ELECTROMAGNETIC ENERGY TRANSMISSION AND RADIATION

RICHARD B. ADLER
Professor of Electrical Engineering

LAN JEN CHU
Professor of Electrical Engineering

ROBERT M. FANO
Professor of Electrical Communications

Department of Electrical Engineering

Massachusetts Institute of Technology

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FOREWORD

This book is one of several resulting from a recent revision of the Electrical Engineering Course at The Massachusetts Institute of Technology. The books have the general format of texts and are being used as such. However, they might well be described as reports on a research program aimed at the evolution of an undergraduate core curriculum in Electrical Engineering that will form a basis for a continuing career in a field that is ever-changing.

The development of an educational program in Electrical Engineering to keep pace with the changes in technology is not a new endeavor at The Massachusetts Institute of Technology. In the early 1930's, the Faculty of the Department undertook a major review and reassessment of its program. By 1940, a series of new courses had been evolved, and resulted in the publication of four related books.

The new technology that appeared during World War II brought great change to the field of Electrical Engineering. In recognition of this fact, the Faculty of the Department undertook another reassessment of its program. By about 1952, a pattern for a curriculum had been evolved and its implementation was initiated with a high degree of enthusiasm and vigor.

The new curriculum subordinates option structures built around areas of industrial practice in favor of a common core that provides a broad base for the engineering applications of the sciences. This core structure includes a newly developed laboratory program which stresses the role of experimentation and its relation to theoretical model-making in the solution of engineering problems. Faced with the time limitation of a four-year program for the Bachelor's degree, the entire core curriculum gives priority to basic principles and methods of analysis rather than to the presentation of current technology.

J. A. STRATTON

PREFACE

The addition of a new work to the multitude of excellent textbooks based on classical electromagnetic theory can be justified only insofar as the book represents an effort to solve a new educational problem. Such a new problem has arisen from the postwar growth of the Electrical Engineering field as a whole. This growth has led the Department of Electrical Engineering at the Massachusetts Institute of Technology to develop during the last six years an undergraduate "core curriculum" program comprised of eight individual subjects. These are taken by *all* of its students, regardless of their personal goals for ultimate specialization.

Quite early in our considerations, it was agreed that one of these subjects should deal with the dynamics of distributed systems and the related field theory. A much longer time was involved in defining more precisely the appropriate subject matter, deciding its position in the sequence of others, and, finally, in preparing the material for the unspecialized background and interests of the students. As our ideas crystallized, it became clear that, in spite of our earlier hopes to the contrary, it would be wise to limit the considerations to purely electromagnetic phenomena, and to place the study beyond the junior level. Accordingly, a first-term senior subject was developed, designed to provide for all electrical engineering students the very minimum of contact with electric waves and oscillations that we regarded as acceptable for anyone who would bear the title "Electrical Engineer" some ten or twenty years from now. The present book contains a rather extended form of the material covered in this subject.

We were fortunate in being permitted by the concurrent development of other subjects to assume that the students involved would have these prerequisites: two and a half years of calculus; more than a year of college-level experience with electromagnetic fields, including one term of quasi-statics based completely upon Maxwell's equations in vector-analytical form;[1] and two full years comprising various aspects of linear-system theory, involving considerable exposure to superposition principles, Fourier series and integrals, and matrix notation. We could not, however, count upon any previous experience with partial differential equations.

It has, broadly speaking, been our effort to gather up the important linear-system concepts of the time and frequency domains, and the energy-power relations from the theories of lumped circuits and quasi-static fields, and extend them to cover "distributed" situations, in which *space* assumes an importance equal to that of the other independent variables. Our objective in this connection has been the formulation of a complete four-way viewpoint—time, frequency, energy (or power), and space.

Nevertheless, the traditional emphasis on the solution of formal boundary-value problems in a variety of coordinate systems would be inconsistent with the balance we wished to achieve between physical understanding, pure technique, and limited specialization of interest. We have, instead, adopted the approach of presenting first the solutions of field or wave problems in the source-free, unbounded space, and then examining the properties of these solutions in considerable detail, with a view to discovering the kinds of boundary conditions that can be met by combinations of a few of them. Moreover, only the simplest of the infinite set of solutions are considered carefully, because it is our conviction that the most important situations that arise in practice under the heading of "boundary-value problems" are those requiring either a sound qualitative understanding of what can (or cannot) happen under the given circumstances, based on very general principles, or, in more particular quantitative cases, the design of a set of *boundaries* that will yield some rather simply stated field configuration within a certain region of space.

The method of presentation described above reflects our commitment to organize the material around principles of lasting value, using examples of the most elementary character which will still illustrate the points involved. In this context, practical devices are often replaced

[1] The subject referred to is based upon portions of Robert M. Fano, Lan Jen Chu, Richard B. Adler, *Electromagnetic Fields, Energy, and Forces*, John Wiley and Sons, New York, 1960.

by reminiscent configurations of boundaries, requiring only very limited combinations of elementary solutions.

The foregoing philosophy of teaching field theory may make more difficult the use of this book as a professional technical reference work; if so, it represents one result of the difficult conscious compromises made necessary by the nature of our educational objectives.

In any effort to develop a "core-curriculum" subject, ruthless discarding of nonessential material is mandatory so as not to overstep the boundaries of time. Correspondingly, a very clear-cut concept of the principal objectives of the work must be borne in mind continuously. For those students described above, we believe we have established such a teachable core-curriculum subject in the following portions of this book: Practically all of Chapters 1–5; approximately two-thirds of Chapter 6; selected sections on *uniform* plane waves and skin effect from Chapters 7 and 8; sections on the *two-conductor* lossless line from Chapter 9; and practically all of Chapter 10. The classroom part of this subject is built upon the following general ideas:

1. The role of stored energy, dissipation, and energy flux in unifying field problems.
2. The circumstances under which voltage, current, and impedance can be defined, and the importance of the energies in linking the circuit and field descriptions of physical systems.
3. The description of waves and oscillations from multiple points of view; time, space, and (complex) frequency domains; the corresponding analytical techniques and physical interpretation of the mathematics; the behavior of the energies.
4. The role of boundary conditions in dividing a complicated physical problem into simpler tractable pieces, and the ideas relating the forms of the elementary solutions to the appropriate boundary conditions.

Of course it has not been possible to give generalized demonstrations of all the items mentioned in the foregoing list, because most of them represent not definite theorems, but rather viewpoints of great utility in the engineering applications of electromagnetic phenomena. Instead, the simplest possible analytical models, suggestive of practical devices, were chosen in each case to exhibit clearly the concept in question. Conversely, we have tried, where possible, to treat examples from several different points of view to develop flexibility with the concepts and depth of understanding of the examples. This whole procedure turns out to contrast strongly with the more conventional one of organizing the subject matter around "real" devices, the analysis of which often mixes several difficult conceptual and mathematical matters together in such a way as to yield the most "expeditious" treatment of the particular case in question.

However desirable we may believe is the inclusion of a third contact with electromagnetic phenomena at the undergraduate level, we recognize that situations will arise in which this contact must be postponed until a student's early graduate career. By this time, although the student's needs may be essentially the same as those we have described previously, his interest in specialization may well have increased greatly. Moreover, his over-all maturity and mathematical facility are expected to be considerably improved. Similar comments can be made about the exceptionally able undergraduate student who may progress relatively rapidly through the material described above as constituting the core subject. It is essentially for these reasons that we have added to this book the sections on coupled resonators in Chapter 6; the rather lengthy discussions of nonuniform plane waves and guided waves in Chapters 7 and 8; and the section on multiconductor lines in Chapter 9. Some justifications of approximation techniques are also given in much greater detail than is suitable for a basic subject. We have marked with an asterisk (*) any section a major part of which is considered to be beyond the "core" subject, and we have occasionally added footnotes concerning elaborate individual developments within a section that can either be omitted or simplified greatly by reducing the generality. We hope these additions have increased the flexibility of the text for application over a wide range of levels from undergraduate to graduate study.

In a work of this kind, it is impossible to acknowledge the many sources, written and personal, from which the authors have drawn their backgrounds for the task. But we cannot refrain from mentioning specifically the tremendous impact made upon us by our long association with the stimulating environment of the M.I.T. Research Laboratory of Electronics, under the able direction of Prof. Jerome B. Wiesner. This association has made natural for us the blurring of boundaries between education and research considered so important to the educational developments underlying this book.

Similarly, the fact that major portions of the book have been taught for about five years makes it difficult to assess accurately the debt we owe to the staff and students whose points of view have been reflected in the final work. Foremost among these, however, with regard to contributions in the form of early text material, constructive criticism of notes, proofreading of preliminary versions of notes, and extensive laboratory developments, are respectively Professors Herman A. Haus and Edward I. Hawthorne, and Messrs. Frederick Hennie, John Blair, and Ronald Massa.

For the critical reading of the final manuscript, numerous extremely helpful criticisms and suggestions for improvement, and the provision, organization, and solution of a great many problems for the text, we are most deeply grateful to Drs. John Granlund and Morton Loewenthal.

We wish also to thank those who helped by preparing typewritten copy and by performing all the secretarial duties that accompany the development of notes into a text; in particular: Mrs. Bertha Hornby, Miss Dorothea Scanlon, Mrs. Margaret Park, Mrs. Carolyn Bennett, and Miss Nancy Rhodenizer.

This work could not have been undertaken or carried to completion without the encouragement, support, and imaginative leadership of Dr. Gordon S. Brown, whose dynamic efforts and great investment of energy, as Head of the Electrical Engineering Department and our own personal Maxwell demon, made possible the entire core-curriculum program, of which this book represents a tiny fraction.

RICHARD B. ADLER
LAN JEN CHU
ROBERT M. FANO

Cambridge, Mass.
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C O N T E N T S

SECTIONS THAT MAY BE OMITTED FROM A MINIMUM UNDERGRADUATE SUBJECT

CHAPTER ONE

Lumped-Circuit and Field Concepts

It is often necessary in physical situations to deal with the relations between lumped circuit and field concepts. Indeed, the existence of a voltage-current characteristic describing any lumped-circuit element arises from the validity of a particular kind of approximation to the field distribution in the space occupied by and surrounding the corresponding physical device. It is helpful, therefore, to be able to express both lumped-circuit and field behavior in similar language. The concept of energy in its various forms, the principle of its conservation, and the conservation of charge are important aids for achieving this goal.

This chapter is intended to supply a rapid review [1] of familiar circuit and field principles, presented in a form especially suitable for our subsequent use.

1.1 Lumped Electric Circuits

1.1.1 Time Domain

To obtain the dynamic-equilibrium equations for a lumped electric circuit, we use essentially three laws:

(a) Conservation of charge (Kirchhoff's current law).
(b) Single-valuedness of voltage drop (Kirchhoff's voltage law).

[1] For a more thorough treatment, see Robert M. Fano, Lan Jen Chu, Richard B. Adler, *Electromagnetic Fields, Energy, and Forces*, John Wiley and Sons, New York, 1960.

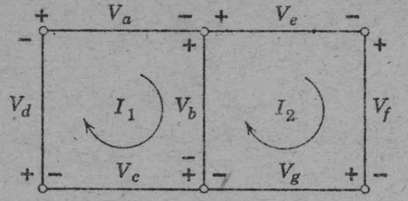

Fig. 1.1. A simple circuit geometry.

(c) Voltage-current relations for the lumped elements:
1. Capacitance.
2. Inductance (and mutual inductance).
3. Resistance.
4. Voltage or current sources.

Charge may be taken to be a fundamental concept that appears in electric systems, as distinct from mechanical ones. Current is the rate of flow of charge along a clear-cut conduction path. In the general case of time-varying charge or current, voltage is a somewhat subtle concept. For our immediate purposes, we wish to emphasize its property of being the instantaneous power divided by the instantaneous current.

With these ideas in mind, it is easy to show that the Kirchhoff laws imply conservation of energy for the whole network. Consider the schematic circuit geometry of Fig. 1.1, in which the detailed character of the branches is not shown, but it is presumed each contains only one of the lumped elements mentioned above. Analysis [1] in terms of loop currents I_1 and I_2 automatically satisfies Kirchhoff's current law. The voltage law yields

$$\begin{aligned} &\text{(a)} \quad V_a + V_d + V_c + V_b = 0 \\ &\text{(b)} \quad V_e + V_f + V_g - V_b = 0 \end{aligned} \tag{1.1}$$

[1] Notation conventions in the text are: italic lightface (A, a) for real scalars; roman lightface (A, a) for complex scalars (phasors); italic boldface (***A***, ***a***) for real space vectors; roman boldface (**A**, **a**) for complex space vectors; and script ($\mathcal{A}$, a) for matrices, with the lower case script (a) reserved for column matrices. The distinctions between real and complex character (italic and roman respectively) and scalar and space-vector character (lightface and boldface respectively) in all four cross combinations are so important to a clear quantitative understanding of the subject matter of this book that they must be carried over carefully into blackboard and problem-solution techniques. To do this most conveniently by hand, we recommend use of underscores and overscores to indicate complex character and real space-vector character respectively, with both together being used for complex space vectors.

After multiplying Eq. 1.1a by I_1, Eq. 1.1b by I_2, and adding the results, we find

$$(V_a + V_d + V_c)I_1 + V_b(I_1 - I_2) + (V_e + V_f + V_g)I_2 = 0 \quad (1.2)$$

Equation 1.2 says that the net power delivered to the network is always zero. Obviously the demonstration is readily extended to an arbitrary network geometry.

The passive lumped elements (C, L, R) either store energy (C, L) or dissipate it (R). From the point of view of lumped circuits, in the strictest sense, we can tell which ones store and which dissipate—but there is no basis for identifying the specific forms of energy involved. Thus, a linear, time-invariant capacitance is defined by the voltage-current relation

$$I = C\frac{dV}{dt} \quad (1.3)$$

where positive I flows in the direction of voltage drop V through the element. Multiplying Eq. 1.3 by V, we find

$$VI = \frac{d}{dt}\left(\frac{1}{2}CV^2\right) \quad (1.4)$$

Equation 1.4 has a left side which is the power delivered to the element. Therefore the quantity $\frac{1}{2}CV^2$ must be energy, and it has proper dimensions. Moreover, it is stored energy because the right side of Eq. 1.4 may be either positive or negative, even though CV^2 is always positive. Without further assumptions, however, we have no right to call it stored electric energy; this nomenclature would invoke a field concept which is beyond the domain of "pure" circuit theory. Nevertheless, let us denote instantaneous energy stored in capacitors by W_e.

A similar derivation would lead to energy W_m stored in linear, time-invariant inductance, of the form $\frac{1}{2}LI^2$. For mutually coupled coils (L_1, L_2, M) carrying currents I_1 and I_2, the stored energy is

$$\tfrac{1}{2}L_1I_1{}^2 + \tfrac{1}{2}L_2I_2{}^2 + MI_1I_2$$

which we include in W_m. The extension to more mutually coupled coils is clear.

The case of resistance is different from that of capacitance or inductance, since its volt-ampere relation is $V = RI$. Thus $VI = RI^2$, and if $R \geq 0$ it follows that $RI^2 \geq 0$. Power is always delivered to a positive resistor—at every instant of time. None ever comes from it. The energy delivered to it has gone down the drain as far as the circuit is concerned. Where has it gone? Again, the answer to this question depends upon other than lumped-circuit concepts. We shall simply call instantaneous power put into (dissipated in) resistors P_d.

Finally, the instantaneous power delivered *by* sources in the network we shall call P.

According to Eqs. 1.2 and 1.4 and the preceding remarks, if for a linear, time-invariant network we include all resistors, capacitors, inductors (including mutual inductance), and sources in the respective energy or power terms, we have

$$P = P_d + \frac{d}{dt}\left(W_e + W_m\right) \tag{1.5}$$

Equation 1.5 expresses the conservation of energy in the network at any instant. It is to be noted that W_e and W_m, being stored energies, are never negative. P_d is positive by virtue of its algebraic form as a sum of RI^2 terms, with all R positive.

1.1.2 Frequency Domain

It might be expected that for a network under any special conditions, e.g., in the sinusoidal steady state, a full discussion of all important energetic matters would stem from appropriate substitutions into Eq. 1.5. This is not so. The reason is that, although Kirchhoff's two laws imply conservation of energy for the whole network, the law of conservation of energy for the whole network, taken with either one of Kirchhoff's laws alone, is not equivalent to the other Kirchhoff law. Reference to Eqs. 1.1 and 1.2 will clarify this fact. Equation 1.2 may be regarded as the result of applying both conservation of energy and conservation of charge (Kirchhoff's current law) to the network of Fig. 1.1. It is only one equation containing two unknown currents. If there were more loops, Eq. 1.2 would still only be one equation, but there would be even more unknown currents in it. Without introducing new assumptions into Eq. 1.2, there is no way of recovering the voltage law relations (Eq. 1.1). The two Kirchhoff laws must therefore say more about a network than is contained in either one of them plus the law of conservation of energy. Hence it may well be possible to state in terms of energy concepts a good deal more about networks than is contained in Eq. 1.5; e.g., though we shall not pursue them here, the Lagrangian and Hamiltonian formulations of network equilibrium have been derived.[1]

[1] Ernst A. Guillemin, *Introductory Circuit Theory*, John Wiley and Sons, New York, 1953, Ch. 10; and David C. White and Herbert H. Woodson, *Electromechanical Energy Conversion*, John Wiley and Sons, New York, 1959, Ch. 1.

Our immediate interest is in the sinusoidal steady-state behavior of a lumped, linear, time-invariant network. If we represent the currents and voltages in Fig. 1.1 by their complex amplitudes V and I, the Kirchhoff voltage law reads:

$$\text{(a)} \qquad \mathrm{V}_a + \mathrm{V}_d + \mathrm{V}_c + \mathrm{V}_b = 0$$

$$\text{(b)} \qquad \mathrm{V}_e + \mathrm{V}_f + \mathrm{V}_g - \mathrm{V}_b = 0 \tag{1.6}$$

Now, defining complex power P as $\frac{1}{2}\mathrm{VI}^*$, which differs from the definition $\frac{1}{2}\mathrm{V}^*\mathrm{I}$ sometimes used, we are led to multiply Eq. 1.6*a* by I_1^* and Eq. 1.6*b* by I_2^*, and to add the results. We find

$$\tfrac{1}{2}(\mathrm{V}_a + \mathrm{V}_d + \mathrm{V}_c)\mathrm{I}_1^* + \tfrac{1}{2}\mathrm{V}_b(\mathrm{I}_1 - \mathrm{I}_2)^* + \tfrac{1}{2}(\mathrm{V}_e + \mathrm{V}_f + \mathrm{V}_g)I_2^* = 0 \tag{1.7}$$

which expresses a new law of conservation of complex power in the network as a whole. Note that Eq. 1.7 is *not* just a special case of Eq. 1.2.

The complex power delivered to any passive branch of the circuit is easily related to the energy stored or dissipated in it—in particular the time-average value thereof. For example, in the sinusoidal steady state the volt-ampere relation of a constant capacitor is

$$\mathrm{I} = j\omega C\mathrm{V} \tag{1.8}$$

making

$$\frac{1}{2}\mathrm{VI}^* = -j\frac{\omega C}{2}|\mathrm{V}|^2 \tag{1.9}$$

But $|\mathrm{V}|^2/2$ is just the time-average value of $V^2(t)$ when $V(t)$ is sinusoidal. Therefore if we denote by a special bracket $\langle\ \rangle$ the time-average value of the enclosed symbol, we have for the time-average value of the instantaneous stored energy W_e given in connection with Eq. 1.4,

$$\langle W_e\rangle = \tfrac{1}{2}C\langle V^2(t)\rangle = \tfrac{1}{4}C|\mathrm{V}|^2 \tag{1.10}$$

In view of Eq. 1.10, Eq. 1.9 becomes

$$\tfrac{1}{2}\mathrm{VI}^* = -2j\omega\langle W_e\rangle \tag{1.11}$$

Similarly, for pure inductance the complex power is $+2j\omega\langle W_m\rangle$, where

$$\langle W_m\rangle = \tfrac{1}{4}L|\mathrm{I}|^2$$

in the sinusoidal steady state. If two coils are coupled (L_1, L_2, M), the time-average energy stored is

$$\langle W_m\rangle = \tfrac{1}{4}L_1|\mathrm{I}_1|^2 + \tfrac{1}{4}L_2|\mathrm{I}_2|^2 + \tfrac{1}{2}M\ \mathrm{Re}\ (\mathrm{I}_1\mathrm{I}_2^*)$$

and again the complex power is simply $+2j\omega\langle W_m\rangle$.

For resistors, the complex power is the time-average power dissipated, $\frac{1}{2}R|\mathrm{I}|^2$, which we express as $\langle P_d \rangle$.

Thus, writing the complex power *produced* by *all* sources in the network as $\langle P \rangle + jQ$, we recast Eq. 1.7 to read

$$\langle P \rangle + jQ = \langle P_d \rangle + 2j\omega\langle W_m - W_e \rangle \tag{1.12}$$

in which the average power and energy terms on the right must of course include sums over all passive elements in the circuit.

In connection with Eq. 1.12, an interesting result arises when only one source is driving the network. Let V_s and I_s be the complex source voltage and current respectively, such that $(\mathrm{V}_s/\mathrm{I}_s)$ is the impedance Z presented to the source by the network. Then we have

$$\tfrac{1}{2}\mathrm{V}_s\mathrm{I}_s{}^* = \langle P_d \rangle + 2j\omega\langle W_m - W_e \rangle \tag{1.13}$$

which, upon division by $\mathrm{I}_s\mathrm{I}_s{}^* = |\mathrm{I}_s|^2$, becomes

$$\frac{\mathrm{V}_s}{\mathrm{I}_s} = \mathrm{Z} = \frac{2\langle P_d \rangle}{|\mathrm{I}_s|^2} + j4\omega\left(\frac{\langle W_m \rangle}{|\mathrm{I}_s|^2} - \frac{\langle W_e \rangle}{|\mathrm{I}_s|^2}\right) \tag{1.14}$$

In other words, the real part of the input impedance is twice the average power dissipated in the network for an ampere of input current, whereas the reactance is proportional to the difference between the average inductive and average capacitive energies stored by an input ampere. It is to be observed that, with a fixed input current $\mathrm{I}_s = 1$ amp, $\langle P_d \rangle$, $\langle W_m \rangle$, and $\langle W_e \rangle$ will in general be complicated functions of frequency according to the redistribution of branch currents and voltages throughout the network. One should therefore not be misled by the fact that ω enters Eq. 1.14 explicitly in only a simple way.

1.2 Electromagnetic Fields

1.2.1 Time Domain

Having reviewed briefly purely lumped circuits, we turn now to review principles of the electromagnetic field in stationary media. At the outset we have six dependent field variables, all of which are in general functions of independent variables denoting space and time. These field variables are:

1. Free-charge density ρ (scalar), coulombs/m^3.
2. Current density $\boldsymbol{J}$ (vector), amp/m^2.
3. Electric field intensity $\boldsymbol{E}$ (vector), v/m.
4. Magnetic flux density $\boldsymbol{B}$ (vector), webers/m^2 = v-sec/m^2.