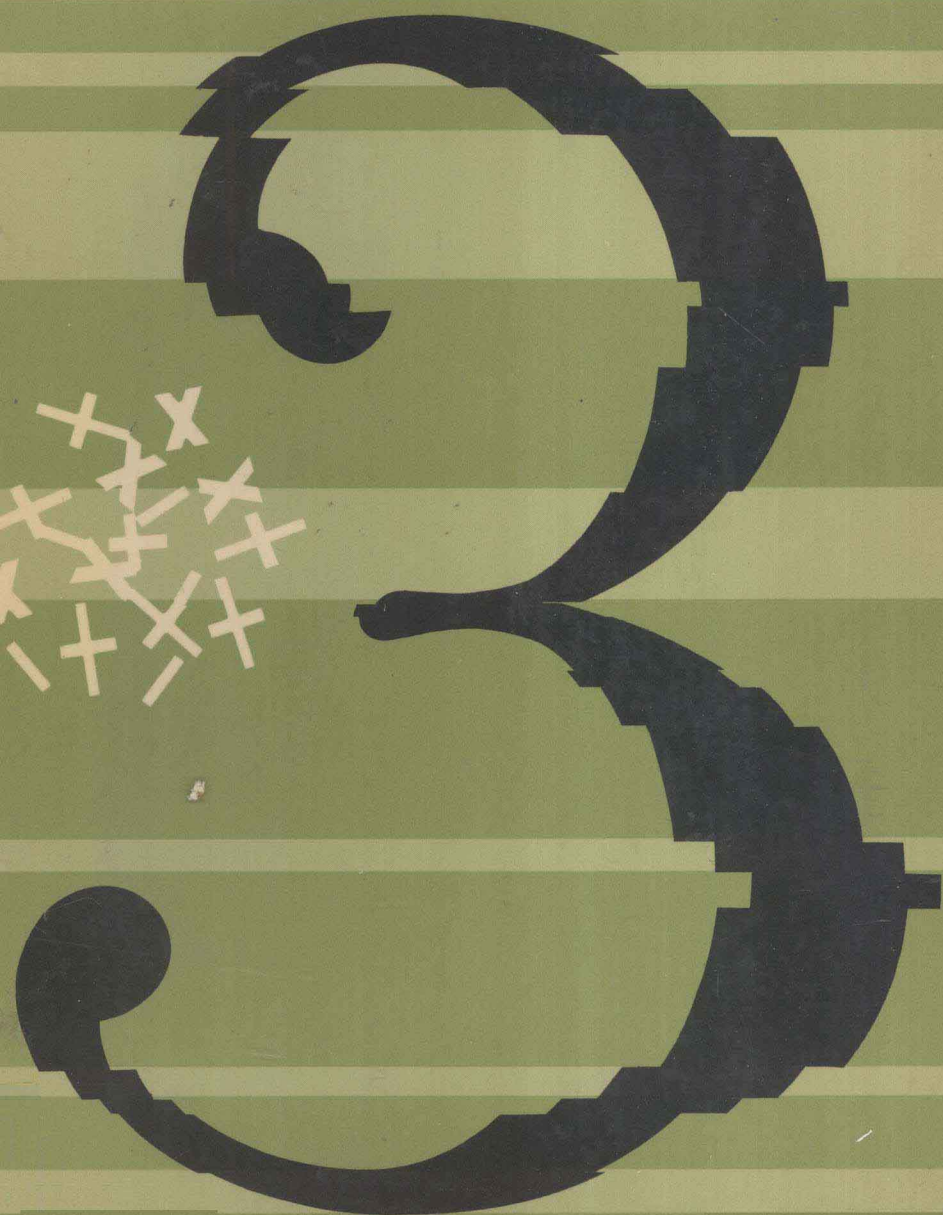


# CONCEPTS AND METHODS OF ARITHMETIC

BY MARVIN C. VOLPEL



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by MARVIN C. VOLPEL

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## PREFACE

The past few years have seen many changes in the teaching of arithmetic. Foremost among these changes is the emphasis upon understandings and meanings. The modern philosophy of the teaching of arithmetic stresses the "why" of the operation as much as the "how." Arithmetic learnings take place when the learner understands the arithmetic processes. Rote learning—learning by rule—has been relegated to the past and the discovery technique has become the accepted mode of instruction.

Because arithmetic has become a respectable subject for students at all levels, and because there is great need for improvement in the teaching of arithmetic in the public schools today, the writer believes that a course in arithmetic which will develop concepts, clarify principles, and explain the structure of the numeration system should be a requirement for all teachers.

Teachers must know the meaning of such ideas as area, interest, decimal fraction, division, million, per cent, and so on. They must understand the principle of equal additions, the law of commutativity, the Golden Rule for fractions, the distributive law with respect to multiplication, and significant figures. They must develop an appreciation of the various algorithms for the operations involved with numbers, the values of approximation and estimation in solving problems, and above all, an awareness of the new approach to the interpretations of numbers and operations with them.

A course which stresses the new philosophy cannot be completely devoid of suggestions for teaching arithmetic. However, the primary function of this text is to explain concepts and principles and rationalize operations with numbers and to suggest ways of learning arithmetic. Since it is difficult to divorce techniques from subject matter, naturally some suggestions have crept into the text. The text is exhaustive in its treatment of topics in order that the reader will grasp a fuller meaning of the concepts with which he may have only a meager understanding. It is not intended that teachers, in turn, should

compel students to learn all of the material presented about particular topics or to solve all of the exercises. Topics are treated fully as background material for the teacher and to show the reader that there are other ways of doing examples.

The text attempts to present arithmetic in a new setting with attention given to the laws which permit operations with numerals and at the same time provides rationalization of mechanical processes. Throughout the text the author reiterates these laws in new and different settings; this re-use of these principles enables the reader to secure a greater grasp of the ideas which are basic to arithmetic.

Some of the features of the book are:

1. It is written in an easy-reading style.
2. It contains clear and logical interpretations of processes and operations.
3. It contains an abundant number of illustrations of operations.
4. It incorporates many suggestions for teaching arithmetic.
5. It provides for maintaining skills already established.
6. It is arranged logically.
7. It stresses the use of laws and principles throughout.
8. It provides numerous checks for testing results.
9. It contains many challenging exercises.

The text was written especially for those who are preparing to enter the teaching profession, but it is suitable for use in secondary schools and colleges where there is a demand for a respectable course in arithmetic presented from an advanced point of view.

MARVIN C. VOLPEL

Towson, Maryland  
January, 1964

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