Digital Signal Processing

Theory, Design, and Implementation

ABRAHAM PELED
BEDE LIU

DIGITAL SIGNAL PROCESSING

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ABRAHAM PELED
IBM Corporation

BEDE LIU
Princeton University

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Abraham Peled
Bede Liu

Preface

Res sunt futurae digitales

The emergence of ***digital signal processing*** as a major discipline began in the mid 1960s when high speed digital computers became widely available for serious research and development work. Many concepts that form the theoretical basis of digital signal processing, such as the Z-transform and the Fourier analysis, had been familiar, however, to engineers for a long time. In the ensuing years, this field has matured considerably. Its development is intimately tied with advances in the computer field.

The past decade has been marked with phenomenal progress in computer technology. With each stride forward, computers became more accessible and more affordable to an everincreasing user community, and the users discovered more new applications, generating new demands for even more sophisticated technology. These developments have had a profound impact on almost all scientific disciplines, and the field of digital signal processing benefitted greatly from these developments.

In digital signal processing, we deal with signals and systems that are the discrete-time counterpart of the more familiar continuous-time systems. The field may be subdivided into two interrelated areas: digital filtering and spectral analysis. Digital filters can perform the same function that analog filters do, while the analog approach, in some cases, may be difficult or unfeasible to implement practically. The use of digital filters offers important engineering advantages, such as perfect reproducibility and a guaranteed level of performance, the increased ease in changing the filter characteristics, and the possibility of time sharing the same hardware system among a multiplicity of filtering functions. These advantages alone would, in many cases, make digital filtering an attractive alternative to analog processing. There is a further important advantage: the possible modularized hardware for customized large scale integration. Digital spectral analysis has been given a tremendous boost by the introduction of the fast Fourier transform (FFT). These computationally efficient algorithms have gained widespread use in many diverse scientific disciplines, making possible accuracies and resolution that could not even be contemplated before with an analog approach.

The proliferation of the use of digital signal processing can be witnessed by its appearance in a variety of areas of scientific endeavor such as biomedical engineering, seismic and geophysical research, image processing and pattern recognition, radar and sonar detection and countermeasures, acoustics and speech research, and telecommunications. In many of these applications, there is a real need for the signal processor to operate at sufficiently high speed as to permit real time processing. At the same time, these processors must be economically competitive to be within the reach of a large user community. The field is expanding rapidly, creating a need for graduating engineers with some exposure and skill in the theory, design, and implementation of digital signal processors. There are a growing number of schools offering a course on digital signal processing; however, a suitable textbook, with emphasis on practical design and implementations, is still lacking. In addition, since the area of digital signal processing is relatively new, many of the practicing engineers today may find themselves thrust into a job that requires considerable knowledge on digital signal processing, but have only a limited time to acquire it. The book by Gold and Rader [1], published in 1969, served these needs for some time. However, there have been many new developments since its publication. The more recent books are directed primarily at the graduate level [2—5].

In this textbook, we have attempted to present a balanced blend of theory and hardware implementation techniques which, in our opinion, constitute the essential body of knowledge in digital signal processing. The material included will enable the reader to enter this important field, and to follow the published literature and the new developments in this area. We have directed this book at both the undergraduate engineering students and the practicing engineers. It can serve as a textbook for a one semester senior course on digital signal processing with a practical bent. The inclusion of the two actual projects in Appendixes 5.1 and 5.2 should help students, who are engaged in similar projects, gain valuable *hands on* experience in this area. The practicing engineer will find in this text the basic theory that he needs for a better understanding of this topic, as well as a large number of specific references to a more detailed treatment of the various subjects. Furthermore, the computer programs included and the detailed hardware implementation discussions should prove to be useful and directly applicable to some of his current problems at work.

The six chapters of the book are organized with minimum interdependence, so that each can be used separately as a reference by persons working in the field for the various topics that these chapters discuss. Chapter 1 contains the essential theoretical background needed for the understanding of the main aspects of digital signal processing, and their relation to the more familiar analog signal processing. Chapter 2 presents the main design methods of digital filters, including some computer programs for their design. In addition to the design of *standard* filters, we also consider the design of digital filters for interpolation and decimation, a process with no counterpart in analog signal processing. Through a number of examples, we demonstrate the effect that finite word length of the filter coefficients has on the filter characteristics, and alert the reader to this important aspect of digital filtering. Chapter 3 is devoted to the fast Fourier transform and its application to power spectra measurement and to performing linear convolution. Chapter 4 deals with the problems of the hardware implementation of general purpose digital signal processors, which are essentially specialized computers. Chapter 5, on the other hand, deals with the hardware implementation of dedicated hardware special purpose digital signal processors which, although less flexible, offer a high degree of cost effectiveness. Finally, in Chapter 6, we discuss some additional implementation considerations, arising from the use of finite word length, such as scaling and limit cycles, which have both theoretical and practical importance. These should be understood and appreciated in order to design a successful processor.

In summary, this book presents, in a concise yet reasonably complete and directly usable form, the main body of knowledge in the area of digital signal processing. It is mainly aimed at the undergraduate engineering student to ease his entrance into this expanding field, and at the practicing engineer to facilitate his ever more difficult job of staying abreast his field of endeavor.

Abraham Peled
Bede Liu

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Contents

CHAPTER 1

An Introduction to Digital Signal Processing

1.1 INTRODUCTION

In this chapter we introduce the reader to the basic theory of digital signal processing, and we also review briefly those topics in signal and system theory that are pertinent to the study of this field. The material included in this chapter is intended to provide the necessary background essential for an understanding of the digital processing of signals that are derived from analog waveforms. The signals and systems treated in this chapter are entirely deterministic in nature. There is a parallel body of knowledge concerning stochastic signals. Readers who are familiar with them will have no difficulty to expand in that direction.

In Section 1.2 we review briefly some aspects of continuous time linear system theory and proceed to define a discrete time system counterpart. In Section 1.3 we introduce the Z-transform, which plays a role similar to that of the Laplace transform in continuous systems. Section 1.4 contains an introduction to digital filtering. This is followed by Section 1.5, where the digital filtering of analog signals is discussed. In Section 1.6 we consider the problems of performing the digital filtering with limited accuracy and alert the reader to some of the problems arising there. Section 1.7 is devoted to the discrete Fourier transform, its properties, and its relationship to the familiar Fourier transform.

The theoretical treatment in this chapter is, by necessity, brief. Although it will provide the reader with a sufficient background for the understanding of the rest of the book and the central area of digital signal processing, additional reading on some of the topics treated in this chapter is advised, should a deeper understanding be desired.

1.2 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS AND SYSTEMS

This section begins with a review of the elementary properties of analog and digital signals, and the response of linear systems to these signals. *Analog signals* are those operating in the continuous-time domain, while *digital signals* operate in discrete-time. The term *digital* also implies that these signals have values limited to discrete levels. The conversion of analog signals to digital signals is taken up in a later section. Only deterministic or nonrandom signals will be discussed in this section.

A continuous-time or analog signal may be described by a function of time, say $f(t)$. Under quite general conditions that are almost always met in engineering practice, we may take the Fourier transform of $f(t)$. That is, the integral

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt \tag{1-1}$$

can be evaluated for all, or almost all, real values of ω, thus defining a function of ω. Often $F(\omega)$ is referred to as the *spectrum* of the signal $f(t)$; or more precisely, $|F(\omega)|^2$ is called the *power spectrum* of the signal. There is an inverse relationship to Eq. 1-1,

$$f(t) = (1/2\pi) \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\, d\omega \tag{1-2}$$

The two functions $f(t)$ and $F(\omega)$ form a Fourier transform pair. The analytic evaluation of the integrals is facilitated sometimes by regarding ω or t as a complex variable and using techniques of contour integration. The subject of numerical computation of $F(\omega)$ from a given $f(t)$ is discussed in Section 1.7.

Consider next the response of a linear time-invariant system to a signal as depicted in the block diagram of Figure 1.1. The output $g(t)$ and the input $f(t)$ are related by a superposition integral of the form

$$g(t) = \int_{-\infty}^{\infty} h(\tau)f(t-\tau)\, d\tau \tag{1-3}$$

where $h(t)$ is the unit impulse response, which is the response of the system at time t due to a unit impulse input at time 0. With a simple change of variable $\tau'=t-\tau$, Eq. 1-3 can also be written as

$$g(t) = \int_{-\infty}^{\infty} f(\tau')h(t - \tau')\, d\tau' \qquad (1\text{-}4)$$

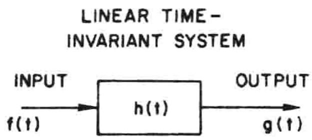

Figure 1.1 Continuous-time signal and system.

The right hand side of Eq. 1-3 or Eq. 1-4 is commonly called the convolution of the two functions f(t) and h(t), often denoted by f(t)*h(t). By a well-known theorem in Fourier integrals, we have

$$G(\omega) = H(\omega)\, F(\omega) \qquad (1\text{-}5)$$

where $G(\omega)$, $H(\omega)$, and $F(\omega)$ are respectively the Fourier transform of g(t), h(t), and f(t). $H(\omega)$ is called the transfer function of the system.

It can be shown that for an input signal of the form $f(t) = A\, e^{j\omega_1 t}$ the output is simply

$$g(t) = A\, H(\omega_1)\, e^{j\omega_1 t} \qquad (1\text{-}6)$$

Since $e^{j\omega_1 t}$ corresponds to a sinusoidal signal of frequency ω_1, we see that the response of a linear time-invariant system to a sinusoidal input is a sinusoidal signal of the same frequency. The amplitude of the output sinusoid is $A\,|H(\omega_1)|$, which is equal to the input amplitude multiplied by the magnitude of the complex number $H(\omega_1)$, and the phase by which the output lags the input is simply the argument *Arg* $H(\omega_1)$. $H(\omega)$ is seen to characterize completely the response of the system to pure sinusoidal signals, and is, therefore, called the *frequency response* of the system.

We now turn our attention to discrete-time signals. A *discrete-time signal* is a sequence of numbers, $\{x_n\}$, where the index n may vary over a finite or an infinite range. When it is desired to display explicitly the range of n, say $N \le n \le M$, we shall use the notation $\{x_n\}_{N,M}$ or $\{x_n\}_{n=N,M}$. Although the most commonly encountered discrete-time signals are the samples of analog signals at uniform intervals, these are by no means the only types of discrete-time signals. The arrival times of automobiles at a toll booth, a record stored on a memory device in a computer system, and the attendance at successive home games of the New York Mets during a season are all examples of discrete-time signals. Other common terminologies used for these signals are sampled data signals and digital signals. However, strictly speaking, the term *digital* carries with it the implication that each sample value x_n is also digitized or quantized to a discrete set. In this book, we shall use *discrete-time* and *digital* interchangeably.

A number of commonly encountered elementary signals are illustrated in Figure 1.2. The first signal consists of a single unit sample at n=0. That is,

$$x_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad (1\text{-}7)$$

This particular signal is called a unit sample signal or, more popularly, a unit impulse signal, even though there is no impulse in the signal. The second signal is a unit step,

$$x_n = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad (1\text{-}8)$$

The last one is a sinusoidal signal

$$x_n = \sin(an+b) \qquad -\infty < n < \infty \qquad (1\text{-}9)$$

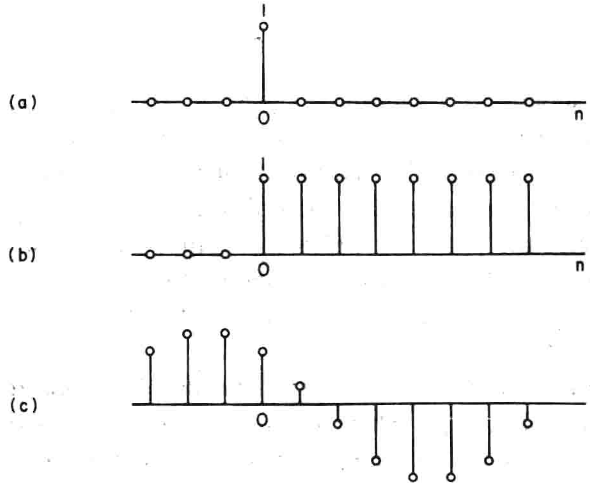

Figure 1.2 Three commonly encountered digital signals: (a) unit impulse, (b) unit step, (c) sinusoidal signal.

A discrete-time system operates on an input digital signal to produce an output digital signal. A linear time-invariant discrete-time system can be described by the input-output relationship

$$y_n = \sum_{m=-\infty}^{\infty} x_m h_{n-m} \tag{1-10}$$

where $\{x_n\}$ and $\{y_n\}$ are, respectively, the input and output signals, and $\{h_n\}$ is the impulse response of the system. That is, h_n is the response of the system at n, due to a unit sample input at 0. By letting $n-m=k$, so $m=n-k$, Eq. 1-10 becomes

$$y_n = \sum_{k=-\infty}^{\infty} x_{n-k} h_k \tag{1-11}$$

This is depicted in Figure 1.3. The right hand side of Eqs. 1-10 and 1-11 is called the convolution sum of the two sequences $\{x_n\}$ and $\{h_n\}$. When the sequence $\{h_n\}$ has only a finite number of nonzero terms, we say the system has a *finite impulse response* (FIR). Otherwise, the system is said to possess an *infinite impulse response* (IIR). If $h_n = 0$ for $n \leq 0$, we say the system is *causal* or *physically realizable*.

A large class of linear time-invariant discrete-time systems can also be described by the linear constant coefficient difference equation

$$y_n = \sum_{k=0}^{M} a_k x_{n-k} - \sum_{k=1}^{L} b_k y_{n-k} \tag{1-12}$$

where $\{x_n\}$ is the input, $\{y_n\}$ is the output, and $a_0, a_1, \ldots, a_M, b_1, \ldots, b_L$ are constants that determine the characteristics of the system. It is possible to convert Eq. 1-12 to an equation of the form of Eq. 1-10. However, we shall defer the discussion of this topic for the time being.

The reader is undoubtedly familiar with the important role that Fourier and Laplace transforms play in continuous-time signals and systems. In an analogous

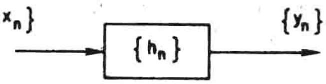

Figure 1.3 A linear time invariant discrete-time system.

manner, the study of discrete-time signals and systems is facilitated by the Z-transform. This is the subject of the next section.

1.3 THE Z-TRANSFORM

(a) *Definition and Some Examples*

Given a sequence $\{x_n\}$, its Z-transform is defined by

$$X(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n} \tag{1-13}$$

where z is a complex variable and plays a role similar to that of the variable s in the Laplace transform. The series on the right side of Eq. 1-13 converges, when z takes on values in a certain region in the complex plane. For many commonly encountered signals the series can be summed in close form. $\{x_n\}$ is called the inverse Z-transform of X(z).

Let us illustrate this with a few examples.

Example 1

$$x_n = \begin{cases} c^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1-14}$$

According to Eq. 1-13

$$X(z) = \sum_{n=0}^{\infty} c^n z^{-n} = \sum_{n=0}^{\infty} (cz^{-1})^n$$

which is a geometric series. It converges if $|cz^{-1}| < 1$, or

$$|z| > |c| \tag{1-15}$$

If Eq. 1-15 is satisfied, then the convergent series has a closed form expression

$$X(z) = \frac{1}{1 - cz^{-1}}, \quad |z| > |c| \tag{1-16}$$

A special case of interest is when c = 1. Then $\{x_n\}$ is simply the unit step, Figure 1.2*b*, and $X(z) = 1/(1-z^{-1})$ for $|z| > 1$.

Example 2

$$x_n = c^{|n|} \qquad -\infty < n < \infty \tag{1-17}$$

with $|c| < 1$. We break the infinite summation of Eq. 1-13 as follows

$$X(z) = \sum_{n=-\infty}^{-1} c^{|n|}z^{-n} + \sum_{n=0}^{\infty} c^{|n|}z^{-n}$$

$$= \sum_{n=-\infty}^{-1} c^{-n}z^{-n} + \sum_{n=0}^{\infty} c^{n}z^{-n}$$

$$= \sum_{n=1}^{\infty} c^{n}z^{n} + \sum_{n=0}^{\infty} c^{n}z^{-n} \quad (1\text{-}18)$$

The first series converges if $|cz| < 1$ or $|z| < 1/|c|$, and the second series converges if $|z| > |c|$, according to the previous example. Since $|c| < 1$ as given, $|c| < 1/|c|$. Thus, if z lies in the ring $|c| < |z| < 1/|c|$, both infinite series in Eq. 1-18 will converge, and we have

$$X(z) = \frac{cz}{1 - cz} + \frac{1}{1 - cz^{-1}}$$

$$= \frac{1 - c^2}{(1 - cz)(1 - cz^{-1})}, \quad |c| < |z| < 1/|c| \quad (1\text{-}19)$$

Example 3

$$x_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad (1\text{-}20)$$

This is the unit impulse sequence, Figure 1.2*a*. We have in this case,

$$X(z) = 1 \quad (1\text{-}21)$$

Example 4

$$x_n = \begin{cases} 1 & n = N \\ 0 & n \neq N \end{cases} \quad (1\text{-}22)$$

This is a shifted unit impulse sequence. Again, straightforwardly,

$$X(z) = z^{-N} \quad (1\text{-}23)$$