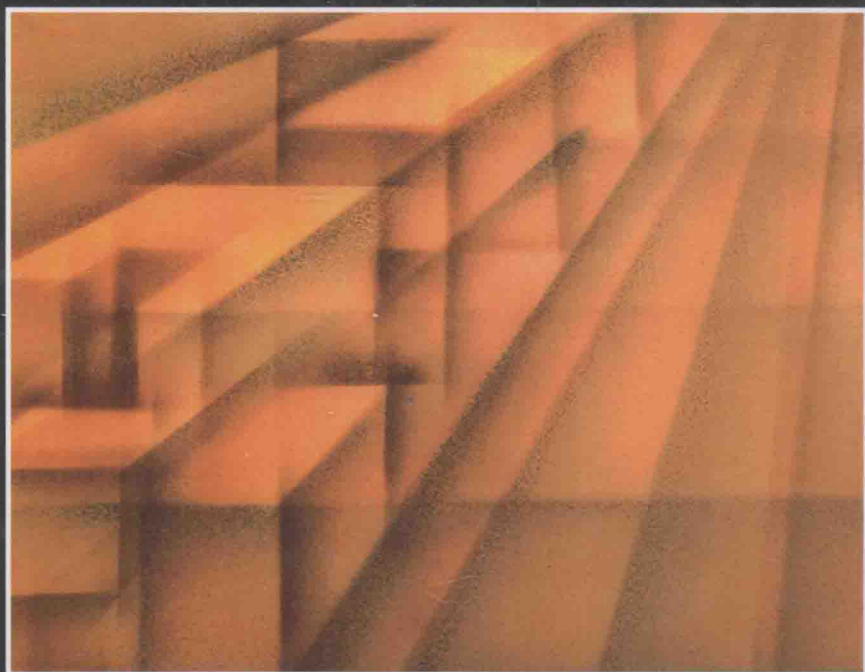
Series A Vol. 72

Series Editor: Leon O. Chua

FRACTIONAL ORDER SYSTEMS

Modeling and Control Applications

Riccardo Caponetto • Giovanni Dongola
Luigi Fortuna • Ivo Petráš



World Scientific

WORLD SCIENTIFIC SERIES ON
NONLINEAR SCIENCE

Series A Vol. 72

Series Editor: Leon O. Chua

FRACTIONAL ORDER SYSTEMS

Modeling and Control Applications

Riccardo Canone

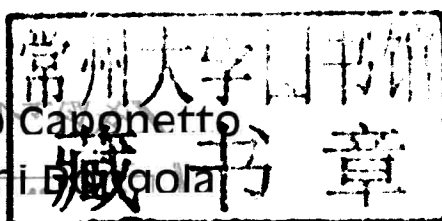
Giovanni D'Amico

Luigi Fortuna

University of Catania, Italy

Ivo Petráš

Technical University of Košice, Slovakia



 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

World Scientific Series on Nonlinear Science, Series A — Vol. 72

FRACTIONAL ORDER SYSTEMS

Modeling and Control Applications

Copyright © 2010 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN-13 978-981-4304-19-1

ISBN-10 981-4304-19-0

Printed by FuIsland Offset Printing (S) Pte Ltd, Singapore

FRACTIONAL ORDER SYSTEMS

Modeling and Control Applications

WORLD SCIENTIFIC SERIES ON NONLINEAR SCIENCE

Editor: Leon O. Chua
University of California, Berkeley

Series A. MONOGRAPHS AND TREATISES*

- Volume 56: Strange Nonchaotic Attractors
U. Feudel, S. Kuznetsov & A. Pikovsky
- Volume 57: A Nonlinear Dynamics Perspective of Wolfram's New Kind of Science
L. O. Chua
- Volume 58: New Methods for Chaotic Dynamics
N. A. Magnitskii & S. V. Sidorov
- Volume 59: Equations of Phase-Locked Loops
J. Kudrewicz & S. Wasowicz
- Volume 60: Smooth and Nonsmooth High Dimensional Chaos and the Melnikov-Type Methods
J. Awrejcewicz & M. M. Holicke
- Volume 61: A Gallery of Chua Attractors (with CD-ROM)
E. Bilotta & P. Pantano
- Volume 62: Numerical Simulation of Waves and Fronts in Inhomogeneous Solids
A. Berezovski, J. Engelbrecht & G. A. Maugin
- Volume 63: Advanced Topics on Cellular Self-Organizing Nets and Chaotic Nonlinear Dynamics to Model and Control Complex Systems
R. Caponetto, L. Fortuna & M. Frasca
- Volume 64: Control of Chaos in Nonlinear Circuits and Systems
B. W.-K. Ling, H. H.-C. Lu & H. K. Lam
- Volume 65: Chua's Circuit Implementations: Yesterday, Today and Tomorrow
L. Fortuna, M. Frasca & M. G. Xibilia
- Volume 66: Differential Geometry Applied to Dynamical Systems
J.-M. Ginoux
- Volume 67: Determining Thresholds of Complete Synchronization, and Application
A. Stefanski
- Volume 68: A Nonlinear Dynamics Perspective of Wolfram's New Kind of Science (Volume III)
L. O. Chua
- Volume 69: Modeling by Nonlinear Differential Equations
P. E. Phillipson & P. Schuster
- Volume 70: Bifurcations in Piecewise-Smooth Continuous Systems
D. J. Warwick Simpson
- Volume 71: A Practical Guide for Studying Chua's Circuits
R. Kiliç
- Volume 72: Fractional Order Systems: Modeling and Control Applications
R. Caponetto, G. Dongola, L. Fortuna & I. Petráš

*To view the complete list of the published volumes in the series, please visit:
http://www.worldscibooks.com/series/wssnsa_series.shtml

Preface

This book is devoted to fractional order systems, their applications to modelling and control. It is based on derivatives and integrals of arbitrary (real) order, fractional differential equations and methods of their solution, approximations and implementation techniques.

The advantages of fractional calculus have been described and pointed out in the last few decades by many authors. It has been shown that the fractional order models of real systems are regularly more adequate than usually used integer order models.

Applications of these fractional order models are in many fields, as for example, rheology, mechanics, chemistry, physics, bioengineering, robotics and many others.

At the same time, fractional integrals and derivatives are also applied to the theory of control of dynamical systems, when the controlled system and/or the controller is described by fractional differential equations.

The main goal of the book is to present applications and implementations of fractional order systems. It provides only a brief theoretical introduction to fractional order system dedicating almost all the space to the modelling issue, fractional chaotic system control and fractional order controller theory and realization.

The book is suitable for advanced undergraduates and graduate students.

It is organized as follows:

Chapter one is a brief introduction to the fractional order systems. Some historical notes, definitions and fundamentals are described.

Chapter two is dedicated to Fractional Order PID Controller defining their stability regions when first order with time delay plant have to be controlled in closed loop.

Chapter three is on fractional order chaotic systems. In this chapter, a survey of well-known chaotic systems is presented. Mathematical models of nonlinear dynamical systems contain the fractional derivatives. Total order of the system is less than three, however, the chaotical phenomena, as for example, in strange attractors can be observed in such systems.

In chapter four the operator s^m , where m is a real number, is approximated via the binomial expansion of the backward difference and then a hardware implementation of differintegral operator is proposed using Field Programmable Gate Array (FPGA). This building block represents the basic element to implement fractional order control systems.

Chapter five is devoted to microprocessor implementation of the fractional order controllers. Fundamentals on discrete approximations of a fractional operator as well as control algorithm for implementation of the controllers are described. Also presented are three examples of the discrete fractional order controllers implemented on PIC, PC with PCL card, and PLC, respectively. A real measurement and obtained results are shown for each particular case. Some concluding remarks close this chapter.

Chapter six is dedicated to the implementation of the fractional order PID controller by using the analog counter part of FPGA that is Field Programmable Analog Array (FPAA).

Chapter seven presents a possible implementation of an Integrated Circuit by using the switched capacitor technology. The aim of the chapter is to start a research activity that can provide an integrated circuit implementing differintegral operators.

Chapter eight concludes this book showing an useful modelling application of fractional order system on Ionic Polymeric Metal Composite (IMPC) membranes. Going beyond the IMPC, the proposed modelling approach shows that it is possible to obtain low order fractional order models instead of bigger order integer one.

More than 140 references are listed and cited in the book, even if it cannot be a complete bibliography for this area of interest. Readers can find many other references related to this topic.

Riccardo Caponetto
Giovanni Dongola
Luigi Fortuna
Ivo Petráš

Acknowledgments

There are several people to whom the authors are obliged for their help and support.

Ivo Petráš (Technical University of Košice, Slovakia) would like to express his thanks to Prof. Igor Podlubny, Prof. Ján Terpák, Prof. Ľubomir Dorčák, and Prof. Imrich Košťál (Technical University of Košice, Slovakia), Prof. Paul O’Leary (Montanuniversitat of Leoben, Austria), Prof. YangQuan Chen (Utah State University in Logan, USA), and Prof. Blas M. Vinagre (University of Extremadura in Badajoz, Spain) for their help, exchange of information and the fruitful discussions. The author would also like to thank a number of colleagues and friends who supported his work. Last but not least, Ivo Petráš is also thankful to his entire family for their understanding and support.

Particular thanks from Giovanni Dongola, Luigi Fortuna and Riccardo Caponetto to Professor Leon Chua from University of California, Berkeley, for his continuous encouragement in their research life.

Contents

<i>Preface</i>	v
<i>Acknowledgments</i>	vii
<i>List of Figures</i>	xiii
<i>List of Tables</i>	xxi
1. Fractional Order Systems	1
1.1 Fractional Order Differintegral Operator: Historical Notes	1
1.2 Preliminaries and Definitions	2
1.3 Laplace Transforms and System Representation	4
1.4 General Properties of the Fractional System	6
1.5 Impulse Response of a General Fractional System	9
1.6 Numerical Methods for Calculation of Fractional Derivatives and Integrals	12
1.7 Fractional LTI Systems	16
1.8 Fractional Nonlinear Systems	20
1.9 Stability of Fractional LTI Systems	20
1.10 Stability of Fractional Nonlinear Systems	30
2. Fractional Order PID Controller and their Stability Regions Definition	33
2.1 Introduction	33
2.2 Problem Characterization	35
2.3 Theory for Analyzing Systems with Time Delays	36
2.3.1 Hermite-Biehler Theorem	37
2.3.2 Pontryagin Theorem	38

2.4	Stability Regions with $PI^\lambda D^\mu$ Controller	38
2.5	Results	41
3.	Fractional Order Chaotic Systems	53
3.1	Introduction	53
3.2	Concept of Chua's System	54
3.2.1	Classical Chua's Oscillator	54
3.2.2	Chua-Hartley's Oscillator	56
3.2.3	Chua-Podlubny's Oscillator	56
3.2.4	New Fractional-Order Chua's Oscillator	56
3.3	Fractional-Order Van der Pol Oscillator	59
3.4	Fractional-Order Duffing's Oscillator	60
3.5	Fractional-Order Lorenz's System	62
3.6	Fractional-Order Genesio-Tesi System	65
3.7	Fractional-Order Lu's System	66
3.8	Fractional-Order Rossler's System	67
3.9	Fractional-Order Newton-Leipnik System	68
3.10	Fractional-Order Lotka-Volterra System	69
3.11	Concept of Volta's System	72
3.11.1	Integer-Order Volta's System	72
3.11.2	Fractional-Order Volta's System	73
4.	Field Programmable Gate Array Implementation	77
4.1	Numerical Fractional Integration	77
4.2	Grünwald-Letnikov Fractional Derivatives	78
4.3	The "Short-Memory" Principle	81
4.4	FPGA Hardware Implementation	82
4.4.1	FPGA Introduction	82
4.4.2	Remarks on the Fractional Differintegral Operator	83
4.4.3	FPGA Implementation of the Fractional Differintegral Operator	85
5.	Microprocessor Implementation and Applications	91
5.1	Introduction	91
5.2	Fractional Controller Realized by PIC Processor	98
5.2.1	Fractional-Order Integrator	99
5.2.2	Measured Results	99
5.3	Temperature Control of a Solid by PC and PCL 812	99

5.3.1	Model of Controlled System	99
5.3.2	Controller Parameters Design and Implementation	101
5.3.3	Experimental Setup and Results	104
5.4	Temperature Control of a Heater by PLC BR 2005	107
5.4.1	Model of Controlled System	107
5.4.2	Controller Parameters Design and Implementation	108
5.4.3	Experimental Setup and Results	110
5.5	Concluding Remarks	113
6.	Field Programmable Analog Array Implementation	115
6.1	The FPAAs Development System	115
6.2	Experimental Results	121
7.	Switched Capacitor Integrated Circuit Design	127
7.1	Introduction	127
7.2	Passive and Active Filters	128
7.3	Switched Capacitors Filters	129
7.4	Design of Sampled Data Filters	130
7.4.1	The Impulse Invariance Method	130
7.4.2	The Matched-z Transformation Method	131
7.4.3	Backward Euler Approximation of Derivatives . .	132
7.4.4	Forward Euler Approximation of Derivatives . . .	133
7.4.5	The Bilinear Transformation Method	134
7.4.6	The Lossless Discrete Integrator Transformation .	135
7.5	Switched Capacitor Fundamental Circuits	135
7.5.1	Resistor Realized by Backward Euler Transformation	136
7.6	Circuitual Implementation of the Fractional Order Integrator	136
7.7	Switched Capacitors Implementation of Fractional Order Integrator	139
7.8	Results	139
8.	Fractional Order Model of IPMC	145
8.1	Fractional Model Identification Introduction	145
8.2	Ionic Polymer Metal Composites (IPMC)	146
8.3	Actuation Mechanism on IPMCs	149
8.4	State-of-the-Art for IPMC Models	151

8.5	Experimental Setup	152
8.6	Marquardt Algorithm for the Least Squares Estimation	155
8.7	Fractional Models for IPMC Actuators	158
8.7.1	Comparison Between an Integer Model and a Fractional Model of IPMC Actuators	158
8.7.2	Fractional Models for the Electrical and Electromechanical Stages of IPMC Actuators	160
	<i>Bibliography</i>	167
	<i>Index</i>	177

List of Figures

1.1	Magnitude Bode Plot of fractional system $F(s) = 1/(s + 1)^m$ with $m = 1$ (solid), $m = 0.5$ (dashed), $m = 1.5$ (dotted)	7
1.2	Phase Bode Plot of fractional system $F(s) = 1/(s + 1)^m$ with $m = 1$ (solid), $m = 0.5$ (dashed), $m = 1.5$ (dotted)	8
1.3	Impulse response of a fractional system for different values of m (dashed), $m = 0.5$ (solid), $m = 1$ (dotted), $m = 1.5$	9
1.4	Step response of a fractional system for different values of m (dashed), $m = 0.5$ (solid), $m = 1$ (dotted), $m = 1.5$	10
1.5	The impulse response $f(t)$ as m varies from 0.25 to 2 in 0.25 increments	11
1.6	The Mittag-Leffler function, $E_m[-t]$ as m varies from 0.25 to 2 in 0.25 increments	12
1.7	The step response as m varies from 0.25 to 2 in 0.25 increments	13
1.8	Analytical solution of the FODE (1.78) where $u(t) = 0$ for 50 s with zero initial conditions	26
1.9	Riemann surface of function $w = s^{\frac{1}{10}}$ and roots of equation (1.82) in complex w -plane	28
1.10	Stability regions of the fractional order system	31
1.11	Double scroll attractor of Chen's system (1.93) projected into 3D state space for 30 sec	32
2.1	Open-loop step response	35
2.2	Feedback control system	36
2.3	K_d vs η as computed by applying the first procedure to the $PD^{1/3}$ controller ($K_i = 0, a = 0, b = 3, c = 1$)	43
2.4	Trend of $\delta_i^*(\omega)$ for different values of K_d for the $PD^{1/3}$ controller ($K_i = 0, a = 0, b = 3, c = 1$)	44

2.5	Stability region of the $PD^{1/3}$ controller ($K_i = 0, a = 0, b = 3, c = 1$)	45
2.6	Trend of $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ for different values of K_d for the $PD^{1/3}$ controller ($K_p = 0, K_i = 0, a = 0, b = 3, c = 1$)	46
2.7	Stability region of the $PD^{2/3}$ controller ($K_i = 0, a = 0, b = 3, c = 2$)	48
2.8	Stability region of the $PD^{1/2}$ controller ($K_i = 0, a = 0, b = 2, c = 1$)	48
2.9	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{1/3}$ with (a) $K_p = -0.9, K_d = 2$, (b) $K_p = 0, K_d = -1.9$, (c) $K_p = 0, K_d = 3.2$, (d) $K_p = 3, K_d = -3.7$, (e) $K_p = 3, K_d = 0.3$, (f) $K_p = 4.5, K_d = -2.7$	49
2.10	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{1/3}$ with (a) $K_p = -1.1, K_d = 2$, (b) $K_p = 0, K_d = -2$, (c) $K_p = 0, K_d = 3.3$, (d) $K_p = 3, K_d = -3.8$, (e) $K_p = 3, K_d = 0.4$, (f) $K_p = 4.6, K_d = -2.7$	49
2.11	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{2/3}$ with (a) $K_p = -0.9, K_d = 1$, (b) $K_p = 0, K_d = -2.4$, (c) $K_p = 0, K_d = 2.7$, (d) $K_p = 2, K_d = -2.2$, (e) $K_p = 2, K_d = 1.7$, (f) $K_p = 3.3, K_d = -0.3$	50
2.12	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{2/3}$ with (a) $K_p = -1.1, K_d = 1$, (b) $K_p = 0, K_d = -2.5$, (c) $K_p = 0, K_d = 2.8$, (d) $K_p = 2, K_d = -2.3$, (e) $K_p = 2, K_d = 1.8$, (f) $K_p = 3.4, K_d = -0.3$	50
2.13	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{1/2}$ with (a) $K_p = -0.9, K_d = 2$, (b) $K_p = 0, K_d = -2.2$, (c) $K_p = 0, K_d = 2.9$, (d) $K_p = 2, K_d = -2.8$, (e) $K_p = 2, K_d = 1.5$, (f) $K_p = 3.6, K_d = -1.2$	51
2.14	Step responses of the closed-loop system where $C(s) = K_p + K_d s^{1/2}$ with (a) $K_p = -1.1, K_d = 2$, (b) $K_p = 0, K_d = -2.3$, (c) $K_p = 0, K_d = 3.1$, (d) $K_p = 2, K_d = -2.9$, (e) $K_p = 2, K_d = 1.6$, (f) $K_p = 3.7, K_d = -1.2$	51
3.1	Practical realization of Chua's circuit	55
3.2	Piecewise-linear $v - i$ characteristic of the nonlinear resistor	55
3.3	Photo of oscilloscope screen: Strange attractor of the Chua's system (3.12)	58

3.4	Strange attractor of the fractional-order Chua's system (3.13) with total order $\bar{q} = 2.90$ for the parameters: $\alpha = 10.1911$, $\beta = 10.3035$, $\gamma = 0.1631$, $q_1 = q_2 = 0.98$, $q_3 = 0.94$, $m_0 = -1.1126$ and $m_1 = -0.8692$	59
3.5	Phase plane (y_1, y_2) plot (limit cycle) for FrVPO with fractional-order $q = 0.9$ and parameter $\epsilon = 1$. Initial conditions were: $\bar{y}_0 = [0, -2]$	61
3.6	Phase plane (x, y) plot for the Duffing's system (3.20) with parameters $\alpha = 0.15$, $\delta = 0.3$, $\omega = 1$, and initial conditions $(x(0), y(0)) = (0.21, 0.13)$	62
3.7	Phase plane (x, y) plot (attractor) for the fractional order Duffing's system (3.21) with parameters $\alpha = 0.15$, $\delta = 0.3$, $\omega = 1$, derivative orders $q_1 = 0.9$, $q_2 = 1.0$, and initial conditions $(x(0), y(0)) = (0.21, 0.13)$	63
3.8	Simulation result of the Lorenz's system (3.23) in $x - y$ plane for initial conditions $(x(0), y(0), z(0)) = (0.1, 0.1, 0.1)$	64
3.9	Simulation result of the Lorenz's system (3.23) in $x - z$ plane for initial conditions $(x(0), y(0), z(0)) = (0.1, 0.1, 0.1)$	65
3.10	Simulation result of the Genesio-Tesi system (3.25) in $y - z$ plane for initial conditions $(x(0), y(0), z(0)) = (-2, 0.5, 2)$	66
3.11	Projection onto $x - y$ plane of Lu's fractional-order system (3.26) for parameters $a = 36, b = 3, c = 20$ and orders $q \in (0.985, 0.99, 0.98)$ for simulation time 60 sec	67
3.12	Simulation result of Rossler's fractional-order system (3.27) in state space for parameter $a = 0.5$ and order $q = 0.9$ for simulation time 120 sec, for initial conditions $(x(0), y(0), z(0)) = (0.5, 1.5, 0.1)$	68
3.13	Projection onto $x - y$ plane of the fractional-order Newton-Leipnik system (3.29) for parameters $a = 0.8, b = 0.175$ and orders $q \in (0.99, 0.99, 0.99)$ for simulation time 200 sec	69
3.14	Projection onto $y - z$ plane of the fractional-order Newton-Leipnik system (3.29) for parameters $a = 0.8, b = 0.175$ and orders $q \in (0.99, 0.99, 0.99)$ for simulation time 200 sec	70
3.15	Phase plane (x, y) plot (limit cycle) for the Lotka-Volterra system with order $q = 1.0$ and parameter $a = 2, b = 1, c = 3, d = 1, r = 0$	71
3.16	Phase plane (x, y) plot (limit cycle) for the Lotka-Volterra with fractional-order $q = 0.9$ and parameter $a = 2, b = 1, c = 3, d = 1, r = 0$	72

3.17	Chaotic attractor of Volta's system (3.33) projected into 3D state space for initial conditions $(x(0), y(0), z(0)) = (8, 2, 1)$ and $T_{sim} = 20$ sec	73
3.18	Chaotic attractor of Volta's system (3.35) projected into 3D state space for initial conditions $(x(0), y(0), z(0)) = (8, 2, 1)$, parameters $(a, b, c) = (19, 11, 0.73)$, orders $(q_1, q_2, q_3) \equiv (q = 0.98)$ and $T_{sim} = 20$ sec	74
3.19	Chaotic attractor of Volta's system (3.35) projected into 3D state space for initial conditions $(x(0), y(0), z(0)) = (8, 2, 1)$, parameters $(a, b, c) = (5, 85, 0.5)$, orders $(q_1, q_2, q_3) \equiv \bar{q} \in (0.89, 1.10, 0.91)$ and $T_{sim} = 20$ sec	75
4.1	Hardware block realizing the acquisition and the memorization of the input data samples	85
4.2	State-Machine managing the acquisition and the memorization of the input signal samples	86
4.3	The area CCB1 contains the State-Machine, the memory, and some blocks used to realize the computation of the coefficients in (4.33)	87
4.4	Top hardware level that realizes the computation of the Fractional Differintegral Operator	87
4.5	Input and output data of a Fractional Integral Operator of order 0.7	88
4.6	Input and output data of a Fractional Integral Operator of order 0.3	88
4.7	Bode diagrams of a Fractional Integral Operator of order 0.7	89
4.8	Bode diagrams of a Fractional Integral Operator of order 0.3	89
5.1	Block diagram of the canonical representation of IIR filter form	94
5.2	Feed-back control loop with prefilter	96
5.3	Block diagram of digital FOC based on PIC	98
5.4	Time response to sin function	100
5.5	Time response to unit-step function	100
5.6	Unit-step response of controlled object	101
5.7	General SISO feedback loop system	102
5.8	Block diagram of experimental setup	104
5.9	Simulated unit step response (x : Time [sec]; y : Amplitude [V])	105
5.10	Measured unit step response (x : Time [sec]; y : Amplitude [V])	106
5.11	Unit step responses (measured and model)	107

5.12	Unit step responses (measured)	108
5.13	Experimental setup in laboratory	111
5.14	Bode plots (simulated)	112
5.15	Unit step responses (simulated)	112
5.16	Unit step responses with prefilter (measured)	113
6.1	Block scheme of the device AN221E04	117
6.2	Scheme of the pole-zero CAM bilinear filter	119
6.3	FPAA schema of $PI^\lambda D^\mu$ controller	119
6.4	Bode diagram of 0.7 integrator	120
6.5	Interface for changing on-line the parameters of $PI^\lambda D^\mu$	121
6.6	Proportional effect of the $PI^\lambda D^\mu$ controller	122
6.7	Integrative effect of the $PI^\lambda D^\mu$ controller with $\lambda = 0.7$	122
6.8	Derivative effect of the $PI^\lambda D^\mu$ controller with $\mu = 0.2$	123
6.9	Response of the $PI^\lambda D^\mu$, with $K_p = 0.5, K_i = K_d = 0.2, \lambda = 0.7, \mu = 0.2$ to a sinusoidal input with $V_{pp} = 1.8$ and a frequency of $2Hz$	123
6.10	Integrative effect of the $PI^\lambda D^\mu$ controller with $\lambda = 0.3$ at 20Hz	125
6.11	Derivative effect of the $PI^\lambda D^\mu$ controller with $\mu = 0.4$ at 20Hz	125
6.12	Derivative effect of the $PI^\lambda D^\mu$ controller with $\mu = 0.3$ at 100Hz	126
6.13	Integrative effect of the $PI^\lambda D^\mu$ controller with $\lambda = 0.3$ at 5KHz	126
7.1	A switched capacitor configuration simulates the function of a resistor	129
7.2	Mapping resulting from backward Euler transformation of the derivative	133
7.3	Mapping resulting from forward Euler transformation of derivatives	134
7.4	(a) Resistance, (b) Euler behind configuration	136
7.5	Circuitual implementation of Oustaloup interpolation of a fractional order integrator	137
7.6	Switched capacitors implementation of a fractional order integrator	140
7.7	R_{on} definition	141
7.8	Bode diagrams of fractional integrator of order 0.5	142
7.9	Waveforms of an integrator of order 0.5 at $100Hz$	142
7.10	Waveforms of an integrator of order 0.5 at $5Hz$	143
7.11	Bode diagrams of fractional integrator of order 0.2	143
7.12	Waveforms of an integrator of order 0.2 at $100Hz$	144