SECOND EDITION

ALGEBRA
FIRST COURSE

MAYOR and WILCOX

JOHN R. MAYOR

American Association for the Advancement of Science and The University of Maryland

MARIE S. WILCOX

Head of Mathematics Department, Thomas Carr Howe High School, Indianapolis, Indiana

SECOND EDITION

ALGEBRA FIRST COURSE

PRENTICE HALL, INC., Englewood Cliffs, N. J.

THE PRENTICE-HALL MATHEMATICS SERIES

ALGEBRA, FIRST COURSE, John R. Mayor and Marie S. Wilcox

ALGEBRA, SECOND COURSE, John R. Mayor and Marie S. Wilcox

ANALYTIC GEOMETRY AND AN INTRODUCTION TO CALCULUS, A. Clyde Schock and Bernard Warshaw

BASIC GENERAL MATHEMATICS, Margaret Joseph, Mildred Keiffer, and John R. Mayor

GOING PLACES WITH MATHEMATICS, M. Peters

MATHEMATICS: ADVANCED COURSE, W. W. Elliott, Edward R. C. Miles, and Thomas D. Reynolds

MATHEMATICS, FIRST COURSE, John A. Brown, Bona Lunn Gordey, Dorothy Sward, and John R. Mayor

MATHEMATICS, SECOND COURSE, John A. Brown, Bona Lunn Gordey, Dorothy Sward, and John R. Mayor

NEW APPLIED MATHEMATICS, Sidney J. Lasley and Myrtle F. Mudd

TODAY'S GEOMETRY, Lee R. Spiller, Franklin Frey, and David Reichgott

TRIGONOMETRY, Paul K. Rees and Dorothy Rees

UNDERSTANDING ARITHMETIC, Maytscherl W. Reckless

ALGEBRA, FIRST COURSE (2nd Edition)
John R. Mayor and Marie S. Wilcox

LIBRARY OF CONGRESS CATALOG CARD NO. 61-5162
Printed in the United States of America
02195-E

TO THE TEACHER

The teacher of mathematics knows that the first year of algebra can be exceedingly important in the mathematical development of the students under his guidance. Success in algebra will bring the student real understanding of what he has learned in arithmetic as well as appreciations, desirable attitudes, and new ways of thinking that can be of great value to him all his life. A first year of algebra, well done, is essential to progress in mathematics and in many other areas of learning. *Algebra, First Course* is intended to assist the student in achieving these ambitious goals and to make this year of study enjoyable for student and teacher alike.

The text reflects the ideas and spirit of modern mathematics whenever they can be introduced appropriately at this level. In this revised edition, a chapter on sets appears as Chapter 2, and the elementary language of sets has been used throughout. For example, the student can become familiar with solution sets of sentences, sets of points on a line, and sets of points in a half-plane. Inequalities have an important place in a number of the chapters, and the idea of absolute value is introduced. Elementary relations and functions are treated in the modern manner.

In all chapters, the ideas of algebra are developed from arithmetic. While there is no chapter on review of arithmetic, the text provides many review exercises closely related to arithmetic. These exercises at the same time develop topics in algebra and make them meaningful. Frequent references to topics in arithmetic, many applications from geometry, careful treatment of elementary trigonometry, and the introduction of some fundamental ideas from analytic geometry should enable the teacher to develop algebra as a logical whole, while laying a solid foundation for the study of high school mathematics subjects which follow.

We have tried to make it possible for student and teacher to pursue the discovery approach to the learning of algebra, and have avoided generalizations and rules that are so stressed in many texts that the student can scarcely make an observation of his own. In *Algebra, First Course*, the explanatory paragraphs, illustrative examples, and numerous exercises are designed to enable the student, with appropriate teacher leadership, to make the ideas of algebra his own ideas from the very beginning.

Suggested Readings. A feature of this text is the carefully selected supplementary reading lists at the ends of chapters. The student in first-year algebra should be encouraged to make use of the high school library. These references can also be used by the teacher as a source of enrichment materials for the better student and as a means of interesting the student who has little natural interest in mathematics. Some of the references are expository in character and will be of value to the slower student in acquiring understanding of topics considered in class.

Cumulative Reviews. The text includes not only a chapter review and chapter test but also a cumulative review at the end of each chapter, starting with Chapter 3. Thus the student has a regular opportunity to refresh his knowledge of topics studied from the beginning of the course. These review exercises can also serve as additional exercises, where needed, as topics are studied within the chapters.

Supplementary Chapter. Chapter 12 includes a variety of topics that can appropriately be studied in a first-year algebra course. Some sections of this chapter can be used at the beginning of the course, and others can provide an extension of units where the teacher feels that additional time is needed. The material is also appropriate for supplementary work on the part of the student who progresses more rapidly.

Many people have assisted in the development of this text. Our thanks go to the hundreds of students in our classes who, by their successes and difficulties, their enthusiasm, and their indifference, have helped us to know how to teach algebra and to be more certain about which topics in algebra are exciting for students and which must be made so. We are also deeply grateful to hundreds of teachers throughout the country from whom we have learned much in discussion of our common problems.

Special thanks for preparation of the manuscript are due to William B. Wilcox and Darlene Mayor. For constructive and valuable criticism of the manuscript, and for using problems in their classes, we are grateful to Ruby Wells, New Albany High School, New Albany, Indiana; John A. Brown, University of Delaware (formerly of Wisconsin High School, Madison, Wisconsin); and Mary Smuck, Thomas Carr Howe High School, Indianapolis, Indiana. In addition, we appreciate the helpful reviews of the manuscript by George E. Hawkins, Lyons Township High School, La Grange, Illinois; Charles W. Peterson, Newton High School, Newton, Massachusetts; Elizabeth Roudebush Mitchell, Seattle, Washington; and Merle M. Simpson, Lincoln, Nebraska.

John R. Mayor
Marie S. Wilcox

CONTENTS

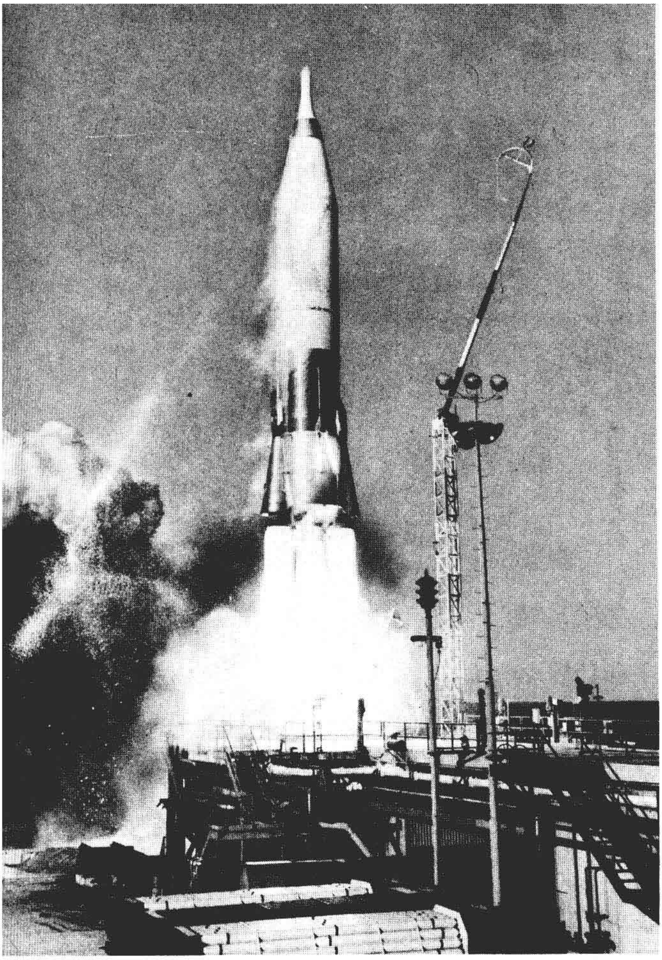
New Kinds of Numbers 1

1.1. Numbers in Arithmetic. Probably the first questions your teacher or parents asked you about numbers when you were very young started with the words, "How many?" How many pennies do you have? How many chairs are at the table? How many cats are in the picture? You answered these questions by learning to count.

Counting seems to be a very simple process to you now, but a great many years ago men could not even count. A shepherd might keep track of his flock by making a pile of pebbles, one for each sheep, or by making notches in a tree, or knots in a rope. To see if all his sheep were in the fold, he would match the objects, one pebble for one sheep. We call this a "one-to-one correspondence." Although the idea of such a correspondence is very old, it is used extensively in modern mathematics.

Later in history, men learned to count, but at first knew only very small numbers. It is said that some ancient peoples could count only to two. When they referred to a number of objects greater than two, they used a word which we translate to mean "a lot." In other ancient countries, men counted to two or three and then had a word which meant "heap" or "many" for larger numbers.

Other peoples learned to count to four and five, and later formed number systems based on these numbers. Some say that those who



An Atlas intercontinental ballistic missile starts a long flight.

counted in groups of fours used a one-to-one correspondence with the spaces between the fingers. It is also thought that four may have been chosen by some people because many animals have four legs. Five was used because of the one-to-one correspondence with the fingers on one hand.

When a tribe counted in groups of fives, they no longer had just a word for each number through five and then a word for "many." The next number after five would be a combination of the words for "five and one," then "five and two," and so on. This would be much like our teens. *Fourteen*, for instance, is "ten and four."

You could probably count to a billion now if you needed to do so, but you have learned mathematics, which makes counting unnecessary in most problems. In the early grades in arithmetic, you were asked to answer the question, "What number is the result of adding 4 and 5?" Your teacher may have asked the question by writing $4 + 5 = ?$, or even $4 + 5 = x$. In the last case, you would be expected to tell what numeral should be in the place of the x to make a true statement. You may have found the answer to this kind of problem by counting at first, but you soon found that you could group or combine numbers by a process called *addition*.

Later came questions like "What are four two's?" or "Six ten's equal what number?" You found that you could multiply as well as add the same kind of numbers that you had used in counting, that is, the natural numbers. The natural numbers with zero added form a set of numbers $\{0, 1, 2, 3, \ldots\}$ which we call the whole numbers. Whole numbers are also called integers. In arithmetic you found that the answers to problems of addition and multiplication with whole numbers were whole numbers.

In the study of division, this was not always true. When you were asked to divide a small number by a larger one, you needed to learn about a new kind of number, a *fraction*, to state your answer.

Notice that in this article we said, "What numeral should be in the place of x," not what number should be in the place of x. A number is an idea. We represent this idea by writing the symbol, 9, which is called a numeral. This 9 is not really the number in much the same way that a picture of a chair is not really a chair. However, you might point to a picture and say, "That is a chair." In mathematics we often call a numeral the number that it represents. You should realize that a difference exists.

EXERCISES

1. Using tablets and pencils, show how you could tell that you had the same number of tablets and pencils without counting either.
2. How could you tell that you had the same number of pupils and chairs in the room without counting?
3. Is there a one-to-one correspondence between the pupils and the algebra books in this room?

1.2. A New Kind of Number. The answer to the question, "What number added to 5 gives 8?", which might have been written "$5 + x = 8$," was not difficult for you in arithmetic because you understood addition. The answer in either case, of course, would be 3.

However, in arithmetic you did not learn an answer to the question, "What number added to 5 gives 2?" or "$5 + x = 2$." In algebra a new kind of number will be used to answer this question. We shall say that the answer is "negative 3." "Negative 3" is written: -3. We can say that $5 + (-3) = 2$.

Similarly, we shall make use of other negative numbers, one to correspond to each of the numbers we used in arithmetic, except zero. This will give meaning to expressions like:

$3 + (-1) = 2$	$6 + (-5) = 1$
$2 + (-2) = 0$	$10 + (-6) = 4$
$11 + (-3) = 8$	$12 + (-7) = 5$
$25 + (-4) = 21$	$8 + (-8) = 0$
$\frac{3}{5} + (-\frac{2}{5}) = \frac{1}{5}$	$4\frac{2}{3} + (-1\frac{1}{3}) = 3\frac{1}{3}$

These new numbers, (-1), (-2), (-3), (-4), $(-\frac{2}{5})$, and so forth, are called *negative* numbers. When negative numbers are used, it is desirable to write the number 5, for example, as $+5$, and to read this expression as "positive 5." It is generally understood, however, that if a number is written without a sign in situations in which positive and negative numbers are being used, the number is positive. The first two lines of the above expressions could be written:

$+3 + (-1) = +2$	$+6 + (-5) = +1$
$+2 + (-2) = 0$	$+10 + (-6) = +4$

With these new numbers, in algebra we shall always have an answer to questions like "What number added to $+17$ gives $+4$?" or "If $+29 + x = -13$, what number does x represent?"

Because $5 + 7 = 7 + 5$, for example, we say that the order of the numbers to be added makes no difference. This property is called the *commutative property of addition.* We accept the commutative property of addition with negative numbers as well as with positive numbers and zero, so that we can say

$$+7 + (-5) = (-5) + 7 = +2$$
$$+11 + (-2) = (-2) + 11 = +9$$
or
$$+3 + (0) = (0) + 3 = +3$$

While it would be correct to write both positive and negative numbers with parentheses, as

$$(-5) + (+7) = +2,$$

we can see that there would be no advantage in doing this. We shall usually not use the parentheses, ( ), with negative numbers. The parentheses have been used here mainly because negative numbers are new to us.

EXERCISES

1. Read the following numbers, using the words *positive* and *negative:* $+5$, -6, -3, 0, $+10$, -2, -27, $+153$, $+18$, -16

2. Answer the following questions:

   (a) What number added to $+14$ gives $+10$? *Answer:* -4
   (b) What number added to $+6$ gives $+3$?
   (c) What number added to $+16$ gives $+2$?
   (d) What number added to $+7$ gives 0?
   (e) What number added to $+117$ gives 0?
   (f) What number added to $+50$ gives $+25$?
   (g) What number added to $+1{,}000$ gives $+100$?
   (h) What number added to $+73$ gives $+61$?
   (i) What number added to $+81$ gives $+39$?
   (j) What number added to $+\frac{5}{4}$ gives $+\frac{3}{4}$?

3. The letter x may be replaced by a numeral in each of the following expressions. This number is called the *value* of x. What is the value of x in each expression?

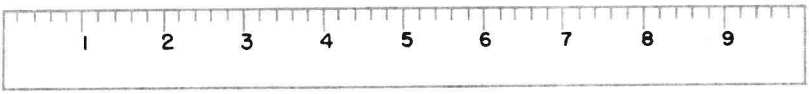
   (a) $+8 + x = +1$
   (b) $+7 + x = +6$
   (c) $+8 + x = 0$
   (d) $+5 + x = 3$

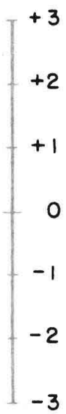
4. What number added to each of the following numbers will give $+7$?
   $+8$, $+11$, $+5$, $+10$, $+15$, $+14$, $+6$, $+29$

(*Hint:* Write your answer in the form: $+8 + (-1) = +7$.)

5. Add each of the following numbers to the numbers in Exercise 4 and state the results: (a) 0 (b) -4 (c) $+5$

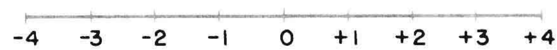
1.3. The Number Line. You have seen points on a scale numbered. For instance, on an ordinary twelve-inch ruler, you will note numbers at points with equal distances between them.





On many thermometers there are a zero point and negative as well as positive numbers. There are equal spaces between the numbers. We speak of a temperature of $-2°$. We mean by "-2" a point two of these equal spaces below the zero point.

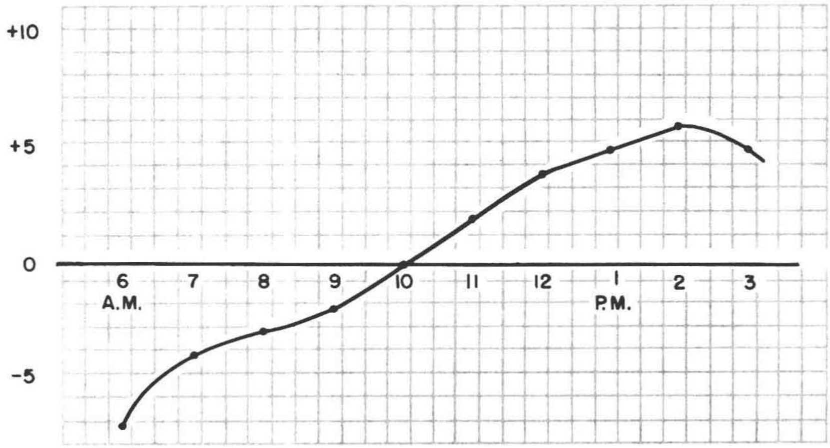
In mathematics we may place points on a line to make a scale, using both positive and negative numbers and zero. Such a line is called the number line. Below is a horizontal line with a scale of this kind, and to the left is a vertical line.



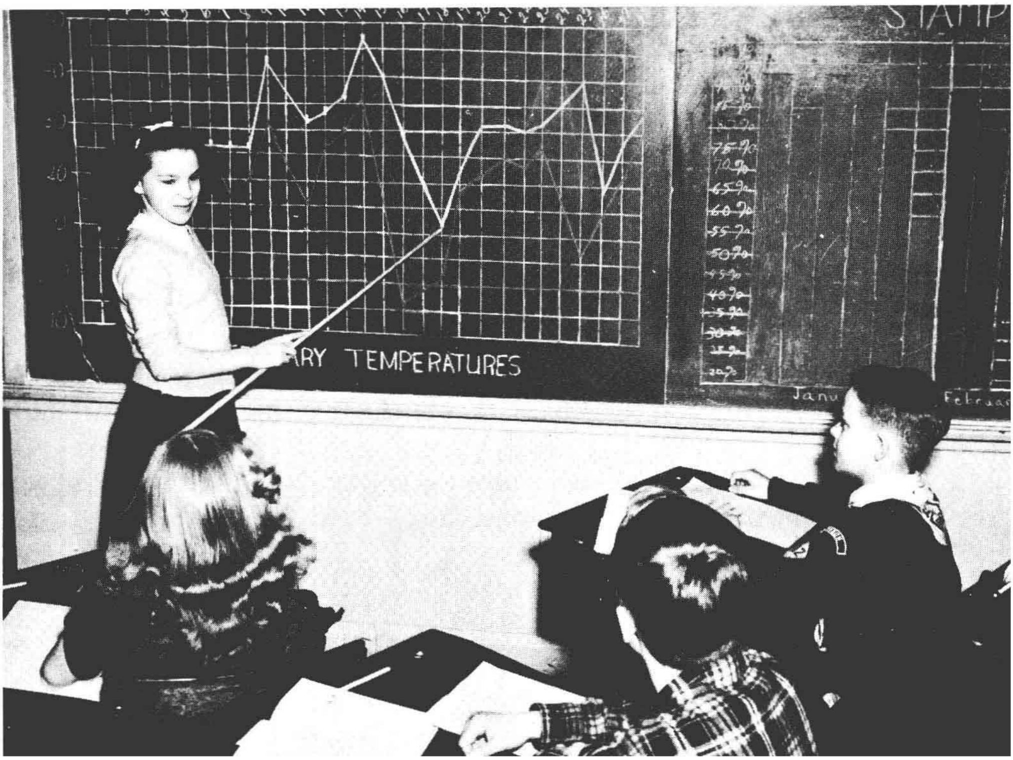
On the horizontal scale, the positive numbers are to the right and the negative numbers are to the left. On the vertical scale, the positive ($+$) direction is up and the negative ($-$) direction is down.

If we would extend each of the lines in our figure indefinitely in both directions, and would continue to mark off equal distances on each line, we could set up a one-to-one correspondence between the integers (positive, negative, and zero) and the marks on the line. Each integer is called the coordinate of the point with which it is associated.

The number line is useful in making certain types of graphs. Below is a graph showing the temperature each hour for certain hours of the day on a cold day. The vertical scale includes both positive and negative numbers.



A student demonstrates a temperature graph.



EXERCISES

1. The temperature at 1 P.M., in the graph on page 7, was $+5°$. Write with positive and negative numbers the temperature readings for each hour as shown in the graph.
2. Draw a horizontal and a vertical scale as shown in the graph for Exercise 1, and make a graph to show the following temperature readings at the given hours on a certain day.

Hour	*Temperature*
6 A.M.	$-15°$
7	-10
8	-4
9	-1
10	0
11	$+8$

Hour	*Temperature*
12 noon	$+11°$
1 P.M.	$+16$
2	$+20$
3	$+22$
4	$+20$
5	$+18$

3. Make a graph as in Exercise 2 to show the following temperature readings.

Hour	*Temperature*
8 A.M.	$-8°$
9	-7
10	-7
11	-6
12 noon	-2

Hour	*Temperature*
1 P.M.	$0°$
2	$+3$
3	$+6$
4	$+7$
5	$+5$

1.4. Other Applications of Positive and Negative Numbers. A report shows that the change in the cost of living during a certain period was -3%. The 3% shows the amount of the change, and the negative sign shows the kind of change or the direction of the change. The cost has gone down 3% since the beginning of the period which was being considered.

A temperature reading of $-5°$ means that the temperature is 5 degrees below a fixed mark on the thermometer. The fixed mark is $0°$.

There are many other uses for positive and negative numbers. Some of them are illustrated in the following exercises.