Probabilistic Methods in the Theory of Structures

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Preface

This book is written both to serve as a first-course text on probabilistic methods in the theory of structures and to provide a more advanced treatment of random vibration and buckling. It is intended in particular for the student in aeronautical engineering, mechanical engineering, or theoretical and applied mechanics, and may also be used by practicing engineers and research workers as a reference. In fact, it combines the features of a textbook and a monograph.

Probability theory and random functions are playing an ever more prominent role in structural mechanics due to the growing realization that many mechanical phenomena can be satisfactorily described by probabilistic means only. In the last 25 years, much work has been done and many studies have been published on this subject. However, despite significant advances, the probabilistic approach to the theory of structures has not yet found its proper place in engineering education.

Chapter 1 introduces the role of probabilistic methods in the theory of structures. Chapters 2 through 4 deal exclusively with elements of the theory of probability for a single random variable. This apparent preoccupation with the single random variable stems from my own feeling that it would be unfair to offer the reader a mere taste of the theory of probability and then immediately confront him or her with a wide range of applications. Chapter 5 is devoted to the reliability of structures described by a single random variable. Chapter 6 discusses elements of the theory of probability of two or more random variables, while Chapter 7 examines the reliability of such multivariable structures. Chapter 8 introduces the theory of random functions. Chapter 9 deals with random vibration of single- and multidegree-of-freedom structures, and Chapter 10 with random vibration of continuous systems. These chapters concentrate on the role of modal cross correlations in random vibration analysis, usually overlooked in literature, as well as treat point-driven structures, and random vibration and flutter. These chapters constitute, among others, a prerequisite to study the fatigue life of structures—a topic which is outside the scope of this book. The reader interested in this subject is referred to other sources where it is adequately treated. Finally, Chapter 11 is devoted to the Monte Carlo method for treating problems incapable of exact solution. Special emphasis is placed on buckling of nonlinear structures where random imperfections may be responsible for drastic reduction of the buckling loads.

Ample examples are included in the book, because it is my experience that much of the material in question may be taught most effectively by this means. An additional purpose of the examples is to examine the validity of some widely accepted simplifying assumptions concerning the probabilistic nature of the output quantities and to observe the errors that these assumptions may cause. Numerous exercises are provided with each chapter, to deepen the reader's grasp of the subject and widen his or her perspectives.

The material in Chapters 1 to 5, together with Sections 11.1–11.3, are suitable for a one-semester, first-level course at the junior or senior level. Prerequisite courses for this part are calculus, differential equations, and mechanics of solids. For departments whose curriculum requires a course in the theory of probability, the first four chapters may be rapidly recapitulated, in which case the one-semester course may include also Chapters 6 and 7, as well as Sections 11.4 and 11.5. The material in Chapters 8 through 11 is open to both the analytically minded senior and the graduate student and may form an advanced course on random vibration and buckling. The additional prerequisite for this part is knowledge of matrix theory and the basics of vibration and buckling of structures, although the necessary material is reviewed at the beginning of each chapter.

It is my agreeable duty to thank the Department of Aerospace Engineering of Delft University of Technology for their invitation to present a series of lectures (from which this text grew) to their students and scientific staff during my sabbatical leave in the academic year 1979–1980—an experience of endless Dutch courtesy and good will. My sincere thanks are due to the Dean, Prof. Ir. Jaap A. van Ghesel Grothe, and to Professor of Aircraft Structures Dr. Johann Arbocz for their constant encouragement and help. Appreciation is expressed to the staff members and the students of Delft, and especially to Ir. Johannes van Geer, Ir. Willie Koppens, and Ir. Kees Venselaar for their able assistance in a number of calculations and constructive suggestions concerning the lecture notes. I acknowledge the help of Ir. J. K. Vrijling of Delft in writing Sec. 4.18. I also thank the Department of Aeronautical Engineering, Technion-Israel Institute of Technology, in whose encouraging atmosphere I was able to bring this work to completion. I am also most indebted to Eliezer Goldberg of Technion, for his kind help in editing the text, to Marijke Schillemans and Dvora Zirkin for typing much of the original manuscript, to Alice Aronson and Bernice Hirsch for typing Chapter 9, and to Willem Spee and Irith Nizan for preparing the drawings.

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Contents

chapter 1

Introduction

For an adequate description of structural behavior, probabilistic methods must be resorted to. Properly speaking, an element of probability is embodied even in the deterministic approach, which claims to "simplify" the structure by eliminating all aspects of uncertainty. Under the deterministic approach, external loading and the properties of the structure are represented as though they were fully determined, and available (often highly sophisticated) tools yield, with sufficient accuracy, the strains and stresses in systems with complex configurations. At the same time, these stresses are compared with allowable ones obtained by dividing their ultimate levels by a "safety factor," so as to yield a level below that of failure, a practice that recognizes the uncertain, and random, features of the stress distribution in the material. This is how a probabilistic consideration is admitted "via the back door"; indeed, the safety factor has often been referred to as the "ignorance factor."

The quality of "randomness" is characteristic both of loads borne by structures and of the properties of the structure themselves. No two structures, even if they have been produced by the same manufacturing process, have identical properties. Thin-walled structures are often sensitive to imperfections—deviations from their prescribed geometry—in the sense that the buckling load of an imperfect structure may be lower than that of its ideal counterpart by several percentage decades. The shape and magnitude of these initial imperfections vary widely from case to case, since differences are inherent in any manufacturing process, which is itself subject (by its very nature) to a large number of random influences. These and other examples clearly indicate that it is impossible to investigate structural behavior without resorting to probabilistic methods.

The need for a probabilistic approach does not obviate the classical treatment of the behavior of an ideal structure with given properties, subjected to given loading. In fact, the solution to a deterministic problem may very often prove useful in a probabilistic setting. For example, assume that the properties of a structure are fully determined, while the external forces or moments are random. We begin by constructing explicit equations of motion (or equilibrium) in terms of these forces and moments, which are then used as input in determining the probabilistic characteristics of the response (output). Where the exact relationship between input and output is unavailable, or its application proves too cumbersome, statistical simulation (such as with the Monte Carlo method) is the logical remedy, now that high-speed digital computers are so readily available. The first step of this method consists in simulating the random variable; the second step is numerical solution of the problem for each realization of the random variable; the third and last is statistical analysis (computation of the characteristics of the output by averaging over the ensemble). Thus, one of the cornerstones of the Monte Carlo method is the solution of a deterministic problem.

The deterministic and probabilistic approaches to design differ in principle. Deterministic design is based on total "discounting" of the contingency of failure. The designer is trained in the doctrine that with the relevant quantities properly chosen, admissible levels would never be exceeded; it is postulated that, as it were, the structure is immune to failure and will survive indefinitely. This approach dates back to antiquity, when design analysis and control were unknown and everything centered on the personal responsibility of the artisan. Its earliest written record is probably Hammurabi's Code, according to which, if a house collapses and the householder is killed the builder is liable to the death penalty.

Deterministic design has now reached a very high level of sophistication, and modern computation techniques make it possible to determine stresses, strains, and displacements in highly complex structures. However, problems of structural design always involve an element of uncertainty, unpredictability, or randomness: No matter how much is known about the phenomenon, the behavior of a structure is incapable of precise prediction. In these circumstances there always exists some likelihood of failure, that is, of an unfavorable state of the structure setting in. Even with safety factors—empirical reserve margins—failures did and still do occur. There can in principle be no "never-fail" structure; it is a question only of a higher or lower probability of failure. Accordingly, probabilistic design is concerned with the probability of failures or, preferably, of nonfailure performance, the probability that the structure will realize the function assigned to it—in other words, with *reliability*. The *McGraw-Hill Dictionary of Scientific and Technical Terms* gives the following definition of this basic concept: "Reliability—the probability that a component part, equipment, or system will satisfactorily perform its intended function under given circumstances, such as environmental conditions, limitations as to operating time, and frequency and thoroughness of maintenance, for a specified period of time." The reliability approach was initiated by Maier and Khozialov and carried on by Freudenthal, Johnson, Pugsley, Rzhanitsyn, Shinozuka, Streletskii, Tye, and Weibull. The contributions of Ang and Tang, Augusti, Barrata and Casciati, Benjamin and Cornell, Bolotin, Ferry Borges

Fig. 1.1. Section of Hammurabi's stela at high magnification (photoassembly by courtesy of J. Kogan).

and Castanheta, Ditlevsen, Haugen, Kogan, Lind, Moses, Murzewski, Rackwitz, Rosenblueth, Schuëller, and Veneziano should also be mentioned.

The development of high-power rocket jet engines and supersonic transport since the 1950s has brought out new problems of mechanical and structural vibration, namely the response of panel-like structures to aerodynamic noise and to a turbulent boundary layer, with the attendant aspects of acoustic fatigue and interior noise, all of which are incapable of deterministic solution. The probabilistic methods for these and other problems are embodied in a new discipline called "Random Vibration," dealt with by numerous research centers and their offshoots, which have come into being throughout the world in the last 20 years. Of these, the teams of Caughey (Caltech), Crandall (M.I.T.), Lin (University of Illinois, Urbana-Champaign), and Shinozuka (Columbia University) in the United States; of Bolotin (Moscow Energetics Institute) and Pal'mov (Leningrad Polytechnic) in the Soviet Union; of Clarkson (Southampton) and Robson (Glasgow) in the United Kingdom, of Ariaratnam (Waterloo) in Canada; and of Parkus (Vienna) in Austria are perhaps the most well-known.

The probabilistic approach proved extremely useful in analysis of flexible buildings subjected to earthquakes (Cornell; Newmark and Rosenblueth; Vanmarcke) or wind (Cermak); offshore structures subjected to random wave loading (BOSS conference); ships in rough seas (Ekimov; Price and Bishop); structures undergoing fatigue failure (Bogdanoff, Freudenthal, Gumbel, Payne, Weibull); structures subjected to environmental temperatures (Heller); structurally inhomogeneous media (Beran, Kröner, Lomakin, Shermergor, Volkov); stability of stochastic systems (Khas'minskii, Kozin, Kushner); probabilistic identification of structures (Hart, Ibrahim, Masri and Caughey); and other fascinating problems.

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*Many highly interesting studies simply could not be mentioned here, since the subject is much too vast. Since a complete bibliography on probabilistic methods in mechanics could fill by itself a hefty volume, I have confined myself mostly to books and reviews, so as to give some idea of what has been done. A list of cited references and of recommended further reading is given at the end of each chapter.

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chapter 2

Probability Axioms

2.1 RANDOM EVENT

We will associate mechanical phenomena with a complex of conditions under which they may proceed, assuming that this complex is realizable (or rather reproducible, at least conceptually) an arbitrarily large number of times in essentially identical circumstances, with an observation or a measurement taken at each such realization. Such a process of observation or measurement will be referred to as a trial or an *experiment*. In this sense an experiment may consist in checking whether stresses in a structure exceed some specified value, or in determining the profile of imperfections of its surface, or else (in modern supersonic aircraft) in determining the noise level. We define an *event* as an outcome, or a collection of outcomes, of a given experiment (a positive or a negative conclusion; readings of the scanning mechanism; the final result of a highly complex calculation). The outcome of a deterministic phenomenon is totally predictable and is, or can be, known in advance: Deterministic phenomena are either *certain* or *impossible*, depending on whether, inevitably, they do or do not occur in the course of the given experiment.

For example, consider a perfectly elastic beam with symmetric uniform cross section (section modulus S), subject to given constraints under a given transverse load resulting in a maximal bending moment M_{max} ("complex of conditions," Fig. 2.1a and 2.1b). The maximum bending stress, according to the theory of strength of materials, is then given by

$$\sigma_{max} = \frac{M_{max}}{S} \tag{2.1}$$

Another example is a perfectly cylindrical shell made of perfectly elastic material, with radius R, length l, thickness h, Young's modulus E, and Poisson's ratio ν, under uniform axial compression with ends simply supported