Karl-Heinz Steiner

Interactions between Electromagnetic Fields and Matter

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Principal Mathematical Symbols Used in This Work

For a general quantity M, the subscripts and superscripts denote the following associations:

M_i, M_j	with the i-th and the j-th particle, respectively
M_n	with the n-th quantum state
M_E	with the electric field
M_H	with the magnetic field
M_K	with a generalized field K (for E or H)
M_e	with an electric reaction
M_m	with a magnetic reaction
M_D	with the dipole moment
M_Q	with the quadrupole moment
M_O	with the octopole moment
M^l	with the linear process
M^s	with the nonlinear phase-sensitive process
M^i	with the nonlinear phase-insensitive process
M_b	with a backward wave
M_f	with a forward wave
$M_\nu, M(\omega_\nu)$	with the frequency of ω_ν

Other denotations and designations are:

M_s, M_o	= a static quantity
M_d	= a dynamic quantity
M^o	= an unperturbed quantity
$M^{(\nu)}$	= $\vec{M} \cdot \vec{e}_\nu$ = the scalar product of $\vec{M}$ and the unit vector $\vec{e}_\nu$
$M^{[\mu]}$	= M in individual steps of expansion
M	= $M' - iM''$ = real and imaginary parts of M
$\|M\|$	= magnitude
$\vec{M}$	= vector
$\|\|M_{kl}\|\|$	= matrix
$[M_{ij}]$	= tensor of second rank
$[M_{ijk}]$	= tensor of third rank
$\|M\rangle$	= ket M
$\langle M\|$	= bra M
$\langle M\rangle$	= expectation value of M.

The multitude of the most variegated quantities required for the presentation made it mandatory that a given symbol had to be reused for various quantities. In so doing, great attention was paid, however, to ensure that such quantities appear in separate fields so that confusion should be virtually impossible.

A = vector potential,
amplitude function,
field constant of a reflected wave in a multilayer structure

A_n, A_μ = normalization constants

a = power series coefficient of a potential function

B = magnetic flux density,
field constant of a transmitted wave in a multilayer structure

b = see a

C = general field constant

c = velocity of light in the vacuum

D = dielectric displacement

d = damping constant

E = energy,
electric field strength

e = unit vector

F = force

f = distribution function

g = gyromagnetic ratio,
complex frequency response

H = Hamiltonian,
magnetic field strength

h = Planck's constant ($\hbar = h/2\pi$)

i = imaginary unit

K = generalized field symbol (for E or H)

k = Boltzmann's constant,
propagation coefficient

L = length

l = orbital angular momentum

m = particle mass

N = number of particles per unit volume

n = normal vector

P = power per unit volume

p = momentum

p_{pq} = induced transition probabilities between states p and q

q = particle charge

R = radius in the macroscopic R-space

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r = radius in the microscopic r-space
r^a, r^b = reflection coefficients at the boundaries a and b, respectively
s = spin angular momentum
T = absolute temperature
T_{pq} = relaxation constants between states p and q
t = time
t_{pq} = thermal transition probabilities between states p and q
V = potential energy
v = velocity of light in the substance
W = perturbation energy
x = damping reduction factor
α = coefficient of characteristic curves,
friction coefficient (loss correction)
β = propagation coefficient
β_o = Bohr magneton
Γ = propagation coefficient
γ = generalized dimensional moment parameter ($\pi = \gamma\xi$)
ΔE = unperturbed energy difference
Δk = phase mismatch
$\Delta\omega$ = bandwidth (angular frequency –)
δ = transmissivity coefficient
δ_{pq} = Kronecker – δ
ϵ = relative permittivity
ϵ_o = vacuum permittivity
η = generalized symbol for ϵ or μ
η_o = generalized symbol for ϵ_o or μ_o
κ = propagation coefficient
λ = wavelength,
ordering parameter with perturbation calculations
μ = relative permeability
μ_o = vacuum permeability
ν = frequency
ξ = generalized dimensionless moment parameter ($\pi = \gamma\xi$)
Π = polarization
π = moment
ρ = density operator
φ = scalar potential,
phase angle
χ = susceptibility
ψ = wave function, state vector
ω = angular frequency

Some Universal Constants (SI-units)

$\epsilon_0 = 8.855 \times 10^{-14}\ \mathrm{AsV^{-1}\ cm^{-1}}$

$\mu_0 = 1.257 \times 10^{-8}\ \mathrm{VsA^{-1}\ cm^{-1}}$

$q_0 = -1.602 \times 10^{-19}\ \mathrm{As}$

$m_0 = 9.108 \times 10^{-28}\ \mathrm{g} = 9.108 \times 10^{-35}\ \mathrm{VAs^3\ cm^{-2}}$

$\hbar = h/2\pi = 1.054 \times 10^{-34}\ \mathrm{VAs^2}$

$g_0 = \dfrac{\mu_0 q_0}{m_0} = -2.211 \times 10^7\ \mathrm{cmA^{-1}\,s^{-1}}$ (spin)

$\beta_0 = \dfrac{\mu_0 q_0 \hbar}{2m_0} = -1.165 \times 10^{-27}\ \mathrm{Vscm}$

$k = 1.380 \times 10^{-23}\ \mathrm{VAsK^{-1}}$

$c = 3 \times 10^{10}\ \mathrm{cms^{-1}}$ (more exactly: $c = 2.998 \times 10^{10}\ \mathrm{cms^{-1}}$)

$\dfrac{q_0}{m_0} = -1.759 \times 10^8\ \mathrm{Asg^{-1}} = -1.759 \times 10^{15}\ \mathrm{cm^2\,V^{-1}s^{-2}}$

$\dfrac{\beta_0}{\hbar} = -1.105 \times 10^7\ \mathrm{cmA^{-1}s^{-1}}$

$kT_0 = 4.043 \times 10^{-21}\ \mathrm{VAs}$

$T_0 = 293\ \mathrm{K}$

$\dfrac{\hbar}{kT_0} = 1.638 \times 10^{-13}\ \mathrm{s}$

Preface

This book was written by an engineer. The author's interest centered, and still centers, on all the principles and methods by means of which, in their specific application to the techniques employed at radio and light frequencies and specifically in communications, electromagnetic fields can be produced or amplified from very low levels, as well as on principles that can be used for modulation, rectification, etc.

All these effects are based on interactions between electromagnetic fields and matter. In the absence of matter, the linear laws of the vacuum field apply – except in the case of extreme high-energy problems –: the field exists, but no additional effects appear.

The laws of classical physics suffice in most cases for a description of the various interaction phenomena. This also applies to interactions that, strictly speaking, are based on typical mechanisms of quantum mechanics: provided matter is held concentrated generally in an n-pole structural element (diodes, transistors, tunnel-elements, etc.) there is no absolute need for the engineer or physicist to care about what may happen inside these elements. Knowledge concerning the combined effect of the individual parameters as derived from a single measurement at the terminals suffices for their purposeful application.

When, however, matter is used as an extended medium, and when the related frequencies become so high that a quasistatic analysis is no longer possible, or when, quite generally, the mechanisms of the interactions are to be covered in greater detail, one must resort for a description to the theory where matter is on its own – quantum theory.

It was here in particular that the discovery of the maser [1]) principle, corresponding to the first aspect mentioned above, also induced the engineer to deal more thoroughly with the interaction phenomena, beyond a purely phenomenological line of thought. The application of this concept to the infrared and visible ranges of the spectrum later on led to the generation of coherent waves at these frequencies for the first time (laser [2])), whilst use of the laser as a source of excitation simultaneously opened up the possibility of realizing at these frequencies the multitude of nonlinear interaction processes familiar from the classical range.

If the analysis of the interaction phenomena is restricted to the complex with energy-storing matter – as will be done throughout this book – the processes known from classical concepts in conjunction with concentrated components are nonlinear processes, for they are the ones that are of essential practical importance. In the extended medium, however, with the lack of frequency-limiting elements such as package capacitances and lead inductances, further processes of interest develop. The frequencies of the process fields now can reach coincidence with the natural resonances of matter itself. This lends practical import also to the linear interaction process while simultaneously the wide spectrum of nonlinear processes is enriched by additional variants.

This book has as its goal a presentation as comprehensive as possible of the mechanisms of the most variegated processes. Their discussion, firstly not closely associated with any practical problems, should here be useful not only to the engineer, but to every natural scientist. It is hoped at least, although, in particular the quantum-mechanical treatment of the various phenomena has been derived from the mentality of an engineer.

The book has six chapters. The first two give an introduction and survey with the means of classical mechanics. In particular the first chapter begins, by considering, firstly, a cluster of charged mass points in the external field, with a determination of the quantities that decisively control the interaction pattern between fields and matter. The chapter terminates with a derivation and interpretation of the individual electric and magnetic moments. The second chapter continues with classical introductions. The picture of matter is subjected to advanced development by dealing with some elementary microscopic models such as the central-field model and the oscillator model. The typical electric and magnetic characteristic curves are derived for the individual models. These relationships already allow many conclusions to be made concerning the interaction pattern with electromagnetic fields. With a general macroscopic characteristic curve a survey is subsequently given of the entire multitude of the various processes. The presentation covers the principal interaction effects up to, and including, the cubic processes. A reader who wants to begin by obtaining a general survey should start here. The systematic treatment of the subject matter made it apparent that it would not be advisable to place this part at the beginning of the book.

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1) Microwave Amplification by Stimulated Emission of Radiation

2) Light Amplification by Stimulated Emission of Radiation

The third chapter introduces the nonclassical part. As a first step the principal fundamentals of quantum mechanics are sketched to the extent required for the set of topics that are of interest. Considerable room has been devoted to the various representation methods of the perturbed particle system – methods that allow the interactions between fields and matter to be covered. Following a general discussion of the methods a specific method finally is chosen that is adapted to the scope of this book in its entirety. Its specialization to the problems of interest is the starting point for the further specialized analysis, while simultaneously terminating this third chapter.

Chapters four and five discuss special interaction processes. The fourth chapter is here closely related in particular with the second chapter, because a general static characteristic curve of matter is here determined, now indeed in terms of the formalism of quantum mechanics. Here, as well as there, all static and quasistatic processes can be easily described, processes whose frequencies are far away from any eigenresonances of matter. It is, in particular, these considerations which establish a direct link with the original classical methods. Whilst including them they give them a more profound interpretation at the same time.

Chapter five deals with the principal resonance processes. At least one of the process frequencies here coincides with a natural resonance of matter. The presentation covers interactions via the linear resonance process (maser, laser), typical interactions via nonlinear resonance processes (multi-quantum absorption and emission, respectively, Raman processes), but also processes such as frequency multiplication, frequency division, parametric mixing processes etc., processes that otherwise take place away from the resonance ranges as a rule. Chapter five ends with an attempt to give an interpretation concerning common traits of the various types of processes.

In the sixth and last chapter the studies of the microscopic reactions are followed by a discussion of a few selected processes concerning their behavior in macroscopic structures. The theoretical analysis here reverts again to the classical presentation: the relationships gained from quantum-mechanical investigations are linked with Maxwell's theory. Specializations allow reactions in microwave structures as well as in structures for optical frequencies to be obtained from the so found relationships. A presentation in a way as uniform and fundamental as possible cannot, however, take into account the most varying forms of macroscopic structures, but must confine itself to a single fundamental and straightforward one. The essential properties in macroscopic structures are so outlined uniformly for all processes under discussion.

The six chapters of the book are followed by an appendix that further illustrates the general relationships by reference to two elementary models of matter. The electron spin has been chosen for this purpose as a basic example of magnetic interactions in the microwave range, as well as the anharmonic oscillator which is to be representative of the hydrogen molecule. It gives a basic example of typical interactions in the range of the infrared spectrum.

The bibliography is compiled as a unit at the end of the book. It includes the literature related to the subject matter, but no general fundamental literature. The latter is quoted in footnotes in the individual chapters. The classical complex of interactions is covered in the collection merely in its mile-stones, and the reader is referred to books that already are of a synoptical character. More detailed attention has been paid to the nonclassical complex. In conjunction with a survey of the historical evolution, it is in particular the experimental work that should be highlighted to some extent because there is no room available for this in a purely theoretical presentation such as this book is.

The brief survey concerning the contents of the book, with the concept of a 'characteristic curve' of a structural element and its adoption to a system of matter already reveals an engineers mentality. The mixing of the line of thought of physicists and engineers goes on throughout the book. Such mixed presentation will indeed cause some confusion to one of the parties while it should aid simultaneously in opening up an access road to the other party. This problematic situation will come about whenever originally separated disciplines meet at a joint frontier.

The scope of the problems touched, starting as it did, from a cluster of charged mass points in the external field up to the macroscopic structures of a frequency doubler or a Raman laser as examples demanded concessions to be made in the presentation. A systematic course, without excessive encumbrance, had to be taken in a way that was as short as possible. In particular in the classical part, this even gave rise to a few modifications of the theory which, however, will be justified when one keeps the entire scope of the book in mind.

Presentation and derivation of the individual processes were, however, made in great detail. A reader merely interested in the final results may, of course, skip over the intermediate steps. Attention to detail is appropriate and necessary whenever general physical results become the object of an engineer's work. This also explains the large number of specializations of the general relationships, last but not least in order to establish from the special examples links between quantum-mechanical results and familiar classical concepts.

The international system of units is used throughout the book with freely selectable orders of magnitude of the basic units.

It is with great pleasure that I express my gratitute to the few who have assisted me with advice and encouragement in the writing of this book. In particular, I desire to thank Dr.-Ing. habil. W. Kautter, Munich, as the translater, Prof. Dr. H. G. Andresen, Mainz, for discussions on certain problems, and Prof. Dr. H. Stumpf, Tübingen, as editor and critic. My gratitude is likewise due to Dr. G. Schollmeier, Munich, and G. R. Jones, M. Sc., London, for help in reading the manuscript.

It is my sincere hope that the book will help all those who wish to make a detailed study of the problems treated.

Lannion (France), autumn 1972 *K.-H. Steiner*

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Appendix

Introductory Remarks

1. Storing Matter and Dissipating Matter

Matter can be roughly visualized as a cluster of charged particles. Taken as a whole, they will usually show a neutral electric reaction, the partial charges compensating for each other. From the paired character of the charges required for compensation, two elementary models can be developed. In the first model, the individual charge pairs establish an internal bond each as shown in Fig. 1 as a primitive cluster of atoms: in the central fields of the nuclei electrons are orbiting around. In the second model (Fig. 2), electrons are moving freely between the nuclei. When external fields act on such systems of particles, they will become superimposed on the internal fields, affecting the patterns of motion. In the bonded-particle model, the effect of the fields results in changes in the inner bonding forces: field energy is stored in the particle system. In the second model, the freely movable electrons receive additional kinetic energy from the external field which is transferred to the ion cores by collisions: field energy is dissipated. This results in a rough classification of matter: a distinction is made between matter with energy-storing properties – "storing matter" – and matter with energy-dissipating properties – "dissipating matter."[1] The first category comprises accordingly substances with bonded charge carriers, the second covers substances with free charge carriers.

The following considerations relate all to the class of "substances with bonded charge carriers."

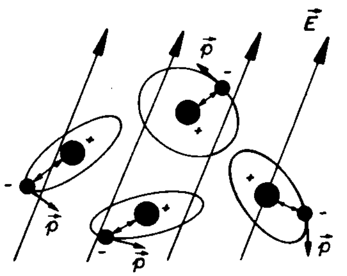

Fig. 1.
Model of bonded particles in an external field.

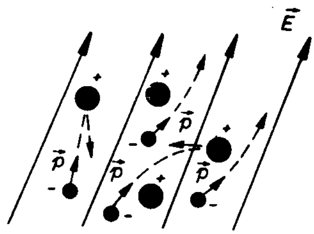

Fig. 2.
Model of free particles in an external field.

[1]) Sometimes the terms *non-dissipative* and *dissipative* are used instead of the terms *storing* and *dissipating* which are employed in this book.

2. Some Phenomenological Considerations to the Reactions of Matter

The change in the bonding forces under the action of external fields produces a polarization of the particle system. Phenomenologically, the response of matter is described by the two constitutive relations

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{\Pi}_e(\vec{E}), \tag{1}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{\Pi}_m(\vec{H}). \tag{2}$$

Their explicit field-dependent terms stand for the vacuum contributions to D and B, the other terms are the contributions of matter properly speaking: Π_e and Π_m denote the electric and the magnetic polarization. By way of these relations, the reactions of matter enter Maxwell's theory.

The polarizations can be further analyzed as:

$$\vec{\Pi}_e(\vec{E}) = N\vec{\pi}_e(\vec{E}) = \varepsilon_0 \chi_e(\vec{E})\,\vec{E}, \tag{3}$$

$$\vec{\Pi}_m(\vec{H}) = N\vec{\pi}_m(\vec{H}) = \mu_0 \chi_m(\vec{H})\,\vec{H}. \tag{4}$$

The respective first equations establish a dependence of the polarizations on moments through the number of particles per unit volume, N. The granular structure of matter thus comes also into evidence in the phenomenological representation. The respective second equations, for which field dependence of the polarization already is a requisite condition, define the susceptibilities; these in turn determine the relative permittivities and permeabilities ϵ and μ, respectively:

$$\vec{D} = \varepsilon_0 \left(1 + \chi_e(\vec{E})\right)\vec{E} = \varepsilon_0 \varepsilon(\vec{E})\,\vec{E}, \tag{5}$$

$$\vec{B} = \mu_0 \left(1 + \chi_m(\vec{H})\right)\vec{H} = \mu_0 \mu(\vec{H})\,\vec{H}. \tag{6}$$

As a rule, all parameters of matter are field-dependent quantities; the special type of field dependence is determined by the specific substance in each case. *Through the intermediary of these relationships, the entire multitude of linear and nonlinear processes in the interactions between fields and matter has its origin.*

In the following, the matter parameters of these phenomenological considerations will be the subject of detailed investigations. The approach is characterized by

1) The interpretation of their physical background.
2) The determination of the principal individual processes which come about from them.