

# HIGH ENERGY PHYSICS 1985

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MARK J BOWICK  
FEZA GÜRSEY  
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**MARK J BOWICK**  
**FEZA GÜRSEY**

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## QUANTUM COSMOLOGY

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## 1. INTRODUCTION

The traditional enterprise of cosmology has been to construct a model of the universe which agrees with our observations on the largest scales and which, when evolved backwards according to the laws of fundamental physics, gives a consistent historical picture of how the universe came to be the way it is today. Our observations tell us that the universe consists of matter and radiation. The matter that we see in galaxies is distributed roughly homogeneously and isotropically on the largest scales. The cosmic background radiation, in which we see a picture of the universe at an early stage, is remarkably isotropic. As a first approximation, we are thus led naturally to the Friedman-Robertson-Walker cosmological models in which the symmetries of homogeneity and isotropy are enforced exactly. Evolved backward in time using Einstein's gravitational theory and the laws of microscopic physics these models provide a consistent history of the universe. Among other things, they describe the evolution of the background radiation, the origin of the primordial elements, the evolution of the fluctuations which became the galaxies, and perhaps the origin of the baryons. The initial condition implied by the extrapolation is an early state in which the matter is in thermal equilibrium with high temperature and density, distributed homogeneously and isotropically but containing the seeds of condensations later to become galaxies.

The Friedman-Robertson-Walker models are successful and they are simple. Their success and simplicity raise the issue of why does the universe have the properties it does? Can we explain the Friedman-Robertson-Walker models? This is a very different kind of issue from the essentially descriptive questions traditionally asked in cosmology. In effect one is asking for a theory of initial conditions. These lectures are about one approach to this problem: the search for a theory of initial conditions in the application of quantum gravity to cosmology - in two words they are about quantum cosmology.

The lectures are not intended as a review of all models of the universe which involve quantum mechanics or even of those which deal directly with the issue of initial conditions. The subject, although already large, is not yet connected enough to make such a review feasible in the space available.<sup>1)</sup> Rather we shall explore a specific proposal for the quantum state of the universe developed by Stephen Hawking and his collaborators.<sup>2)</sup> In the process we shall be able to review much about the general issues.

To state the proposal for the quantum state of the universe we shall need some of the framework of quantum gravity. This we describe in Section 3. We shall develop the idea and compare its predictions with observations in Section 6, but, in order to know where we are going, we shall first review the observations we hope to explain in Section 2.

## 2. THE UNIVERSE TODAY AND THE PROBLEM OF ITS INITIAL CONDITIONS

### 2.1 Observations

The variety and detail of the observations now available which bear on the structure of our universe in the large is one of the most impressive achievements of contemporary astronomy. The relationships between these observations are complex and deriving an understanding of the universe in the large from them is a complex theoretical story. Emerging from this analysis, however, is a picture of striking simplicity on the largest scales. In this section we shall summarize this picture in a few "observational facts" and briefly indicate the nature of the supporting evidence for each one. These are the facts one seeks to explain in a theory of initial conditions. We can only adumbrate the arguments for these observations here and cannot hope to give a complete list of references to them. For greater detail and references the reader is encouraged to consult the many reviews of this subject.<sup>3)</sup>

#### Fact (1). Spacetime is four dimensional with Euclidean topology

This is so built into our fundamental physics that we usually take it as granted. It is important to remember however that all aspects of geometry have an observational basis.

#### Fact (2). The universe is large, old and getting bigger

At moderate distances galaxies recede from each other according to Hubble's law

$$\left( \begin{array}{c} \text{velocity} \\ \text{of recession} \end{array} \right) = H_0 \text{ (distance apart) } , \quad (2.1)$$

where  $H_0$  is somewhere between 40 and 100 (km/sec)/Mpc.



[A pc is  $3.09 \times 10^{18}$  cm. A Mpc is  $10^6$  pc.] Inverted, Hubble's law gives us a connection between distance and redshift. Since  $H_0$  is uncertain distances are often quoted as a multiple of  $h^{-1}$  where  $h$  is  $H/[100(\text{km/sec})/\text{Mpc}]$ . The background radiation originates at distances of order  $c/H_0 \sim 3000 h^{-1}$  Mpc (the Hubble distance) from us and at times of order  $1/H_0 \sim 10^{10}$  yrs. (the Hubble time) ago. These are the largest scales which are directly accessible to observation today. It is perhaps a trivial observation, but these are not the scales of elementary particle physics.

Fact (3). The universe contains matter and radiation distributed homogeneously and isotropically on the largest scales

Direct evidence for the homogeneity of the universe is hard to come by. Ideally one would like to make a three dimensional map showing the distribution of galaxies and this involves measuring distances. Such surveys have been made but only out to limited distances ( $\sim 100$  Mpc). The test that probes homogeneity on the largest scales is the oldest - counts of galaxies vs. limiting flux. One can easily show that if there are several populations of objects distributed uniformly in flat three dimensional space, then the number of objects counted with flux  $f$  greater than some limiting flux,  $f_0$ , should vary with  $f_0$  as

$$N(f > f_0) \propto f_0^{-3/2} \quad , \quad (2.2)$$

with calculable corrections for spatial curvature. Modern surveys<sup>4)</sup> which probe out to depths comparable to the Hubble distance yield approximate agreement with this law.

If we accept the Copernican principle that we are not at a preferred position in the universe, and there is no evidence that we are, then evidence for isotropy becomes

evidence for homogeneity. Evidence for the isotropy of the universe comes from angular surveys of its major constituents.

Most directly there are the galaxies. A plot on the sky of the Shane-Wirtanen catalog of the  $10^6$  galaxies contained in an effective depth of several hundred Mpc is as close as we can come to a picture of the universe today.<sup>5)</sup> It is roughly isotropic on large scales but clearly exhibits structure. A quantitative measure of the isotropy is the galaxy-galaxy angular correlation function. This is the excess probability for finding a second galaxy at some fixed angle from any given one. It is conveniently quoted in terms of a spatial correlation function  $\xi(r)$  which would produce the same result assuming homogeneity.  $\xi(r)$  is about unity at  $7 h^{-1}$  Mpc and decreases to a few 1/10ths by  $20 h^{-1}$  Mpc. The galaxy distribution is thus essentially isotropic at large scales.

Radio sources are distributed across the sky in an essentially uniform way. The diffuse X-ray background is isotropic to a few percent at angular scales of  $5^\circ$ . Since a significant fraction of this radiation comes from distant quasars this becomes a test of isotropy on large scales.

The temperature distribution of the  $2.7^\circ\text{K}$  cosmic background radiation provides the most accurate test of the isotropy of the universe and the one which probes this feature on the largest scales and therefore the earliest times. The anisotropy in the temperature,  $\Delta T/T$ , has been well measured on a number of different angular scales.<sup>6)</sup> There is a purely dipole anisotropy which is attributable to the motion of our solar system with respect to the rest frame of the radiation. If this is subtracted out no anisotropy has been detected in the residual component on any scale.<sup>7,8)</sup> The current best limit on the quadrupole anisotropy, for example, is  $(\Delta T/T)_{\text{quadrupole}} \lesssim 7 \times 10^{-5}$ .

Figure 1 shows this graphically. It is a map of the sky with the dipole anisotropy subtracted out, produced by Lubin and Villela<sup>8)</sup> at 3mm. where the background radiation dominates all other sources. It is thus in effect a snapshot of the universe at an age of  $10^5$  years when the background radiation was emitted. It is essentially featureless.

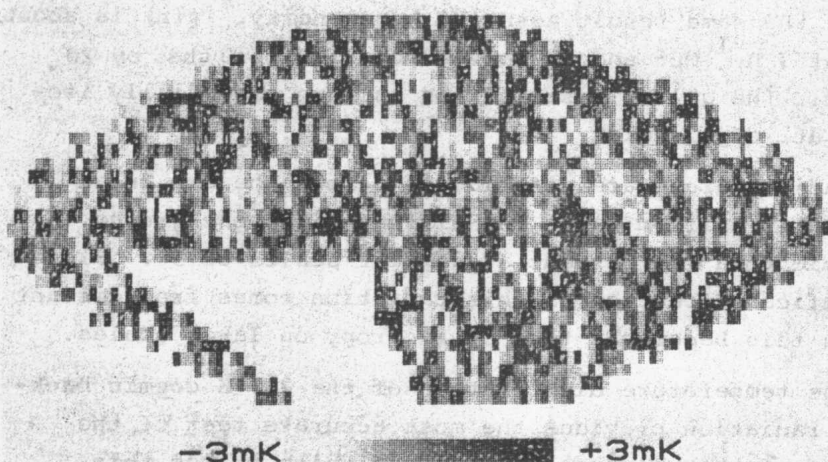


Figure 1. The sky at 3mm. This figure shows the map of the sky at 3mm. observed by Lubin and Villela<sup>8)</sup> with the dipole anisotropy removed. The shading in the rectangles, each a few degrees on a side, indicates the temperature deviation from the mean. Since the background radiation is the dominant source of radiation at this wavelength, this is essentially a picture of the universe 300,000 years after the big bang and it is remarkably isotropic.

The observed approximate homogeneity and isotropy on large scales suggest that the Friedman-Robertson-Walker models, in which these symmetries are enforced exactly, should give a good first approximation to the dynamics of the universe. The metric of a spacetime geometry with homogeneous and isotropic spatial sections can, in suitable coordinates, be described by the line element

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2 \right], \quad (2.3)$$

where  $d\Omega_2^2$  is the metric on the unit two sphere. The spatial geometry is open with negative curvature if  $k = -1$ , open and flat if  $k = 0$  and closed with the geometry of a three-sphere if  $k = +1$ .

All the geometrical information about the dynamics of the universe is contained in the scale factor  $a(t)$ . Einstein's equation for perfect fluid matter with energy density  $\rho$  and cosmological constant  $\Lambda$  implies

$$\left( \frac{\dot{a}}{a} \right)^2 = -\frac{k}{2} + \frac{\Lambda}{3} + \frac{8\pi G}{3} \rho. \quad (2.4)$$

This equation plus the constitutive relations of the matter are enough to extrapolate the dynamics of the universe forward and backward in time given the constants  $k$  and  $\Lambda$  and the present values of  $a$  and  $\rho$  or equivalently the present values of  $\rho$  and  $\dot{a}/a$ . The present value of  $\dot{a}/a$  is the Hubble constant  $H_0$ . It is uncertain because the extragalactic distance scale is uncertain, but most determinations fall in the range 40 - 100 (km/sec)/Mpc.

Eq. (2.2) shows that, were  $\Lambda = 0$ , the density today would have to be greater than the critical value

$$\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G}, \quad (2.5)$$

to have a closed ( $k=+1$ ) universe. It has become conventional to quote  $\rho$  and even  $\Lambda$  in terms of their dimensionless ratios to this critical density. For example, the present density  $\rho_0$  defines the ratio  $\Omega_0 = \rho_0/\rho_{\text{crit}}$  and the cosmological constant the ratio  $\Omega_\Lambda = (\Lambda/8\pi G)/\rho_{\text{crit}}$ . We now briefly describe the observational evidence for each of these quantities.

The density in luminous matter, found essentially by counting galaxies whose redshifts and therefore distances are known, corresponds to an  $\Omega$  of about .01. There is considerable evidence, however, that the universe contains significant amounts of non-luminous matter. The rotational velocity of a galaxy at a given radius from its center can be used to estimate the mass interior to that radius. These velocities do not fall with radius as would be predicted from the density of luminous matter in galaxies. They remain constant as far out as can be measured indicating the presence of a dark component perhaps 10 times more massive than the luminous one. Dynamical analysis of the infall of galaxies towards the center of the Virgo supercluster (of which we are an outlying member) argue for  $\Omega_0 \sim .3$  if there is no dumped matter which is non-luminous.<sup>9)</sup> Models of the nucleosynthesis of deuterium in the early universe together with its measured abundance today suggest that the  $\Omega$  corresponding to the density of baryons today is about .1. These arguments suggest a value of  $\Omega_0$  of a few tenths. They cannot rule out, however, a larger  $\Omega_0$  if there is non-luminous, non-baryonic matter which is not clustered with the galaxies or if there is matter clustered like the galaxies but non-luminous.

It is difficult to measure  $\Omega_\Lambda$  from anything other than direct observation of the cosmological expansion. However, it cannot be many orders of magnitude larger than unity or it would imply observable deviations from Newtonian dynamics

in clusters of galaxies.<sup>10)</sup> Thus there is no direct evidence that  $\Lambda = 0$  today. Even an  $\Omega_\Lambda$  of 1, however, corresponds to a cosmological constant which is very small on the scale of the Planck mass  $m_p = (\hbar c/G)^{1/2}$

$$\Lambda \approx 8.8 \times 10^{-122} m_p^2 \Omega_\Lambda h^2 \quad (2.6)$$

The available information on the density of energy in the universe is not enough to tell us whether the spatial geometry of the universe is open or closed. It is, however, close to the flat geometry which is the borderline between the two. We might therefore summarize this information in a fourth "observational fact":

Fact (4). The spatial geometry is approximately flat

Fact (5). The spectrum of density fluctuations

The universe is not exactly homogeneous and isotropic. Matter in galaxies is very clumped as measured by the ratio of the difference in their density to the mean density,  $\delta\rho/\rho$ . The evidence from the background radiation is that earlier the universe was much smoother. The present large scale structure arose from this earlier, smoother distribution through gravitational attraction. At present, direct observations of the background radiation give only upper limits on fluctuations both as to amplitude and spectrum. The amplitude required for those scales where  $\delta\rho/\rho \sim 1$  now (superclusters of galaxies) may be found by extrapolating backwards in time using linear perturbation theory and is  $(\delta\rho/\rho) \sim 10^{-4}$  at the time the background radiation was emitted. This is consistent with the upper limits. Information on the spectrum can be obtained by assuming appealing candidates at decoupling and extrapolating them forward non-linearly and comparing with the existing large scale structure. The spectrum such that all fluctuations have the same amplitude at the time their

scales coincide with the Hubble scale, called the Zel'dovich spectrum, is a popular candidate consistent with all current observations.

Fact (6). The entropy of the universe is low and increasing in the direction of expansion

Today, essentially all of the entropy of matter is in the background radiation. The ratio of the density of entropy  $s$  to the density of baryons  $n_b$  is

$$s/kn_b \sim 10^9, \quad (2.7)$$

so that the total entropy within a Hubble distance is approximately  $S/k \sim 10^{87}$  (the word approximately refers to the exponent!). This is a large number but a small fraction of the entropy which could be obtained by clumping all the matter within the Hubble distance into a black hole.<sup>11)</sup> A black hole of mass  $M$  has entropy  $4\pi kGM^2/(\hbar c)$  so that with a reasonable estimate for the mass within the horizon

$$S/k \sim 10^{120}. \quad (2.8)$$

The 33 orders of magnitude discrepancy between fact and possibility is another way of saying that the universe is still in a reasonably well ordered state. Entropy is increasing and even on the largest scales we seem to see a steady progression from order when the universe is small to disorder when it is large.

We, of course, have more information about the large scale features of the universe than can be summarized in the above six cosmological facts. We observe specific abundances for the elements, a baryon-antibaryon asymmetry, the thermal spectrum of the background radiation and so on. The above list, however, contains those features whose origin is to be found in the earliest stages of our universe.

## 2.2 Initial Conditions

In most problems in physics we divide the universe up into two parts, the system under consideration and the rest. We use the local laws of physics to solve for the evolution of the system. For example we use Maxwell's equations and Newton's laws of mechanics to predict the evolution of a plasma. The local laws of physics require boundary conditions: sometimes initial conditions, sometimes spatial boundary conditions, sometimes radiative boundary conditions, and often a combination of these. These boundary conditions are set by the physical conditions of those parts of the universe which are not part of the physical system under consideration. There are no particular laws determining these conditions, they are specified by observations of the rest of the universe. The situation is different in cosmology. Boundary conditions are still required to solve the local laws governing the evolution of the universe. They are needed, for example, to solve Einstein's equation (2.2). There is, however, no "rest of the universe" to pass their specification off to. If there is a general specification of these initial conditions it must be part of the laws themselves.

If we extrapolate the Friedman-Robertson-Walker models backward in time we can find initial conditions which give rise to the present universe. What attitude are we to take to these initial conditions? A number of attitudes have been taken. Many of them are summarized in the following four rough categories.

### Attitude 1: That's the way it is.

The universe might have been in any one initial state as well as any other. It happens that the one it is in is homogeneous and isotropic on the scales we observe. That's as far as physics can go. It's not the proper subject of



physics to explain these initial conditions only to discover what they were.

This is a reasonable but not very adventurous attitude. It certainly has no predictive power concerning what we will see when with increasing time we are able to observe larger and larger regions of the universe. I believe we will only be able to say it is correct when all attempts to explain the initial conditions have failed.

Attitude 2: The conditions which determine the universe are not initial conditions but the fact that we exist.

This attitude is related to the set of ideas called the anthropic principle.<sup>12)</sup> The universe must be such as to allow galaxy condensation, star formation, carbon chemistry and life as we know it. This is indeed a restriction on the structure of the universe. Perhaps, if one were given a choice of three or four very different cosmologies one could identify our own using the anthropic principle. As stressed by Penrose,<sup>11)</sup> however, the anthropic principle does not seem strong enough to single out the observed universe from among all possibilities. Suppose, for example, the sun had been located in a cloud near the galactic center and we had not been able to make observations of the large scale structure. Would we have been able to predict the large scale homogeneity and isotropy using the anthropic principle?

Attitude 3: Initial conditions are not needed - dynamics does it all.

The idea is that interesting features like the large scale homogeneity and isotropy will arise from any reasonable initial conditions through the action of physical processes over the course of the universe's history. Even if it started in an inhomogeneous and anisotropic state the universe would evolve towards a homogeneous and isotropic