PROCEEDINGS

SPIE—The International Society for Optical Engineering

Applications of Digital Image Processing XI

Andrew G. Tescher
Chair/Editor

15–17 August 1988
San Diego, California

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APPLICATIONS OF DIGITAL IMAGE PROCESSING XI

Volume 974

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Conference 974, *Applications of Digital Image Processing XI,* was part of a four-conference program on Information Processing held at SPIE's 32nd Annual International Technical Symposium on Optical & Optoelectronic Applied Science & Engineering, 14–19 August 1988, San Diego, California. The other conferences were

Conference 975, *Advanced Algorithms and Architectures for Signal Processing III*
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INTRODUCTION

The 1988 symposium on Applications of Digital Image Processing XI brought together a large international group of engineers and scientists. Following the trend of previous SPIE conferences on the image processing field, this symposium attracted several exciting papers from Europe and other foreign countries, in addition to those from the United States.

The quality of presentations was high and topics included a large number of diverse applications. In particular, the topics covered theoretical approaches, specialized architectures, image coding, and medical and various industrial applications.

The reader will find that the published papers are well representative of the exciting field of digital image processing and they are also indicative of the significant continuing progress in this field. The more recent advances in applications areas, such as in hardware implementations, are particularly noteworthy. An increasing number of papers covered the exciting and growing field of PC-based image processing.

It is a pleasure to acknowledge the technical as well as administrative contributions of the cochairs.

Andrew G. Tescher
Lockheed Palo Alto Research Laboratories

APPLICATIONS OF DIGITAL IMAGE PROCESSING XI

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CONTENTS

(continued)

SESSION 1

Enhancement and Restoration

Chair

Andrew G. Tescher

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Results concerning image restoration applications of the mutual information principle

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ABSTRACT

Results from the mutual information principle, or MIP, approach to image restoration are presented. The MIP approach to image restoration, based on minimizing the mutual information of an image degradation system model subject to known prior statistical knowledge constraints, is briefly reviewed. An implementation of the MIP image restoration equations is then discussed. The results from a number of examples show the MIP technique to be a robust image restoration approach with apparently none of the convergence difficulties associated with similar techniques.

1. INTRODUCTION

Image restoration is a popular problem arising in digital image processing. Here, an estimate of an original image is desired after observing a distorted and noisy version of it. The general model of an image degradation system is shown in Figure 1[1]. The original image, w(i), is distorted by a blurring or distorting function with impulse response h(i) to yield the intermediate result, g(i), the convolution of w(i) and h(i). Random noise, n(i), is then added to g(i) to give the output or observed image, q(i). Mathematically, the equation governing this system is given by

$$q(i) = g(i) + n(i) = w(i) * h(i) + n(i) \tag{1}$$

where * denotes the convolution operator. In these terms, the image restoration problem becomes, given the impulse response of the blurring function, h(i), noise of a known power, n(i), and the observed image, q(i), what is the best estimate of the original image, w(i)?

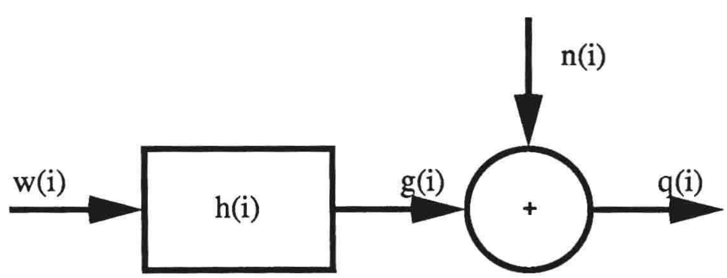

Figure 1. Model of an image degradation system

While classic approaches to the restoration problem have included filtering and Fourier transform methods, many modern approaches are based on some variation of the maximum entropy method. The maximum entropy solution to image restoration was first offered by Frieden. He proposed that the original image, w(i), be chosen such that the entropy of w(i) be maximized, subject to the system constraints[2]. Maximum entropy originally had been used by Jaynes to estimate prior probabilities[3] and then by Burg to estimate power spectra[4]. Since Frieden, maximum entropy techniques in image processing have evolved greatly[5,6].

Another function defined in information theory is the mutual information function. The mutual information function is not as widely known as the entropy function. However, the authors have proposed the concept of minimizing the mutual information function, called the mutual information principle, or MIP, for probability and stochastic systems modelling[7,8] and, recently, have completed work presenting the MIP as a unified approach to the general problem of restoration, with examples in image restoration, spectral estimation, and deconvolution[9]. While others have looked at the mutual information function with regards to optical systems[10,11,12], they have chosen to maximize the mutual information by varying the input over a channel, a channel capacity approach. The MIP minimizes the mutual information, a rate distortion theory approach, having its roots in the authors' previous work[7].

3. THE MIP APPROACH

In terms of the image degradation model of Figure 1, the mutual information, in discrete form, is

$$I(w,q) = \sum_i \sum_j p_{w,q}(i,j) \ln\left[\frac{p_{w,q}(i,j)}{w(i)\, q(j)}\right] \tag{2}$$

where the input image is considered the probability density function w(i), the output image is considered the probability density function q(j), and $p_{w,q}(i,j)$ is the joint probability density function relating the two. Note that treating the images as probability density functions has been used in the literature for some time[2].

The mutual information function relates the input to the output of a system. In this case, I(w,q) is the amount of information that w(i) indicates about q(j) or, since the function is symmetric, I(w,q) is the amount of information that q(j) indicates about w(i). In either case, it is a statistic concerned with total system performance. Another form of the mutual information function that shows its relation to the entropy function is

$$I(w,q) = E(w) - E(w|q) = E(q) - E(q|w) \tag{3}$$

where E( ) is the entropy operator. The mutual information, therefore, can be interpreted as the source or sink entropy less the information gained when w(i) is sent, given q(j) is received, or the information gained when q(j) is received, given w(i) is sent. Quantitatively, a large value for the mutual information would indicate a strong relationship between w(i) and q(j) and a mutual information value of zero would indicate that w(i) and q(j) are statistically independent.

The MIP approach to image restoration chooses the estimate of w(i) such that the mutual information of the system is minimized and the mean square error of the system associated with the estimate of w(i) is equal to the random noise known power. In this context, minimizing the mutual information may be interpreted as choosing the system which destroys the most information consistent with the constraint. It is a conservative model in that it assumes that the system has degraded the input image as much as possible.

The optimization is performed using the classic approach of the calculus of variations. Recalling this technique, as well as the laws of probability, the equation to minimize becomes

$$B = \sum_i \sum_j p_{w|q}(i|j)\, q(j) \ln\left[\frac{p_{w|q}(i|j)}{\sum_k p_{w|q}(i|k)\, q(k)}\right] + \lambda\left(q(j) - \left(\sum_m p_{w|q}(j|m)\, q(m)\right) * h(j)\right)^2 - \ln\left[\frac{f}{q(j)}\right] p_{w|q}(i|j)\, q(j) \tag{4}$$

where λ is a Lagrangian multiplier that ensures that the mean square error will be the proper noise power value, f is a normalization constant, and * is the convolution operator. The minimization proceeds by taking the derivative of equation (4) with respect to $p_{w|q}(i|j)$, setting the result equal to zero, and solving for $p_{w|q}(i|j)$. This results in the MIP estimate for $p_{w|q}(i|j)$

$$\hat{p}_{w|q}(i|j) = \frac{f\, w(i)}{q(j)} \exp\left\{ \lambda (q(i) - g(i)) * h(-i) \right\} \tag{5}$$

Equation (5) can be recognized to be in iterative form. Inserting an iteration index, we obtain

$$\hat{p}^{n+1}_{w|q}(i|j) = \frac{f\, w^{n}(i)}{q(j)} \exp\left\{ \lambda (q(i) - g^{n}(i)) * h(-i) \right\} \tag{6}$$

Multiplying both sides of equation (6) by q(j) and summing over j, we have the MIP estimate for the original image, w(i),

$$\hat{w}^{n+1}(i) = f\, w^{n}(i) \exp\left\{ \lambda (q(i) - g^{n}(i)) * h(-i) \right\} \tag{7}$$

The maximum entropy approach of Burch, Gull, and Skilling[6] results in

$$\hat{w}^{n+1}(i) = k \exp\left\{ \lambda (q(i) - g^{n}(i)) * h(-i) \right\} \tag{8}$$

where k is a constant. Comparing equation (8) with the MIP result of equation (7), the maximum entropy expression is not multiplied by $w^n(i)$. The solution method chosen by Burch, Gull, and Skilling[6] is an iterative modified conjugate gradient method, with three gradient vectors directing the search direction. The gradient vectors, however, are multiplied by the $w^n(i)$.

The solution method chosen for the MIP approach is to substitute into the constraint equations, the result shown in equation (7), and solve the obtained equations simultaneously. Recall, the constraints were that the mean square error associated with the new estimate of w(i) be equal to the random noise known power and the new estimate of w(i) be normalized. Mathematically, these equations are

$$\sigma^2 = \frac{1}{N} \sum_i \left(q(i) - f\, w^{n}(i) \exp\left\{ \lambda (q(i) - g^{n}(i)) * h(-i) \right\} * h(i)\right)^2 \tag{9}$$

and

$$\sum_i f\, w^{n}(i) \exp\left\{ \lambda (q(i) - g^{n}(i)) * h(-i) \right\} = 1 \tag{10}$$

where σ^2 is the known noise power.

Solving equations (9) and (10) simultaneously is no easy task. The mathematics is significantly reduced, if the first two terms of the Taylor series expansion for the exponential is substituted into equations (9) and (10). This simple substitution allows the f and λ of each iteration to be to be solved for directly. For a complete discussion of the Taylor series expansion substitution, the reader is referred to the authors' earlier work[9].

4. EXAMPLES

The first example represents a one-dimensional image consisting of 128 fairly structured data points. The original image is shown in Figure 2. The blurring function for this example is a 24 point wide, flat topped function with rounded sides. The observed image, shown in Figure 3, is the convolution of the original image and the blurring function with random noise added of sufficient power to achieve a signal-to-noise ratio of 30 db. The MIP restoration is shown in Figure 4.

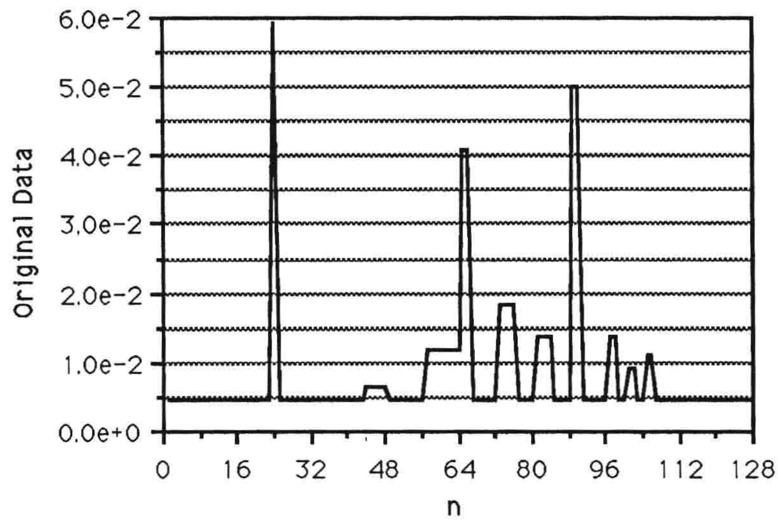

Figure 2. Original image for example one

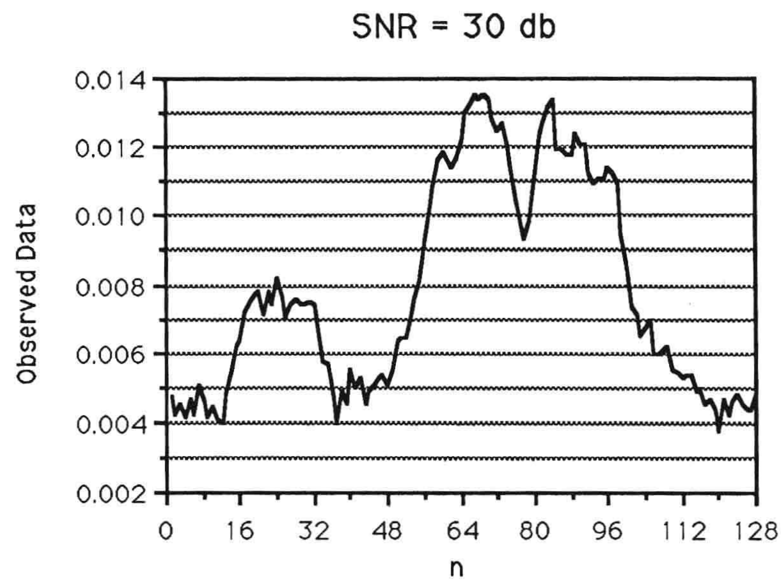

Figure 3. Observed image for example one, SNR = 30 db

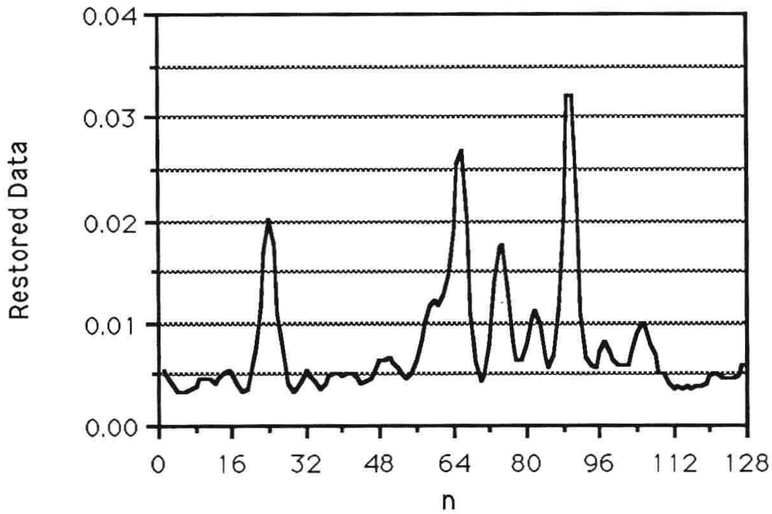

Figure 4. Restored image for example one

Example two represents a simple 16 by 16 two-dimensional image. The original image is shown in Figure 5. The blurring function is a 5 by 5 two-dimensional pulse. The original image is convolved with the blurring function and random noise of sufficient power is added for a signal-to-noise ratio of 20 db to yield the observed image shown in Figure 6. The restored image is shown in Figure 7.

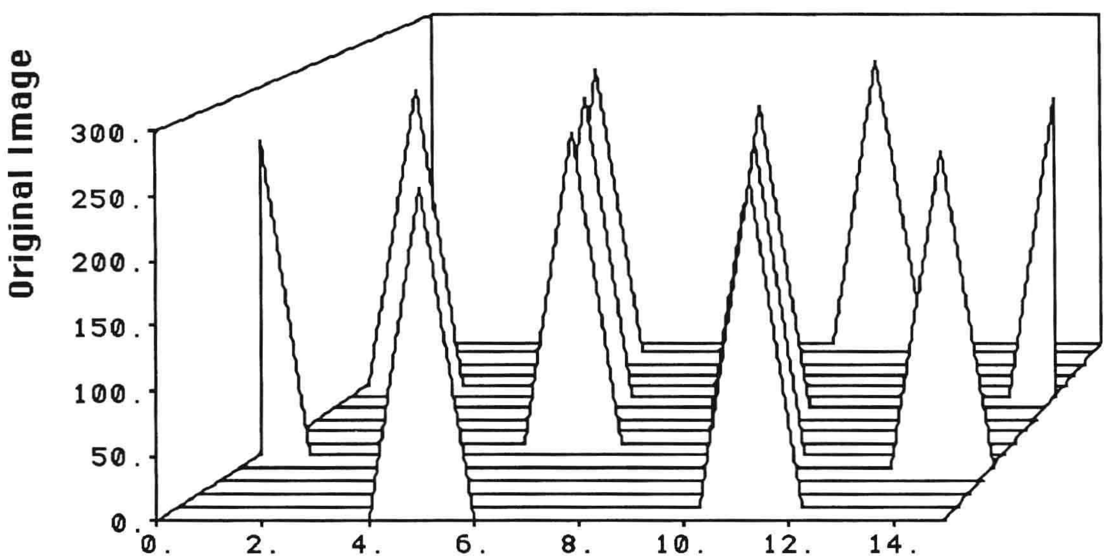

Figure 5. Original image for example two

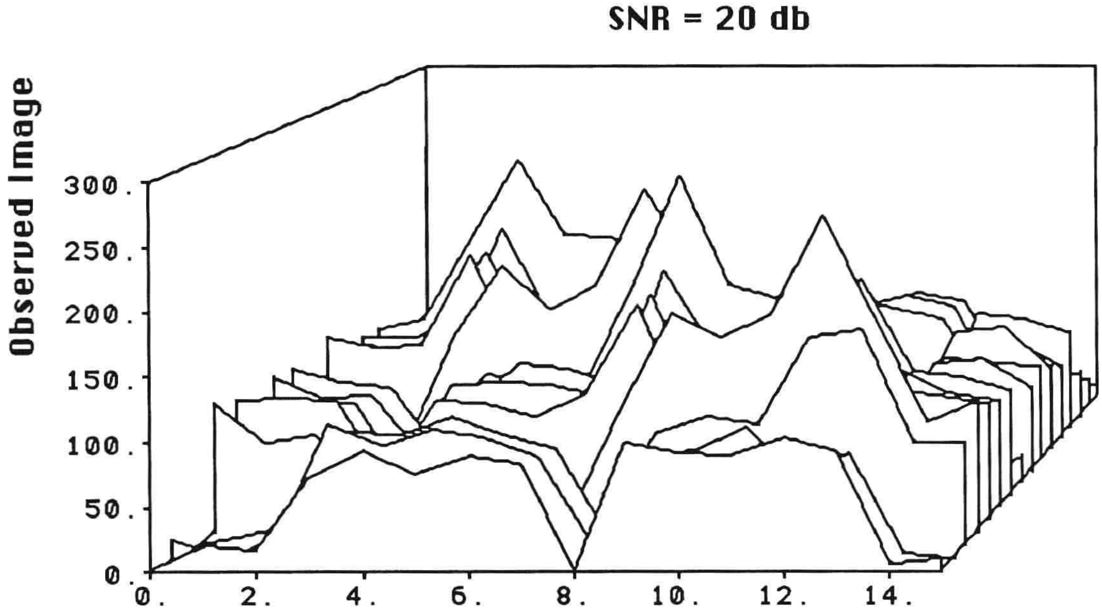

Figure 6. Observed image for example two, SNR = 20 db

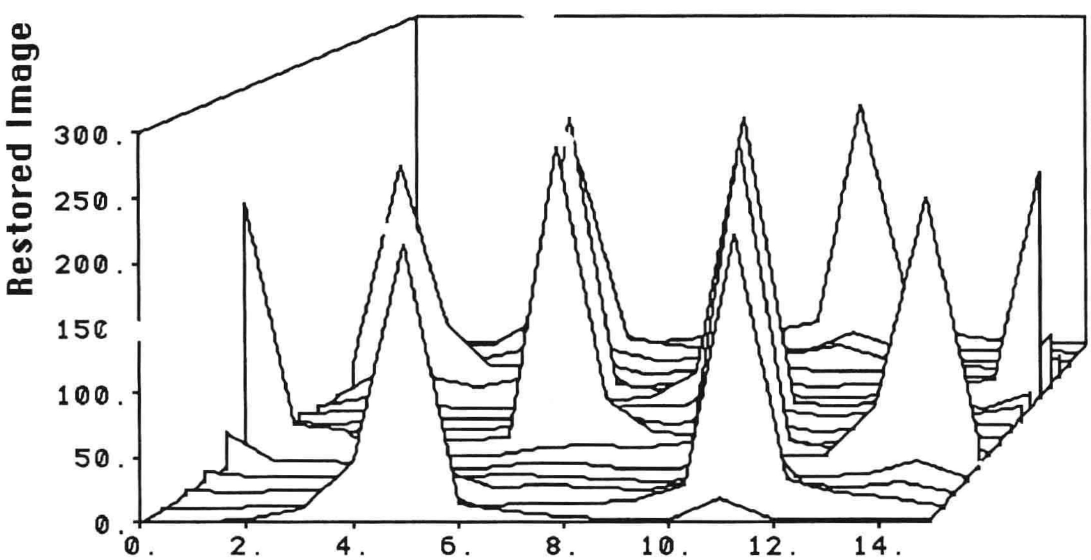

Figure 7. Restored image for example two

The third example is a 64 by 64 two-dimensional digital image. The original image is shown in Figure 8. The blurring function for this example is a 5 by 5 two-dimensional pulse. The observed image is shown in Figure 9. Here, random noise has been added of sufficient power to achieve a signal-to-noise ratio of 50 db. The restored image is shown in Figure 10.

Figure 8. Original image for example three

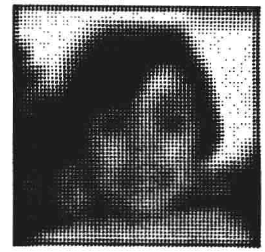

Figure 9. Observed image for example three, SNR=50 db

Figure 10. Restored image for example three

It is should be noted that the minimum mutual information algorithm does not require any averaging of successive estimates to achieve convergence. The estimate obtained for the present iteration is used directly in the next iteration. It is this apparent robust property that is perhaps one of the mutual information principle's greatest advantages.

5. CONCLUSIONS

The mutual information principle is a new and different approach to the field of digital image restoration. Having similar information theory origins as the maximum entropy technique, the MIP offers a unified and general approach to restoration for degradation system problems. From the examples above, it is clear that the mutual information principle approach to image restoration is a sound and promising technique.

6. ACKNOWLEDGEMENTS

The authors are grateful to Robert Gonsalves and the Tufts Electro-Optics Technology Center for the original data used in example one and the use of the image processing equipment to print the results of example three.

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