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COLLEGE PHYSICS

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SCHAUM'S OUTLINE SERIES IN SCIENCE

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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

of

COLLEGE PHYSICS

SIXTH EDITION

BY

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Preface

Lord Kelvin said: *"I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it."* Comprehensive tests indicate definitely that only a small percentage of the subject matter presented in the typical college physics course is ever assimilated by the majority of its students. This book aims to help the student to see clearly the principles of physics and of simple mathematics associated with the problem work of a first year course in college physics.

The previous edition of this book has been very favorably received and adopted as a supplementary problems book by more than five hundred colleges and technical schools. In this sixth edition the scope of the book has been greatly enlarged and most chapters have undergone thorough revision to keep pace with the most recent concepts, methods, and terminology. In mechanics, the English gravitational and mks systems are emphasized, but the cgs system is also used. In electricity, only rationalized mks units are employed. Dimensional units are used throughout in order to stimulate their constant application by the student, and an explanation of the use of units and of dimensions appears in the fifth chapter. Emphasis is placed on the use of the correct number of significant figures in line with scientific and engineering practice. The appendix includes sections on significant figures, trigonometry, exponents, logarithms, and conversion tables.

The author has tried to use both the physical and mathematical methods in giving explanations, using word equations and simple mathematics, instead of mere substitution in a formula. The principles and reasonings affecting the solution of problems are presented directly and in the most simple and familiar terms, and the mathematical deductions stress the significance of the problem as a whole.

The contents are divided into chapters covering duly-recognized areas of theory and study. Each chapter begins with a clear statement of the pertinent definitions, principles, and theorems. This is followed by carefully graded sets of solved and supplementary problems. These are arranged so as to present a natural development of each topic, and they include a wide range of applications in both pure and applied physics. The solved problems serve to illustrate and amplify the theory, provide the repetition of basic principles so vital to effective teaching, and bring into sharp focus those fine points without which the student continually feels himself on unsafe ground. The supplementary problems serve as a complete review of the material of each chapter. This book is in no way a condensation of ordinary text material; it is intended to be a comprehensive problem book.

The author gratefully acknowledges the cooperation and suggestions of many teachers who have been using this book and who have urged him to make such additions and modifications as would better adapt it for college physics courses. Particular thanks are extended to Professor Carel W. van der Merwe of New York University for invaluable assistance and critical review of the entire manuscript.

DANIEL SCHAUM

March, 1961

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Chapter 1

Introduction to Vectors

A SCALAR QUANTITY has only magnitude, *e.g.* time, volume of a body, mass of a body, density of a body, amount of work, amount of money.

Scalars are added by ordinary algebraic methods, *e.g.* 2 sec + 5 sec = 7 sec.

A VECTOR QUANTITY has both magnitude and direction. For example:

1) **Displacement** – an airplane flies a distance of 160 mi in a southerly direction.
2) **Velocity** – a ship sails due east at 20 mi/hr.
3) **Force** – a force of 10 lb acts on a body in a vertically upward direction.

A vector quantity is represented by an arrow drawn to scale. The length of the arrow represents the magnitude of the displacement, velocity, force, etc. The direction of the arrow represents the direction of the displacement, etc.

Vectors are added by geometric methods.

THE RESULTANT of a number of force vectors is that single vector which would have the same effect as all the original vectors together.

THE EQUILIBRANT of a number of vectors is that vector which would balance all the original vectors taken together. It is equal in magnitude but opposite in direction to the resultant.

PARALLELOGRAM METHOD OF VECTOR ADDITION. The resultant of two vectors acting at any angle may be represented by the diagonal of a parallelogram drawn with the two vectors as adjacent sides, and directed away from the origin of the two vectors.

VECTOR POLYGON METHOD OF VECTOR ADDITION. This method of finding the resultant consists in beginning at any convenient point and drawing (to scale) each vector in turn, taking them in any order of succession. The tail end of each vector is attached to the arrow end of the preceding one. The line drawn to complete the triangle or polygon is equal in magnitude to the resultant or equilibrant.

The resultant is represented by the straight line directed from the starting point to the arrow end of the last vector added.

The equilibrant is represented by the same line as the resultant but is oppositely directed, i.e. toward the starting point.

SUBTRACTION OF VECTORS. To subtract vector B from vector A, reverse the direction of vector B and add it vectorially to vector A, i.e. $A - B = A + (-B)$.

A COMPONENT OF A VECTOR is its effective value in any given direction. For example, the horizontal component of a vector is its effective value in a horizontal direction. A vector may be considered as the resultant of two or more component vectors, the vector sum of the components being the original vector. It is customary and most useful to resolve a vector into components along mutually perpendicular directions.

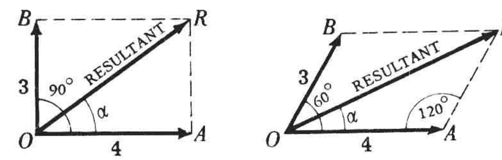
SOLVED PROBLEMS

1. Find the resultant of two forces of 4 lb and 3 lb, acting on a point O at an angle of $a)$ 90°, $b)$ 60°, with each other. Use the parallelogram method.

Graphical Solution:

In each case, choosing a suitable scale, draw the vectors OA and OB to represent 4 and 3 lb respectively, at the given angles with each other. Complete the parallelogram by drawing BR parallel to OA, and AR parallel to OB.

The diagonal OR represents in each case the magnitude and direction of the resultant of the two forces.



(a) (b)

In $a)$, OR is measured by scale and is found to represent 5 lb. Angle α is measured by protractor and is found to be 37°.

In $b)$, OR is found to represent 6.1 lb and angle α is 25°.

By Computation:

$a)$ OAR is a right triangle. Then $OR^2 = 4^2 + 3^2 = 25$ and $OR = 5$ lb.

$$\tan\alpha = \frac{AR}{OA} = \frac{3}{4} = 0.75 \quad \text{and} \quad \alpha = \tan^{-1} 0.75 \text{ (i.e., angle whose tangent is 0.75)} = 37^\circ$$

$b)$ $\angle OAR = 120^\circ$. To compute the magnitude of the resultant OR, use the law of cosines.

$$OR^2 = OA^2 + AR^2 - 2(OA)(AR)\cos 120^\circ = 4^2 + 3^2 - 2(4)(3)(-0.5) = 37 \quad \text{and} \quad OR = 6.1\text{ lb.}$$

To compute angle α, use the law of sines.

$$\frac{\sin\alpha}{AR} = \frac{\sin 120^\circ}{OR} \quad \text{or} \quad \frac{\sin\alpha}{3} = \frac{0.866}{6.1}. \quad \text{Then } \sin\alpha = 0.43 \text{ and } \alpha = \sin^{-1} 0.43 = 25^\circ.$$

Note. $\sin 120^\circ = \sin(180^\circ - 120^\circ) = \sin 60^\circ$, $\cos 120^\circ = -\cos(180^\circ - 120^\circ) = -\cos 60^\circ$.

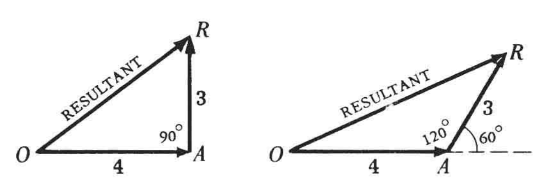
2. Solve Problem 1 by the vector polygon method.

Graphical Solution:

In each case draw OA to represent the 4 lb force. From A plot $AR = 3$ lb in the direction of the 3 lb force. Draw OR to complete the triangle.

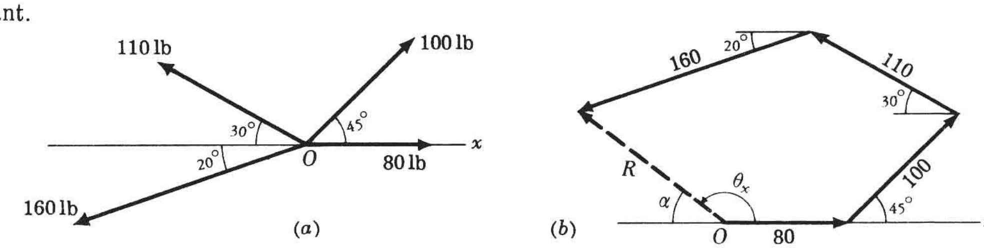
The vector OR is the resultant and is in magnitude equal to 5 lb in (a) and 6.1 lb in (b).

The computation is exactly the same as in Problem 1.



(a) (b)

3. Four coplanar forces act on a body at a point O as shown in Fig.(a). Determine graphically their resultant.



(a) (b)

Solution:

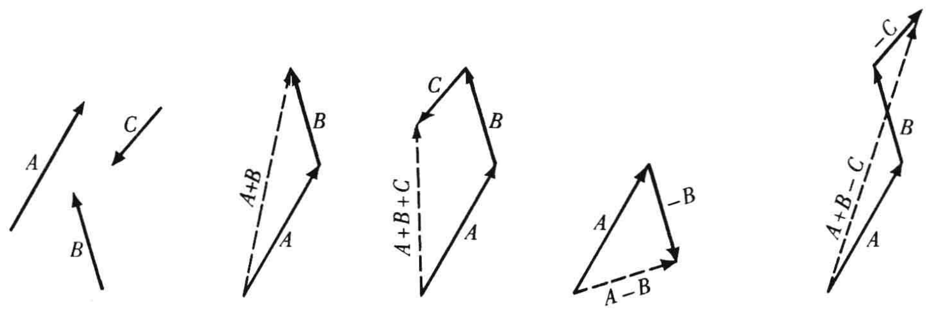
Starting from O, the four vectors are plotted in turn, placing the tail end of one vector at the arrow end of the preceding vector, as shown in Fig.(b). The directed line R, connecting the tail end of the first vector with the arrow end of the last vector, is the resultant.

R is measured to scale and is found to represent 119 lb. Angle α is measured by protractor and is found to be 37°. Hence the resultant R has a magnitude of 119 lb and is directed 37° above the negative x-axis (or at an angle of $\theta_x = 180^\circ - \alpha = 143^\circ$ with the positive direction of the x-axis).

4. Perform graphically the following vector additions and subtractions, where A, B, and C are vectors. *a*) $A+B$, *b*) $A+B+C$, *c*) $A-B$, *d*) $A+B-C$.

Solution:

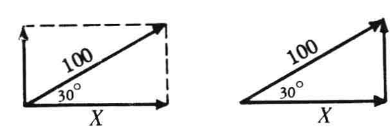
The vector additions and subtractions are shown in Figures (*a*)–(*d*) below. In (*c*), $A-B = A+(-B)$; i.e., to subtract vector B from vector A, reverse the direction of B and add it vectorially to A. Similarly in (*d*), $A+B-C = A+B+(-C)$, where $-C$ is equal in magnitude but opposite in direction to C.



(*a*) (*b*) (*c*) (*d*)

5. The resultant of two forces at right angles is 100 lb. If one of the forces makes an angle of 30° with the resultant, compute that force.

Solution:



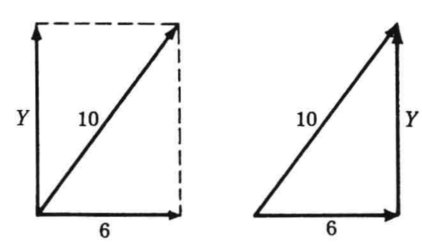
Draw a rectangle with the diagonal making an angle of 30° with the horizontal and representing 100 lb.

Or, draw a right triangle with the hypotenuse making an angle of 30° with the horizontal and representing 100 lb.

$X = 100 \times \cos 30° = 100 \times 0.866 = 86.6$ lb.

6. Two forces at right angles have a resultant of 10 lb. If one of the two forces is 6 lb, compute the other force.

Solution:



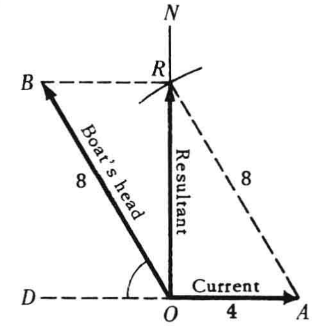
Let Y be the required force. Draw a rectangle with a side representing 6 lb and the diagonal representing 10 lb. Y and 6 lb are at right angles.

Or, draw a right triangle with the hypotenuse representing 10 lb and one arm representing 6 lb.

$Y^2 = 10^2 - 6^2 = 100 - 36 = 64 \qquad Y = 8$ lb

7. A boat travels at 8 mi/hr in still water. At what angle with the shore must the boat be steered to reach a point directly opposite if the velocity of the current is 4 mi/hr ?

Graphical Solution:



Choose a suitable scale. Draw OA to represent the current's velocity, 4 mi/hr.

Draw ON, an indefinite line perpendicular to OA, to represent the resultant direction of the boat. With A as center and 8 mi/hr as radius describe a circle cutting ON in R.

Join AR. Complete the parallelogram AB.

Then OB is the direction of the boat's head, and the required angle DOB is measured to be 60°.

By Computation: $\cos \angle DOB = \cos \angle OAR = 4/8 = 0.5$

Hence $\angle DOB = \cos^{-1} 0.5 = 60°$.

8. A ship is heading due north at 12 mi/hr but drifts westward with the tide at 5 mi/hr. Determine the magnitude and direction of the resultant velocity of the ship.

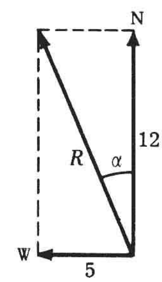
Solution:

The velocity of 12 mi/hr north is added vectorially to the velocity of 5 mi/hr west to give the resultant velocity R of the ship relative to the ground.

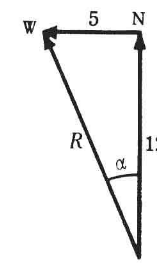
Magnitude of $R = \sqrt{5^2 + 12^2} = 13$ mi/hr

The direction of R is such that it makes an angle α whose tangent is $\tan\alpha = 5/12 = 0.42$; hence $\alpha = 23°$.

The resultant velocity is 13 mi/hr in a direction 23° West of North.



Parallelogram Method



Triangle Method

9. A motorcyclist is riding north at 50 mi/hr and the wind is blowing westward with a velocity of 30 mi/hr. Determine the apparent wind velocity as observed by the cyclist.

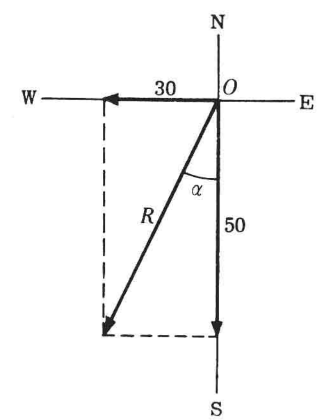
Solution:

When the cyclist is riding north at 50 mi/hr he creates a wind (relative to himself) blowing south at 50 mi/hr which is added vectorially to the west wind of 30 mi/hr to give the resultant velocity R of the air relative to the cyclist.

$R = \sqrt{30^2 + 50^2} = 58$ mi/hr

$\tan\alpha = 30/50 = 0.6$ and $\alpha = 31°$.

The wind seems to blow at 58 mi/hr in a direction 31° W of S.

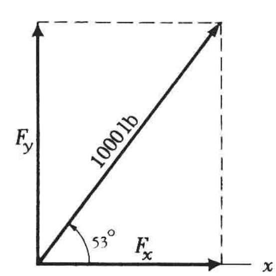


10. Resolve a force of 1000 lb at 53° with the x-axis into horizontal and vertical components.

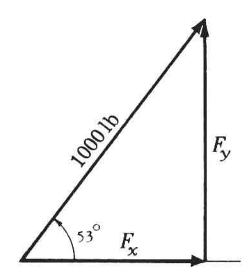
Solution:

Horizontal component $F_x = 1000 \cos 53° = 1000(0.6018) = 602$ lb

Vertical component $F_y = 1000 \sin 53° = 1000(0.7986) = 799$ lb

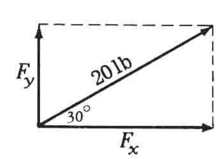


Parallelogram Method

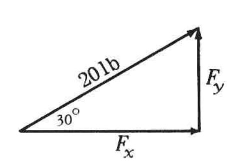


Triangle Method

11. A boy pulls a rope attached to a sled with a force of 20 lb. The rope makes an angle of 30° with the ground. Compute the effective value of the pull tending to move the sled along the ground, and the effective value tending to lift the sled vertically.



Parallelogram Method



Triangle Method

Solution:

The force tending to move the sled along the ground is the horizontal component F_x. The force tending to lift the sled off the ground is the vertical component F_y.

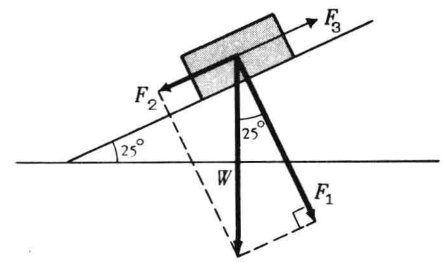
$$F_x = 20\text{ lb} \times \cos 30° = 20\text{ lb} \times 0.866 = 17.3\text{ lb}$$

$$F_y = 20\text{ lb} \times \sin 30° = 20\text{ lb} \times 0.500 = 10.0\text{ lb}$$

12. A block of weight $W = 300$ lb rests on a smooth board inclined 25° with the horizontal.

a) Find the components of W normal (perpendicular) and parallel to the plane.

b) What force F_3 parallel to the plane is required to draw the block up the plane?



Solution:

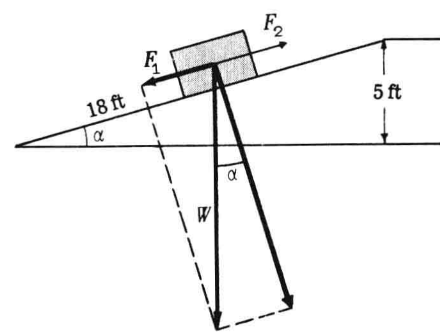
a) Resolve W into two components, F_1 and F_2, respectively normal and parallel to the plane.

F_1 = component of W normal to plane
= $W \cos 25° = 300 \text{ lb} \times 0.9063 = 272$ lb

F_2 = component of W parallel to plane
= $W \sin 25° = 300 \text{ lb} \times 0.4226 = 127$ lb down plane

b) $F_3 = -F_2 = 127$ lb up the plane

13. What least force F_2 parallel to the plane is required to draw a machine of weight $W = 900$ lb up a smooth inclined plane 18 ft long to a platform 5 ft above the ground?

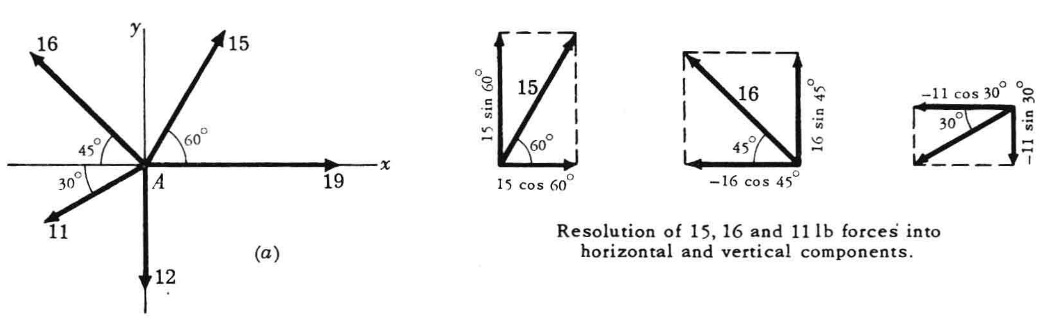


Solution:

Resolve the 900 lb weight into two components, normal and parallel to the plane.

Then $F_1 = 900 \text{ lb} \times \sin\alpha = 900 \text{ lb} \times 5/18 = 250$ lb down the plane, and the required force $F_2 = -F_1 = 250$ lb up the plane.

14. Calculate the resultant of the five coplanar forces (19, 15, 16, 11, 12 lb) acting on an object at A, as shown in Fig.(*a*).



Resolution of 15, 16 and 11 lb forces into horizontal and vertical components.

Solution:

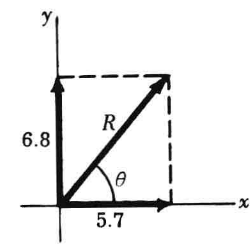
1) Each force is resolved into horizontal and vertical components. (The force of 19 lb horizontally to the right has no vertical component. The vertically downward force of 12 lb has no horizontal component.)

2) The horizontal components to the right are considered positive, and those to the left negative. The upward vertical components are taken as positive, and those downward as negative.

3) The horizontal components are added separately (algebraically), and the vertical components are added separately (algebraically). This gives two resultant components (ΣF_x and ΣF_y) at right angles. The resultant of ΣF_x and ΣF_y is R.

4) The horizontal and vertical components of the five forces are as follows.

	Force	Horizontal Component	Vertical Component
a)	19 lb	19.0	0.0
b)	15 lb	$15\cos 60^\circ = 7.5$	$15\sin 60^\circ = 13.0$
c)	16 lb	$-16\cos 45^\circ = -11.3$	$16\sin 45^\circ = 11.3$
d)	11 lb	$-11\cos 30^\circ = -9.5$	$-11\sin 30^\circ = -5.5$
e)	12 lb	0.0	−12.0
		$\Sigma F_x = +5.7$ lb	$\Sigma F_y = +6.8$ lb

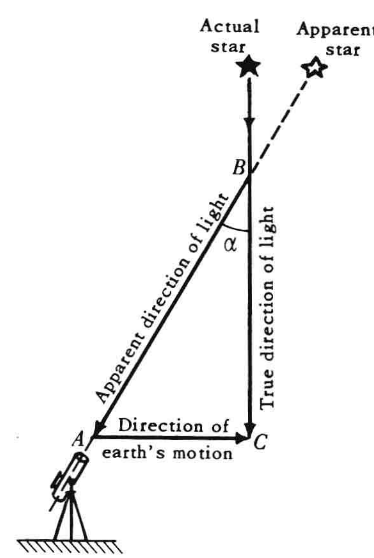


$$\text{Magnitude of } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(5.7)^2 + (6.8)^2} = 8.9 \text{ lb}$$

and $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{6.8 \text{ lb}}{5.7 \text{ lb}} = 1.2$ from which $\theta = 50^\circ$.

The resultant is 8.9 lb at an angle of 50° with the positive direction of the x-axis.

15. A telescope must be inclined 20.5 seconds of arc with the vertical in order to see a fixed star which is vertically overhead. Due to the earth's orbital motion, the telescope has a speed of 18.5 mi/sec at right angles to the direction of the star. From these facts deduce the speed of light.



Solution:

Let c = speed of light, v = speed of telescope. The inclination of the telescope must be such that the time t taken for light to travel a distance BC ($t = BC/c$) equals the time taken by the telescope to travel the distance AC ($t = AC/v$). Then $BC/c = AC/v$ or $AC/BC = \tan\alpha = v/c$ and

$$c = \frac{v}{\tan 20.5''} = \frac{18.5 \text{ mi/sec}}{9.94 \times 10^{-5}} = 1.86 \times 10^5 \text{ mi/sec}$$

This method was used by Bradley in 1728 to determine the speed of light, and $20.5''$ is called the angle of aberration.

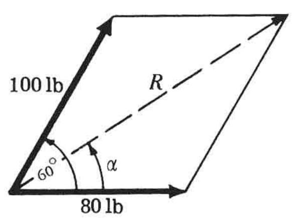
Note. The sines and tangents of very small angles are almost exactly equal numerically to their angles when measured in radians.

$$20.5'' = 20.5'' \times \frac{2\pi \text{ rad}}{360 \text{ deg}} \times \frac{1 \text{ deg}}{3600''} = 9.94 \times 10^{-5} \text{ rad}$$

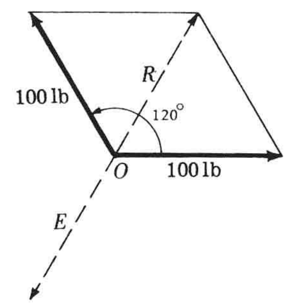
SUPPLEMENTARY PROBLEMS

16. Two forces, of 80 and 100 lb acting at an angle of 60° with each other, pull on an object. What single pull would replace the given forces?
Ans. $R = 156$ lb at $\alpha = 34^\circ$ with the 80 lb force

17. Two forces, of 100 lb each and making 120° with each other, pull on an object. Find the single pull that would *a*) replace the given force system, *b*) balance the given force system.
Ans. *a*) Resultant R = 100 lb at 60° with each given force
b) Equilibrant E = 100 lb directed opposite to the resultant



Prob. 16



Prob. 17

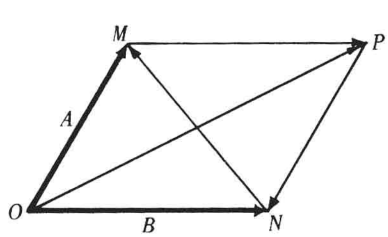
18. A man walks 50 ft east, 30 ft south, 20 ft west, and 10 ft north. Determine his distance from the starting point. *Ans.* 36 ft directed 34° south of east

19. Find the vector sum of the following four displacements: 60 ft north; 30 ft west; 40 ft, 60° W of N; 50 ft, 30° W of S. Solve graphically and by the component method.
Ans. 96.8 ft, 67.7° W of N

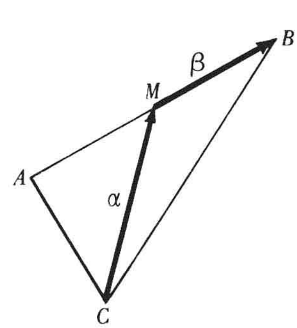
20. Given vector A and vector B, and parallelogram $OMPN$. Express the following vectors in terms of A and B: OP, MP, PN, NM. *Ans.* $OP = A + B$, $MP = B$, $PN = -A$, $NM = -B + A$

21. If CM is a median of triangle ABC, and $CM = \alpha$ and $MB = \beta$, express each of the following directed line segments in terms of vectors α and β: CB, AM, MA, AB, CA.
Ans. $CB = \alpha + \beta$, $AM = \beta$, $MA = -\beta$, $AB = 2\beta$, $CA = \alpha - \beta$

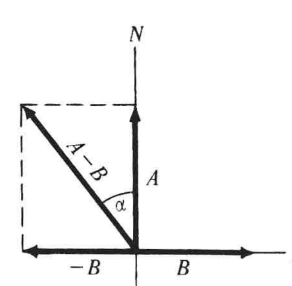
22. Given vector A = 80 ft/sec north and vector B = 60 ft/sec east. Find the vector difference $A - B$.
Ans. 100 ft/sec at $\alpha = 37^\circ$ W of N



Prob. 20

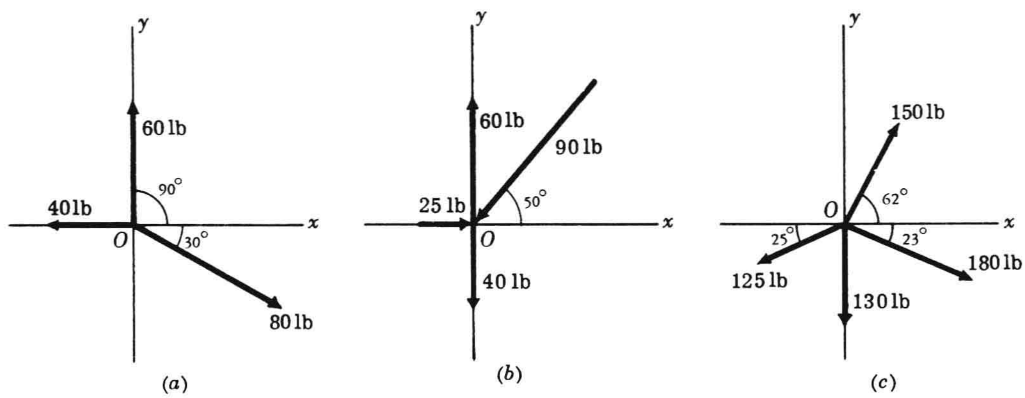


Prob. 21



Prob. 22

23. Find graphically the resultant of each of the three coplanar concurrent force systems shown in Fig. (*a*), (*b*), (*c*) below. *Ans.* (*a*) 35 lb at $\theta_x = 34^\circ$, (*b*) 59 lb at $\theta_x = 236^\circ$, (*c*) 172 lb at $\theta_x = 315^\circ$



Prob. 23

24. A ship is traveling due east at 10 mi/hr. What must be the speed of a second ship heading in a direction 30° east of north, if it is always due north from the first ship?
Ans. 20 mi/hr

25. From an electric car traveling at 15 mi/hr, a ball is thrown at right angles to the motion of the car and with a speed of 20 ft/sec. What is the speed of the ball (relative to the earth) at the beginning of its flight?
Ans. 29.7 ft/sec

26. A boat, propelled so as to travel with a speed of 500 ft/min in still water, moves directly across a river 2000 ft wide, the river flowing at 300 ft/min. How long will it take the boat to cross the river? Towards what point on the opposite shore is the boat headed when it starts?
Ans. 5 minutes, 1500 feet upstream

27. A stationary soldier sees an enemy tank 500 ft away moving at 44 ft/sec on a line perpendicular to his line of sight. (*a*) If the speed of the bullet is 1000 ft/sec, at what horizontal angle with his line of sight must he aim the gun to hit the tank? (*b*) How many feet to one side of the tank must he aim?
Ans. 2.52°, 22 ft

28. Resolve a force of 10 lb into two components at right angles, the line of action of one component making an angle of 45° with the line of action of the 10 lb force. Solve by construction and by computation.
Ans. Each component 7.07 lb

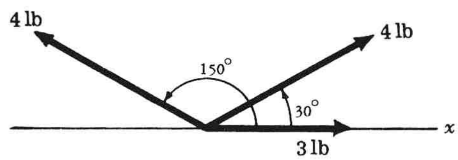
29. Resolve a force of 100 lb into two components at right angles, one component making an angle of 30° with the 100 lb force. Solve by construction and by computation.
Ans. 50 lb, 86.6 lb

30. A telegraph pole is supported by a guy wire which exerts a 250 lb pull on the top of the pole. The guy wire makes an angle of 42° with the pole. Determine the horizontal and vertical components of the pull on the pole. *Ans.* 167 lb horizontal, 186 lb vertical

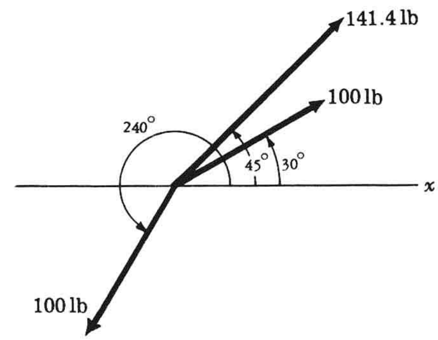
31. A horse exerts a force of 300 lb to pull a barge along a canal, using a 50 ft rope. If the barge is kept 10 ft from the canal bank, compute (*a*) the effective value of the pull tending to move the barge along the canal, and (*b*) the sideways force which must be exerted by the rudder to keep the barge 10 ft from the bank.
Ans. 294 lb, 60 lb

32. What force parallel to the plane is required to draw a 200 lb box with uniform speed up a smooth 32° incline?
Ans. 106 lb

33. Compute the resultant and equilibrant of the following system of coplanar concurrent forces: 3 lb, 0°; 4 lb, 30°; 4 lb, 150°.
Ans. Resultant = 5 lb at 53°, equilibrant = 5 lb at 233°



Prob. 33



Prob. 34

34. Compute the resultant and equilibrant of the following system of coplanar concurrent forces: 100 lb, 30°; 141.4 lb, 45°; 100 lb, 240°.
Ans. Resultant = 151 lb at 25°, equilibrant = 151 lb at 205°

35. Compute the resultant of the following system of coplanar concurrent forces: Forces of 20, 40, 25, 42 and 12 lb, making angles of 30°, 120°, 180°, 270° and 315° respectively with the positive direction of the x-axis.
Ans. 20 lb at 197°

Chapter 2

Equilibrium of a Rigid Body

PARALLEL COPLANAR FORCES

THE MOMENT OF A FORCE, or torque, about an axis is the effectiveness of the force in producing rotation about that axis. It is measured by the product of the force and the perpendicular distance from the axis of rotation to the line of action of the force.

Moment = force × perpendicular distance from axis to action line of force.

When the force is expressed in pounds and the distance in feet, the unit of moment is the pound-foot (lb-ft).

DEFINITION OF EQUILIBRIUM. A body is in **translational equilibrium** if it is at rest, or is moving at constant speed in a straight line. It is in **rotational equilibrium** if it is not rotating, or if it is rotating at constant angular speed about an axis.

A system of forces acting on a body produces equilibrium if, when acting together, the forces have no tendency to produce a change in the body's translatory (straight line) motion, or in its rotary motion.

CONDITIONS FOR EQUILIBRIUM UNDER ACTION OF PARALLEL COPLANAR FORCES.

1) The algebraic sum of the forces acting on the body in any given direction must be zero. This is equivalent to saying that the sum of the upward forces equals the sum of the downward forces, and similarly for those forces along the other directions such as left to right, etc.

   When this condition is satisfied there is no unbalanced force acting on the body, (and therefore the body will have no linear acceleration). In other words, the system of forces will not tend to produce any change in the linear motion of the body.

2) The algebraic sum of the moments of all the forces acting about any axis perpendicular to the plane of the forces must be zero. This is equivalent to saying that the sum of the clockwise moments about any such axis equals the sum of the counterclockwise moments about that axis.

   When this condition is satisfied there is no unbalanced torque or moment acting on the body, (and hence the body will have no angular acceleration). In other words, the system of moments will not tend to produce any change in the angular motion of the body. If it is initially at rest it will not start to rotate, and if it is initially rotating it will maintain the same rate of rotation.

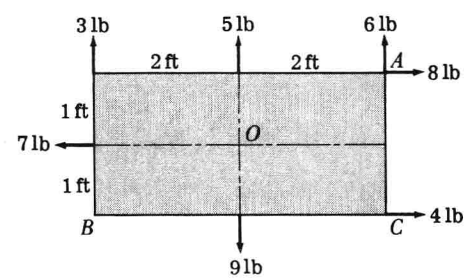
A COUPLE consists of two equal and oppositely directed parallel forces, not in the same straight line. A couple can produce only rotation.

The moment of a couple is equal to the product of one of the forces and the perpendicular distance between them. A couple can be balanced only by another couple of equal moment, in the opposite direction.

THE CENTER OF GRAVITY of a body is the point at which the entire weight of the body may be considered as concentrated, i.e. the line of action of the weight passes through the center of gravity. A single vertically upward force equal to the weight of the body and applied at the center of gravity will keep the body in equilibrium.

SOLVED PROBLEMS

1. Forces of 8, 6, 5, 3, 7, 9 and 4 lb act on a 4 ft by 2 ft rectangle as shown. Determine the algebraic sum of the moments (ΣL) of these forces about an axis *a*) through A, *b*) through B, *c*) through C, *d*) through the center O.



Solution:

Moment = force × perpendicular distance from axis to action line of force.

Consider clockwise moments as negative and counterclockwise moments as positive.

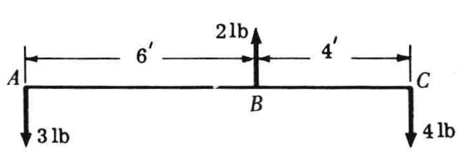
a) ΣL_A = 8 lb × 0 ft + 6 lb × 0 ft − 5 lb × 2 ft − 3 lb × 4 ft −7 lb × 1 ft + 9 lb × 2 ft + 4 lb × 2 ft
= (0 + 0 − 10 − 12 − 7 + 18 + 8) lb-ft = −3 lb-ft

b) ΣL_B = −8(2) + 6(4) + 5(2) + 3(0) + 7(1) − 9(2) + 4(0) = +7 lb-ft

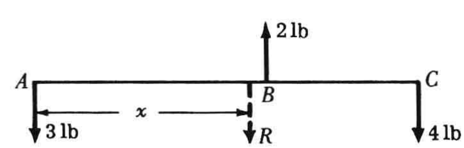
c) ΣL_C = −8(2) + 6(0) − 5(2) − 3(4) + 7(1) + 9(2) + 4(0) = −13 lb-ft

d) ΣL_O = −8(1) + 6(2) + 5(0) − 3(2) + 7(0) + 9(0) + 4(1) = +2 lb-ft

2. A horizontal weightless rod AC, 10 ft long, is acted upon by three vertical forces as shown in Fig. (*a*).
   a) Find the algebraic sum of the forces (ΣF) acting on the rod.
   b) Find the algebraic sum of the moments (ΣL) about an axis through each of the following points: A, B, C.
   c) Determine completely the resultant R and the equilibrant of the given force system.



(*a*)



(*b*)

Solution:

a) Consider upward forces positive. Then ΣF = (−3 + 2 − 4) lb = −5 lb (down).

b) Consider clockwise moments negative and counterclockwise moments positive.

ΣL_A = 3 lb × 0 ft + 2 lb × 6 ft − 4 lb × 10 ft = −28 lb-ft (clockwise)
ΣL_B = +3 lb × 6 ft + 2 lb × 0 ft − 4 lb × 4 ft = + 2 lb-ft (counterclockwise)
ΣL_C = +3 lb × 10 ft − 2 lb × 4 ft + 4 lb × 0 ft = + 22 lb-ft (counterclockwise)

c) From *a*) the value of resultant $R = \Sigma F$ = −5 lb (down). Then from Fig. (*b*),

moment of R about axis through A = sum of moments of given forces about A (ΣL_A)
−5 lb × x = −28 lb-ft, and x = 5.6 ft from A.

The resultant R of the given force system is 5 lb down, its line of action being at a perpendicular distance x = 5.6 ft from A.

The equilibrant (i.e., the force required to produce equilibrium) is 5 lb *up* at a perpendicular distance x = 5.6 ft from A.