

**Nikita Borisov
Philippe Golle (Eds.)**

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Privacy Enhancing Technologies

**7th International Symposium, PET 2007
Ottawa, Canada, June 2007
Revised Selected Papers**



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Nikita Borisov Philippe Golle (Eds.)

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Foreword

The PET community has grown in size considerably since the first PET workshop was held in 2000. With this growth came an increase in the number and quality of submissions. PET has become a premier venue for publishing original research on privacy enhancing technologies, and the current acceptance ratio puts PET in the same league as other highly selective security and privacy venues. To appropriately reflect this evolution, the PET workshop is changing its name to the *Privacy Enhancing Technologies Symposium*.

PET 2007 was held at the University of Ottawa, Canada, on June 20–22, 2007. We received 84 full-paper submissions, of which 16 were selected for presentation at the symposium. PET also included a keynote address and two panel discussions. PET was once again colocated with the IAVoSS Workshop on Trustworthy Elections (WOTE 2007), with a full day of plenary sessions. All participants were free to attend sessions from both events.

The program chairs would like to thank, first of all, the authors, speakers, and panelists for their contribution to the content of the workshop. We would also like to thank the program committee for their hard work of a month of reviews and two more weeks of intense discussions, helping to ensure a program of high scientific quality. As well, we would like to acknowledge the contribution of the external reviewers, who assisted the program committee with the reviews. A special thanks is due to the designers of the Websubmission and Webreview software at K.U. Leuven for allowing us to use their software to help with the selection process, and to Thomas Herlea for his help in getting the software up and running.

Our general chair, Carlisle Adams, did an outstanding job taking care of the local arrangements and making sure the symposium ran smoothly. We also would like to thank Jeremy Clark for designing and maintaining the PET 2007 Website. We are very grateful to Josh Benaloh, the chair of WOTE 2007, for his help in coordinating the two events. Finally, PET 2007 was made possible, and more affordable, thanks to our sponsors: Microsoft, ORNEC, Bell Privacy Centre of Excellence, PGP Corporation, and Google. We are extremely grateful for their generous support.

The Award for Outstanding Research in Privacy Enhancing Technologies was given this year to Stephen C. Bono, Matthew Green, Adam Stubblefield, Ari Juels, Aviel D. Rubin, and Michael Szydlo for their paper “Security Analysis of a Cryptographically-Enabled RFID Device.” The award is sponsored by Microsoft and by the Office of the Information and Privacy Commissioner of Ontario and the winners were selected through an independent prize committee headed by George Danezis to whom we are thankful.

July 2007

Nikita Borisov
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Attacking Unlinkability: The Importance of Context

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Abstract. A system that protects the unlinkability of certain data items (e. g. identifiers of communication partners, messages, pseudonyms, transactions, votes) does not leak information that would enable an adversary to link these items. The adversary could, however, take advantage of hints from the context in which the system operates. In this paper, we introduce a new metric that enables one to quantify the (un)linkability of the data items and, based on this, we consider the effect of some simple contextual hints.

1 Introduction

A number of privacy-preserving systems, such as mix networks, anonymous credential systems, and secret voting schemes, protect the unlinkability of certain data items of interest. Mix networks, in particular, protect the unlinkability of the messages that enter the network with respect to their recipients, the messages that leave the network with respect to their senders, and, hence, the identifiers of communicating parties with respect to communication sessions. Since their introduction [9], a number of different mix network variants have been proposed (see, for example, [4,19,26,33,34]), some of which have also been implemented and deployed. Anonymous credentials, on the other hand, protect the unlinkability of the pseudonyms and the transactions with respect to the users they correspond to. Since their introduction into the digital world [10], a number of anonymous credential systems have been proposed (see, for example, [7,8,11,12,13,14,29,32,38]). Secret voting schemes protect the unlinkability of votes with respect to the users who cast them. Such schemes have evolved from ostracism [24] to sophisticated cryptosystems; for an overview of the current state of the art the reader is referred to [1].

The problem of analysing how well the above types of system protect unlinkability has received some attention during recent years. The focus of most works is, however, on mix networks (see, for example, [2,15,16,25,27,30]). This is not surprising since mix networks provide the basis for anonymous communication

and are, as such, necessary for preserving privacy in a number of settings, including the setting of anonymous credentials [17] and, sometimes, the setting of voting systems (see, for example, [6]).

An adversary that wishes to link the protected items may use information that is leaked by the system during its operation, or hints from the environment in which the system operates. In contrast to existing literature, the focus of this paper is on the latter. That is, we study a number of simple contextual hints and their effect on unlinkability. Our results apply to *all* types of unlinkability-protecting system, including mix networks, anonymous credentials, and secret voting schemes. The rest of the paper is organised as follows. Section 2 introduces the metric for unlinkability that is used throughout the paper. Section 3 examines seven different types of hint and their effect on unlinkability. Finally, Section 4 concludes.

2 Measuring Unlinkability

Consider a set of elements A and a partition $\pi \vdash A$ of that set. Note that we do not distinguish between π and the equivalence relation it defines. In the sequel, we write $a_1 \equiv_\pi a_2$ if the elements $a_1, a_2 \in A$ lie in the same equivalence class of π , and $a_1 \not\equiv_\pi a_2$ otherwise. Let $\tau \vdash A$ denote a ‘target’ partition, chosen uniformly at random. We use entropy as a metric for unlinkability. That is, the unlinkability of the elements in a set A against an adversary \mathcal{A} is defined as

$$\mathcal{U}_A(\mathcal{A}) = - \sum_{\pi \in \Pi} \Pr(\pi = \tau) \log_2(\Pr(\pi = \tau)),$$

where $\Pi = \{P : P \vdash A\}$ denotes the set of partitions of A and $\Pr(\pi = \tau)$ denotes, in \mathcal{A} ’s view, the probability that π is the target partition τ . We further define the *degree* of unlinkability of the elements in A against an adversary \mathcal{A}_H with access to a hint H about τ as

$$\mathcal{D}_A(\mathcal{A}_H) = \frac{\mathcal{U}_A(\mathcal{A}_H)}{\mathcal{U}_A(\mathcal{A}_\emptyset)},$$

where \mathcal{A}_\emptyset denotes the adversary without any hints. That is, \mathcal{A}_\emptyset knows A but has no information about τ . The set of candidate partitions for \mathcal{A}_\emptyset is therefore $\Pi_A(\mathcal{A}_\emptyset) = \Pi$, i.e. the set of all partitions of A . The number $|\Pi_A(\mathcal{A}_\emptyset)| = B_{|A|}$ of such partitions, a Bell number [3,35], is given by the recursive formula

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k \tag{1}$$

where $B_0 = 1$.¹ Since τ is chosen uniformly at random, the unlinkability of the elements in A is therefore at its maximum, i.e. $\mathcal{U}_A(\mathcal{A}_\emptyset) = \log_2(B_{|A|})$ bits. This

¹ The first few Bell numbers are 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147.

is the best case from a privacy point of view: all partitions of A are equally likely to be the target partition τ .

Remark 1: In the setting of unlinkability-protecting systems, the goal of the adversary is to identify a target partition from an ‘anonymity set’ of candidate partitions. The fact that the information-theoretic metric we use for unlinkability is identical to the metric introduced for anonymity in [18,36], is therefore natural.

Remark 2: \mathcal{U}_A is a measure of the information that is contained in the probability distribution that the adversary assigns to the set of all partitions of A . Since we assume that τ is selected uniformly at random, this distribution is, a priori, uniform. However, a hint may enable the adversary to change his view such that, a posteriori, some partitions are more likely than others. The hints we consider in this paper enable the adversary to exclude a number of candidate partitions (i.e. to reduce the size of the ‘anonymity set’) while the remaining partitions remain equally likely.

Example: Consider an anonymous help line where a clerk offers advice over the telephone. Suppose that, one day, the clerk receives four calls, denoted $A = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$. Without any additional information, all $B_4 = 15$ partitions of A constitute valid ways to link these calls. Since without any additional information all these partitions are equally likely, the unlinkability of the calls is, in this case, $\log_2(15) \simeq 3.9$ bits, and the degree of unlinkability is $\log_2(15)/\log_2(15) = 1$.

The clerk, however, has some additional information: he realised that the calls λ_1 and λ_2 were made by men, and that the calls λ_3 and λ_4 by women (however, the clerk does not know whether or not the same person called twice). This hint effectively rules out all partitions where λ_1 or λ_2 appears in the same equivalence class as λ_2 or λ_4 . In particular, only four partitions remain valid, namely $\{(\lambda_1, \lambda_2), (\lambda_3, \lambda_4)\}$, $\{(\lambda_1, \lambda_2), (\lambda_3), (\lambda_4)\}$, $\{(\lambda_1), (\lambda_2), (\lambda_3, \lambda_4)\}$, and $\{(\lambda_1), (\lambda_2), (\lambda_3), (\lambda_4)\}$. Since these four partitions are equally likely, the unlinkability of the calls is, in this case, $\log_2(4) = 2$ bits, and the degree of unlinkability is $\log_2(2)/\log_2(15) \simeq 0.52$.

3 The Importance of Context

In this section, we examine seven types of hint that an adversary may obtain from the operational context of the system. In particular, we examine hints that reveal to the adversary (a) the number of equivalence classes in τ , (b) the cardinality of equivalence classes in τ , (c) the fact that all equivalence classes in τ have a given cardinality, (d) a ‘reference partition’ the equivalence classes of which have exactly one representative in each equivalence class in τ , (e) a set of element pairs that are equivalent in τ , (f) a set of element pairs that are not equivalent in τ , and (g) a combination of (e) and (f).

3.1 The Number of Equivalence Classes

Consider an adversary \mathcal{A}_{H_1} with a hint $H_1 = (\alpha)$, where $\alpha \in \mathbb{N}$ and $1 \leq \alpha \leq |A|$, that reveals how many equivalence classes τ has. \mathcal{A}_{H_1} can restrict its attention

to $\Pi_A(\mathcal{A}_{H_1}) = \{P : P \vdash A, |P| = \alpha\}$, i.e. the partitions that divide A into exactly α equivalence classes. The number of such partitions, which is a Stirling number of the second kind [22], is given by

$$|\Pi_A(\mathcal{A}_{H_1})| = \frac{1}{\alpha!} \sum_{k=0}^{\alpha} (-1)^k \binom{\alpha}{k} (\alpha - k)^{|A|}.$$

Since τ is chosen uniformly at random, the unlinkability of the elements in A is $\mathcal{U}_A(\mathcal{A}_{H_1}) = \log_2(|\Pi_A(\mathcal{A}_{H_1})|)$ bits. Figure 1 shows the degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_1})$ as a function of $|A|$.

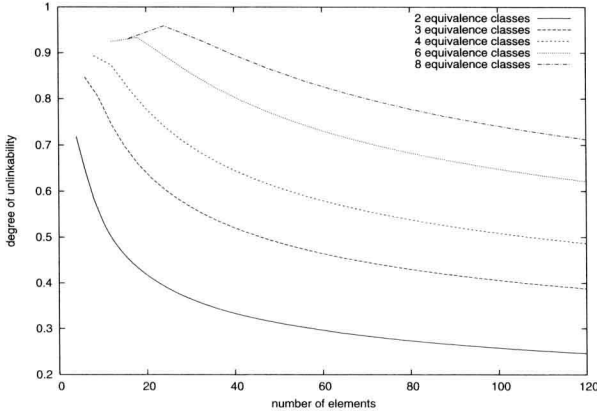


Fig. 1. Degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_1})$ of elements in a set A as a function of $|A|$, if it is known that they must be divided into α equivalence classes

How to obtain this hint: The number α typically is the number of users in a system. In the setting of mix networks, this number may be known to the operator of the network if users are required to register themselves or pay a fee. Otherwise, obtaining such a hint may be tricky due to the possibility of sybil attacks [20]. Whether or not it is straightforward to obtain this hint in the setting of anonymous credentials depends on the application. In the case of cash, for example, the financial institution is very likely to know how many users participate in the system. Similarly, in the case of demographic or personal credentials (such as age certificates or driving licences), the issuing authority is also likely to know the number of users in the system. In the setting of secret voting, there exist multiple ways to obtain the number of voters. The number of cast ballots, for example, may be conclusive about the number of voters.

3.2 The Cardinality of Equivalence Classes

Consider an adversary \mathcal{A}_{H_2} with a hint $H_2 = (\beta_1, \beta_2, \dots, \beta_\alpha)$, where $\sum_{i=1}^{\alpha} \beta_i = |A|$ and $1 < \alpha < |A|$, that reveals the sizes of the equivalence classes in τ . That is,

if $\tau = \{T_1, T_2, \dots, T_\alpha\} \vdash A$, H_2 reveals that $|T_1| = \beta_1$, $|T_2| = \beta_2$, and so on. \mathcal{A}_{H_2} can restrict its attention to $\Pi_A(\mathcal{A}_{H_2}) = \{P : P = \{T_1, T_2, \dots, T_\alpha\} \vdash A, \forall 1 \leq i \leq \alpha, |T_i| = \beta_i\}$, i. e. the partitions that divide A into exactly α equivalence classes with cardinalities $\beta_1, \beta_2, \dots, \beta_\alpha$. The number of such partitions is given by

$$|\Pi_A(\mathcal{A}_{H_2})| = \frac{|A|!}{\prod_{i=1}^\alpha (\beta_i!) \prod_{i=1}^{|A|} (\gamma_i!)} \quad (2)$$

where, for all $1 \leq i \leq |A|$, $\gamma_i = |\{\beta \in H_2 : \beta = i\}|$ (for a proof see Appendix B). That is, γ_i is the number of equivalence classes in τ that have cardinality i . Since τ is chosen uniformly at random, the unlinkability of the elements in A is $\mathcal{U}_A(\mathcal{A}_{H_2}) = \log_2(|\Pi_A(\mathcal{A}_{H_2})|)$ bits. It is perhaps worth noting that there exist hints of type H_2 which do not reveal any information as to whether any two given elements are equivalent or not. This is in contrast to what is claimed in [37] (see Appendix A).

As a special case, consider an adversary \mathcal{A}_{H_3} with a hint $H_3 = (\alpha)$, where $\alpha \in \mathbb{N}$ divides $|A|$, that reveals the fact that τ has α equivalence classes of the same cardinality $|A|/\alpha$. \mathcal{A}_{H_3} can restrict its attention to $\Pi_A(\mathcal{A}_{H_3}) = \{P : P \vdash A, \forall p \in P, |p| = |A|/\alpha\}$, i. e. the partitions that divide A into exactly α equivalence classes of equal cardinality $|A|/\alpha$. The number of such partitions is given by

$$|\Pi_A(\mathcal{A}_{H_3})| = \frac{|A|!}{\alpha! (|A|/\alpha)!^\alpha} \quad (3)$$

(for a proof see Appendix B). Since τ is chosen uniformly at random, the unlinkability of the elements in A is $\mathcal{U}_A(\mathcal{A}_{H_3}) = \log_2(|\Pi_A(\mathcal{A}_{H_3})|)$ bits. Figure 2 shows the degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_3})$ as a function of $|A|$.

How to obtain this hint:² In the setting of mix networks, this hint may be obtained if it is known how many messages each user sends in each session. In the setting of anonymous credentials, it is possible to obtain this hint if it is known how many pseudonyms each user has. In the setting of secret voting, this hint may be obtained if it is known how many ballots each user has cast.

3.3 A Reference Partition

Consider an adversary \mathcal{A}_{H_4} with a hint $H_4 = (\rho)$, consisting of a ‘reference partition’ $\rho = \{R_1, R_2, \dots, R_{|A|/\alpha}\} \vdash A$ such that, for all $1 \leq i \leq |A|/\alpha$, $|R_i| = \alpha$ (note that α divides $|A|$), and that reveals the fact that each of the equivalence classes of τ contains exactly one element from R_i . \mathcal{A}_{H_4} can restrict its attention to $\Pi_A(\mathcal{A}_{H_4}) = \{P : P \vdash A, P \text{ is a transversal of } \rho\}$, i. e. the partitions that divide A into α equivalence classes of equal cardinality $|A|/\alpha$, where each class contains exactly one element from each of $R_1, R_2, \dots, R_{|A|/\alpha}$. The number of such partitions is given by

$$|\Pi_A(\mathcal{A}_{H_4})| = (\alpha!)^{(|A|/\alpha)-1} \quad (4)$$

² This paragraph refers to hints of both type H_2 and H_3 .

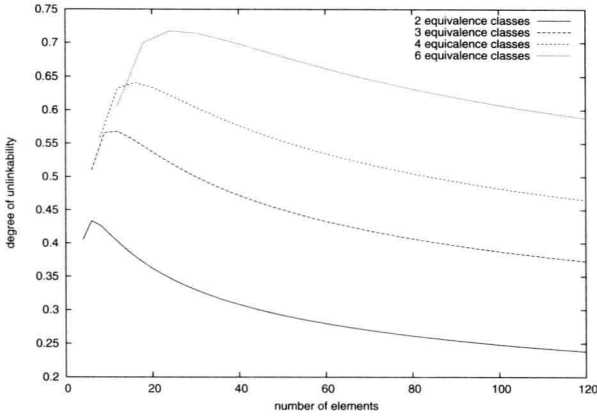


Fig. 2. Degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_3})$ of elements in a set A as a function of $|A|$, if it is known that they must be divided into α equivalence classes of equal cardinality $|A|/\alpha$

(for a proof see Appendix C). Since τ is chosen uniformly at random, the unlinkability of the elements in A is $\mathcal{U}_A(\mathcal{A}_{H_4}) = \log_2(|\Pi_A(\mathcal{A}_{H_4})|)$ bits. Figure 3 shows the degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_4})$ as a function of $|A|$.

How to obtain this hint: In the setting of mix networks this hint may be obtained if each of the α users sends exactly one message through the network in β communication sessions. An adversary that wishes to divide the set of messages that leave the network (there are $\alpha \cdot \beta$ of them) into α subsets of equal cardinality β , such that each subset contains the messages sent by a single user,

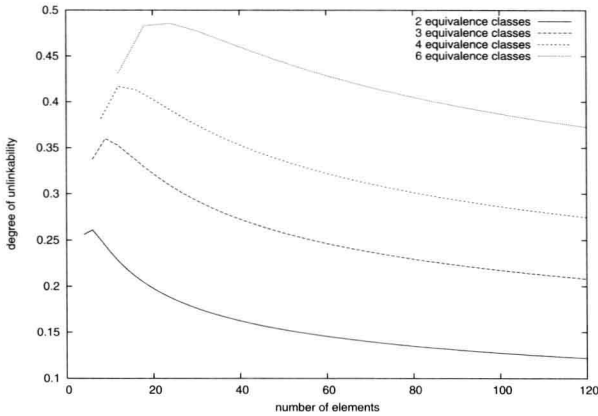


Fig. 3. Degree of unlinkability $\mathcal{D}_A(\mathcal{A}_{H_4})$ of elements in a set A as a function of $|A|$, if it is known that they must be divided into α equivalence classes of equal cardinality $|A|/\alpha$, such that each class contains exactly one element from each equivalence class of a given partition