FREQUENCY RESPONSE FOR PROCESS CONTROL

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McGRAW-HILL BOOK COMPANY, INC.

1959 · New York · Toronto · London

Preface

Several years ago one of the authors of this book was asked to organize a course which applied frequency-response methods to process control. An investigation of existing texts revealed that although these methods were used in books on servomechanisms, they had not been used in books on process control. Because of this, the lecture notes prepared were designed to introduce frequency-response methods in this application.

In 1954, the notes were modified for use in a course given for personnel from process industries. In 1955, they were again modified and reworked in order to obtain a more complete and uniform presentation. Experience gained through continued study and presentation of the course from 1954 to 1958 has resulted in the removal of some original material and the addition of new material. The present book maintains the two-part breakdown of the earlier notes; basic theory is covered in Part I, and Part II is devoted primarily to applications.

In the writing of this book an attempt has been made to be as nonmathematical as possible without sacrificing rigor. In most cases mathematical proofs are given at the end of each chapter so that the reader can learn the results without having to read proofs. Where approximations have been used, care has been taken to point out their limitations.

This book presents new graphical methods which effectively reduce the amount of mathematical computation required. Graphical methods are used extensively in the solution of problems and in the presentation of data.

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In addition to these contributors, the authors would like to acknowledge the cooperation of the following associates who have given generously of their time and knowledge. Their help, in the form of test-data proofreading, and in the preparation of problems and figures, has made the publication of this book possible.

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Since the present book is based on several years of cooperative effort, it is impossible to assign credit for all the individual contributions. Where omissions occur, the authors apologize.

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PART I

BASIC THEORY

1

Introduction

1.1. Historical. For many years process control was an art rather than a science. The instrument man, relying heavily on his own experience, applied trial-and-error methods until he achieved a satisfactory solution to a particular problem. Mathematical analysis was either too simple to be of any real use or too complicated to be applied by the average worker.

The last thirty years have seen more and more stringent demands on automatic control systems. Synthesis of organic compounds such as benzene and toluene, production of synthetic rubbers, catalytic refining of gasoline, and the separation of the isotopes of uranium for atomic energy could never have resulted without precise controls. As processes become more complex, the need grows for methods of analysis which can be easily applied to obtain information about system performance and stability.

For some years audio engineers have used frequency-response methods for studying and describing their equipment. During World War II a considerable amount of analytical work was done in applying these same methods to servomechanisms. Since there is a close analogy between the feedback circuits used in electronic amplifiers and the circuits used in process control, it was a natural development to employ frequency-response methods in process control and in the design of process-control equipment. By now a substantial body of literature exists on the subject.

1.2. The Control Loop. A simple control system is shown in Fig. 1.1. The operator manually adjusts the *manipulated variable* m in an effort to make the *controlled variable* c maintain a desired value.

There are several disadvantages to such a system:

1. In case of frequent disturbances, the operator must check often to see if the controlled variable c is within tolerances. This may be tedious work.

2. If the system is slow in responding to corrective adjustments, there may be a tendency for the operator to overcorrect for changes in the controlled variable, with resultant overshoot of the desired value.

The task of adjusting m in relation to changes in c is usually accomplished more accurately and economically by using an instrument called an *automatic controller.* In the system with automatic control (Fig. 1.2), the value of the controlled variable is measured and fed back for comparison with the value of the *desired value* r (also called *set point*). Consequently, automatic control systems are often referred to as "feedback systems." As shown in Fig. 1.2, the difference between the *measured*

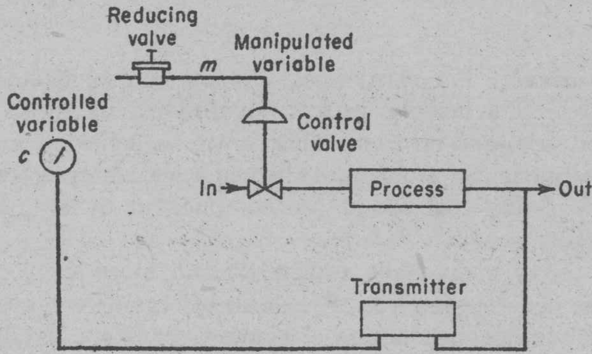


FIG. 1.1. A simple control system.

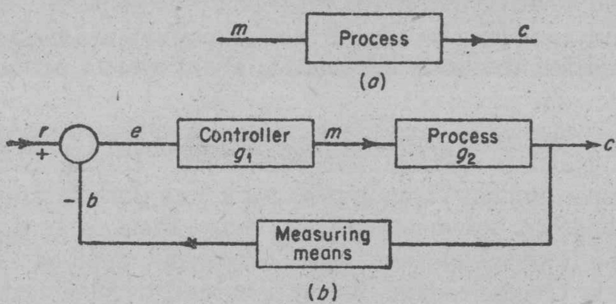


FIG. 1.2. Block diagrams of simple control systems: (*a*) manual control; (*b*) automatic control.

value b (of the controlled variable c) and the desired value r is e, the *actuating signal* of the controller. By convention, $e = r - b$. This signal is modified by the controller to obtain the manipulated variable (*controller output*) m, which in turn acts to change the controlled variable. In industrial control instruments the comparison of r and b to generate the actuating signal e is generally made in the control-instrument case. In block diagrams such as Fig. 1.2, means for summing or taking the difference of two signals are represented by circles and are called *summing points.* The measuring means may also be in the control-instrument case. The closed circuit formed by the process, measuring means, controller, and final control element (often a control valve in fluid processes) is called a *closed loop.*

1.3. Methods of Describing a Control System. Our principal concern is the dynamic behavior of a closed-loop control system. In order to make such a study, we must have available detailed information about the characteristics of the system. There are several methods which are commonly employed in making such a description.

Control systems may be described by their responses, or their curves of controlled variable against time, to various disturbances. Some disturbances commonly applied are shown in Fig. 1.3. The sine-wave, or

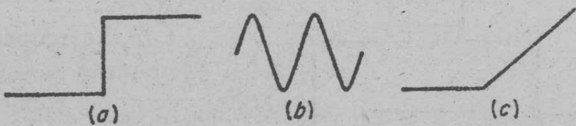


FIG. 1.3. Disturbances used to study processes or components of a control system: (*a*) step; (*b*) sine wave; (*c*) ramp.

sinusoidal, disturbance is the basis of frequency response. This book also makes use of a suddenly applied, sustained disturbance (*step* disturbance) and a disturbance that changes at a constant rate (*ramp* disturbance). Systems may also be described by their responses to other disturbances, such as square-wave, impulse, and random. However, these are not used in this book. Responses to disturbances may be obtained either by experimental tests on an existing system or by solving the differential equations which describe the system.

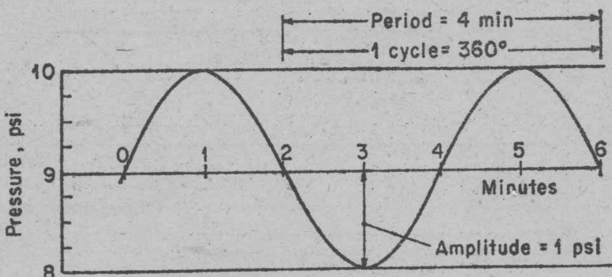


FIG. 1.4. A pressure sine wave with a frequency of ¼ cpm.

1.4. Frequency Response. Analysis by frequency response has been used for some time by workers in the field of servomechanisms who have to design complete systems to meet performance specifications. Some of these methods may be applied to problems in process control.

To determine experimentally the frequency response of a component, its input is made to oscillate continuously with a fixed amplitude, so that the plot of input against time is a sine wave. If the system is linear, the steady-state output will be in a sine wave of the same frequency. The amplification and phase change corresponding to each frequency are recorded and constitute the data of frequency response.

The basis of the analysis is, therefore, a sine wave (Fig. 1.4). Th *amplitude* of this sine wave is the magnitude of the peak in pounds pe square inch (psi) measured from the mean pressure level. The unit of measurement is psi, inches, degrees Fahrenheit, etc., depending upon the physical system under test. The *period* is the time required for one complete cycle. The *frequency of cycling* is equal to unity divided by the period and thus has units of cycles per unit time.

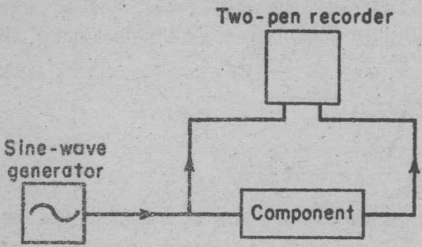
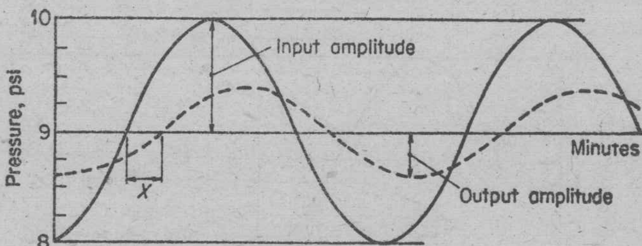


FIG. 1.5. Block diagram of arrangement for obtaining frequency response of an isolated component.

To obtain the frequency response of a component, a sine-wave generator may be connected to the input of the component in the manner shown on Fig. 1.5. The generator is adjusted to produce a sine wave of specified frequency. If the component is linear and stable, the output pen will record a sine wave *after steady-state conditions have been reached.* The dashed line on Fig. 1.6 is such a sine wave.

If the component is linear, both sine waves will have the same frequency or period. In the case shown, the amplitude of the output is smaller than the amplitude of the input wave. The *gain* is defined as



$$\text{Component gain} = \frac{\text{Output amplitude}}{\text{Input amplitude}}$$

$$\text{Phase lag} = \frac{X}{\text{Period}} \text{ times } 360°$$

$$= X \text{ times frequency times } 360°$$

FIG. 1.6. Steady-state output of component in response to a sinusoidal pressure input.

the ratio of the output amplitude to the input amplitude under steady-state conditions. This is also called *magnitude ratio.* Sometimes it is convenient to talk about attenuation, the ratio of the input amplitude to the output amplitude. Thus, *attenuation* is 1/gain.

The waves differ also in that they are "out of phase," which means that they cross the mean pressure level at different times. We speak of a *phase shift.* Since the dashed curve (output) crosses the mean pressure level at a later time than the solid curve (input), the output is said to *lag* the input. If the output wave crosses the mean pressure level before the input wave, the output is said to *lead* the input. This phase lead or lag is commonly measured in angular degrees. A complete cycle is 360°. If x is the difference between the input and output waves, then

$$\text{Phase angle in degrees} = \frac{x}{\text{period}} \times 360^\circ$$
$$= x \times \text{frequency} \times 360^\circ$$

The phase angle will be negative for a phase lag and positive for a phase lead. Thus, if the phase angle of an element is -20° at a certain frequency, the output lags the input by 20° at this frequency. If the sine-wave generator is adjusted so that the input wave has a new frequency, the component will probably have a new value of gain and of phase shift.

In presenting the results of frequency-response tests, it is not necessary to show the pairs of input and output sine waves corresponding to the various frequencies. Since sine waves are completely characterized by their amplitudes and phase angles, there is no particular advantage in reproducing the entire curves shown in Fig. 1.6. The essential facts are the gain and phase at each frequency.

These results, which constitute the frequency response, are commonly presented in graphical form by plotting two curves: gain against frequency and phase angle against frequency (Fig. 2.7, for example). This is in contrast to step-response data, which are plotted against time.

In Fig. 1.5 we showed a block diagram of an arrangement for obtaining the frequency response of an isolated component. Often it is desirable to obtain the frequency response of a component in a system. To illustrate how this may be done, refer to the control system shown in Fig. 1.7. To the control system have been added a sine-wave generator, a shutoff valve in the transmitter line as shown, a motion transmitter attached to the valve stem, and various recorders numbered 1 to 4. Recorder 4 is located near the controller, so that the transmitter-line length is essentially the same as in normal operation. The frequency response of the controller is obtained from recorders 2 and 1, the frequency response of the controller-output transmission line and control valve from recorders 3 and 2, etc. By comparing the sine waves of recorders 4 and 1 we obtain the so-called *open-loop frequency response.* Since valve *a* is closed in Fig. 1.7, the value of the controlled variable cannot influence the control-valve position, and consequently the loop is said to be open. The shutoff valve can be located in any convenient place in the loop.

In analyzing a control loop, we may use frequency responses of segments of the loop to clarify their behavior, or we may combine the frequency responses of noninteracting segments to obtain the open-loop frequency response of the system (this will be done by graphical means in Chap. 2). From the open-loop frequency response it is possible to predict the behavior of the closed loop quite accurately.

By graphical methods we can study the effects of using various controller actions and of changing controller settings. Such studies provide a better understanding of why the different control actions are used, under what conditions they are beneficial, and what happens to the behavior of the system when the settings are changed. The behavior of most controllers can be described more completely in terms of their frequency responses than in the usual control terminology.

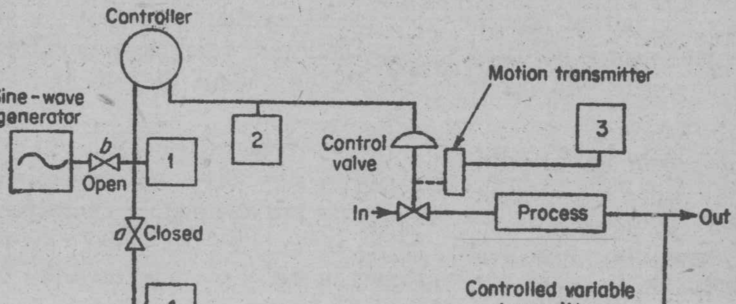


Fig. 1.7. Experimental arrangement for obtaining frequency response of components in a system. The frequency response of the process including the control valve and the transmitter is obtained from recorders 4 and 2.

Frequency response can also help us understand the behavior of processes. It shows why many processes can be adequately described by a few time lags and why the relative values of these lags are most important.

In all processes having the same relative values of lags, good controller settings can be expressed in terms of one of these lags. Consequently, when good controller settings are found for a process, it is unnecessary to determine them for other processes having the same relative values of lags. Later in this book good controller settings are given for many processes in the form of control contours. By studying these contours, we can see the effects of changes in the process.

Much of the usefulness of frequency response is due to the fact that it shows clearly and quickly the interrelations between the various components of a system and the effects of changes in physical constants on performance and stability.

As data are accumulated on components and correlated with physical properties and dimensions, it should be possible to make reasonable predictions about the dynamic behavior of these components. Then frequency-response methods can be applied to proposed systems to determine their responses to expected disturbances. If the predicted system behavior is not satisfactory, necessary changes can be made in the design stage. This is the ultimate goal of many workers in the process-control field.

Frequency response does not cure all the aches and pains of process control. Taking frequency-response data does require special equipment and is time-consuming on slow processes. In contrast, step-response data may be relatively easy to obtain.

1.5. Step Responses of System Components. Frequency-response analysis has a great advantage over step-response analysis, since it is easy to combine the frequency responses of *series-connected, noninteracting* elements. Since frequency response can be determined from step response, frequency-response analysis is often based on step-response data. This is one reason for dealing with step response in this book. Another is that process control systems may be subjected to sustained disturbances or load changes similar to step changes, and we want to know their effect on the controlled variable.

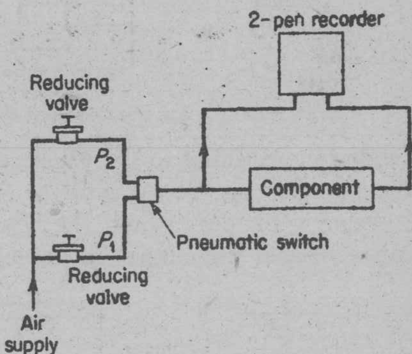


FIG. 1.8. (*a*) An arrangement for obtaining the step response of a pneumatic component.

A step response is the output versus time curve resulting from a suddenly applied, sustained input change. Figure 1.8*a* shows a possible setup for running the step responses of an isolated component. Step changes in the air pressure may be made by switching from pressure P_1 to P_2, or vice versa.

1.6. Step Response of Complete Systems. Step responses of complete systems are seldom useful for determining the characteristics of the individual components. However, as later chapters show, they are used to determine good controller settings.[1,2]* For this purpose the open-loop step response is used. There are many possible experimental arrangements, one of which is shown in Fig. 1.8*b*. With this arrangement, to eliminate the effects of the controller, it is adjusted so that its

* Superscript numbers refer to the references at the end of each chapter.

output is proportional to its input and so that a sustained change of 1 psi to its input results in a sustained change of 1 psi on the valve motor.

A fixed pressure is applied to the controller input, and the system is allowed to come to equilibrium. Then a small step change in pressure is

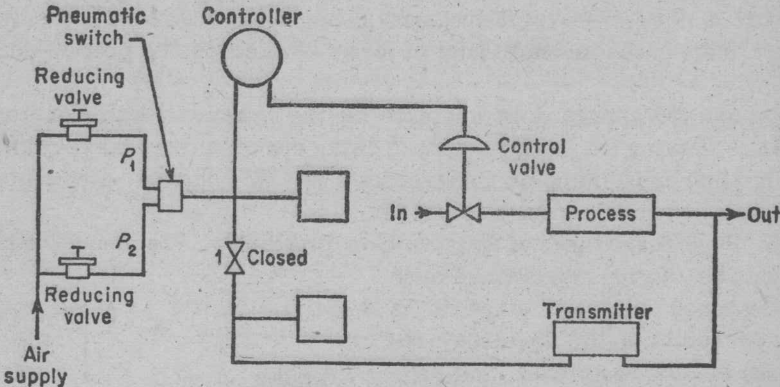


FIG. 1.8. (*b*) An arrangement for finding the open-loop step response.

made by switching to the other reducing valve. In response to the step change, the control valve opens (or closes) an appropriate amount, thus increasing (or decreasing) the flow of energy to the process. The transmitter senses the change in process variable. However, since valve 1 is

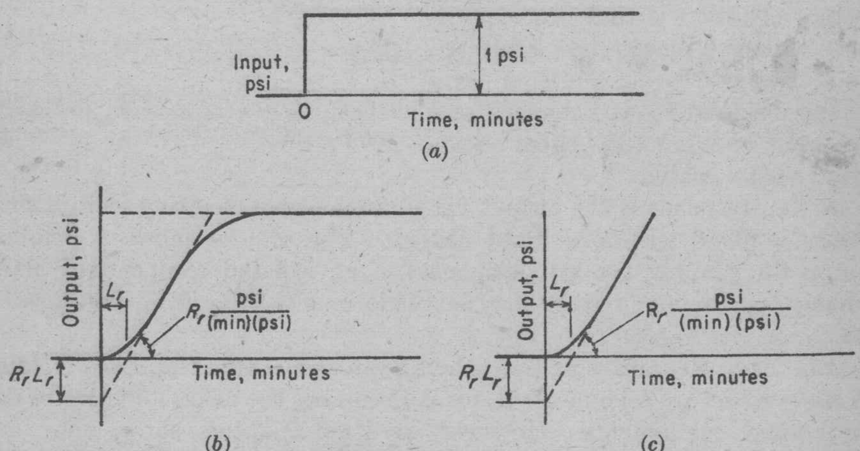


FIG. 1.9. Reaction curves: (*a*) step change in input; (*b*) process with self-regulation; (*c*) process with no self-regulation.

closed, the controller does not receive the signal from the transmitter and does not make a corresponding corrective action. The recorded output of the transmitter is commonly called the *process reaction curve*.

Typical reaction curves are drawn on Fig. 1.9. In Fig. 1.9*b* the reaction levels off at a finite value, and the process is said to have self-regulation. If the curve increases indefinitely with time, as shown in Fig. 1.9*c*, the process is said to have *no self-regulation.*

Quantities that have been used to choose good controller settings are the reaction rate R_r and the effective lag L_r. To obtain them, draw a tangent to the reaction curve where its slope is a maximum. The *reaction rate* R_r is the slope of this line divided by the magnitude of the input step change. Thus this reaction rate has units of psi (output) per minute per psi (input). The intercept on the time axis is called the *effective lag* L_r. The product R_rL_r equals the intercept on the vertical axis divided by the magnitude of the step change.

When a recording controller is used in the control system and the reaction curve is recorded on the controller chart, it is convenient to adjust the controller so that its output changes 1 psi for a 1-in. change in pen position and to measure R_r in units of inches (chart) per minute per psi (input).

In some cases it is not practical to run a reaction curve, since the disturbance would produce intolerable upsets in the process. Another drawback to the use of reaction curves is that it is not easy to get the reaction curve of a combination of components from the step-response curves of the individual components. Consequently, reaction curves are seldom used to predict the behavior of a proposed control system.

1.7. Differential Equations. This classic approach uses relations between rates of change of variables and the variables themselves. The solutions of the equations may be obtained by standard mathematical procedures.

The equations may be derived from experimental data or by physical reasoning. In either case the equations will constitute only an approximate representation of the physical system. Their solutions will describe accurately the dynamic behavior of the system only in so far as the original equations give an accurate description of the system.

This type of analysis is powerful and yields much detailed information. However, such accurate information about performance of the system may be quite unnecessary, especially in the early stages of design. For complicated systems, the solution of the differential equations may be tedious and involve lengthy computations. In that case efficiency may require the use of analogue and digital computers.

Such objections as these eventually led the engineer to develop techniques which depend less on advanced mathematics.

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