FIBER OPTICS

James C. Daly

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Editor

James C. Daly
Professor of Electrical Engineering
University of Rhode Island
Kingston, Rhode Island

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PREFACE

This book is designed to provide communication engineers with the theoretical and practical knowledge needed to understand and design fiber optic communication systems. Since this is a new technology, the University study of most practicing engineers did not include courses on fiber optic communications. These people do, however, have an understanding of communication systems and the fundamentals of electromagnetic theory. This book bridges the gap between classical communication practice and the new techniques required to design fiber optic communication systems.

The book is organized into eight chapters, each written by an expert with first-hand experience. Chapter 1 reviews general properties of fibers, history, and some applications. This includes a review of the electromagnetic base of some important optical phenomena, matrix methods for ray tracing, and Gaussian beams. Chapter 2 covers optical waveguide manufacture. Various types of fibers and the processes used to produce them are explained. This includes preform preparation and fiber drawing. Measurements used to characterize optical and mechanical properties of fibers are covered. Chapter 3 explains propagation in fibers in terms of both wave and ray theory. Dispersion, coupling, modal noise, and the influence of fiber properties on system performance are covered. Sources are discussed in Chapter 4. Characteristics pertinent to optical communication systems, including physical principles and structures of lasers and IRED diodes are explained. Chapter 5 covers detectors. The principles of operation of Si PIN and avalanche photo diodes are discussed. Detectors for long wavelength systems and noise in detectors is covered. Chapters 6 and 7 treat fiber optic communication systems. Chapter 6 covers the important topic of digital communication systems. The operation of various components, such as fibers and lasers, are reviewed in terms relevant to communication engineers. This is followed by an analysis of design criteria for digital receivers. Practical constraints are covered. Analog systems are covered in Chapter 7. This includes a comprehensive treatment of the fundamentals and limitations of analog transmission systems. Noise models and degeneration effects are covered. Practical considerations in implementing systems are included. Chapter 8 treats the use of optical fibers in imaging systems. Imaging theory, fiber fabrication, and imaging applications are discussed.

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THE EDITOR

James C. Daly, Ph.D., is Professor of Electrical Engineering at the University of Rhode Island, Kingston, Rhode Island.

Dr. Daly received his B.S. degree in Electrical Engineering from the University of Connecticut in 1960 and his M.E.E. and Ph.D. degrees from Rensselaer Polytechnic Institute in 1962 and 1967, respectively.

Dr. Daly is a member of the Institute of Electrical and Electronics Engineers and the honor societies, Tau Beta Pi, Eta Kappa Nu, and Sigma Xi.

His current research interest involves VLSI for fiber optic electronics. He has published over twenty papers in the area of optical communications and electronic systems. He holds a patent on an optical waveguide.

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INTRODUCTION

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I. HISTORY

The use of fibers for optical communications was suggested by Kao and Davies in 1968.[1] At that time typical fiber losses were above 1000 dB/km. Kao suggested that purer materials should permit much lower losses. In November of 1970, Kapron et al. reported an observed total attenuation of approximately 20 dB/km in a single mode fiber.[1] Today, fibers operate at a wavelength of 1.5 μm with losses less than 1 dB/km.[3-4] Low losses have been achieved by reducing impurity absorption due to transition metal ions such as iron, chromium, cobalt, and copper. Absorption from OH^- ions due to water impurity is also an important factor. Parts per billion purity of iron and chromium ions is required if their loss contributions are to be kept below 1 dB/km.[5]

A communcation system requires a transmitter, a transmission medium, and a receiver. Fiber optics became a feasible transmission medium in 1970 with the reduction of losses to 20 dB/km. At that time, technology also existed to produce semiconductor detectors suitable for use with optical fibers, but there were no suitable sources. The earlier invention of the laser and the possibility of using it for communication had stimulated fiber optics research. In 1970 laser sources and light emitting diodes (LEDs) had problems of short lifetime and low outputs. Early semiconductor lasers were inefficient and required cooling. A series of improvements has resulted in more efficient semiconductor lasers operating reliably at room temperature with high outputs and lifetimes greater than 10^5 hr. Today, with high quality semiconductor sources, low-loss fiber transmision media and low noise semiconductor detectors, the three elements (transmitter, medium, and receiver) are available for the construction of economical, reliable optical communication systems.

A. Fiber Classes

The information-carrying capacity (bits/sec) of an optical fiber is determined by its impulse response. The impulse response and thus the bandwidth are largely determined by the modal properties of the fiber. The optical fibers, in common use, can be separated into two classes based on their modal properties, (1) single-mode fibers, and (2) multimode fibers. Single-mode fibers are step-index. Multimode fibers can be divided into step- and graded-index. Step- or graded-index refers to the variation of the index of refraction with radial distance from the fiber axis. Fiber types are discussed thoroughly in Chapter 2. Figure 1 of Chapter 2 shows these three types of fibers (1) step-index multimode, (2) graded-index, and (3) single-mode. These fibers consist of a core surrounded by a cladding. The higher index of refraction of the core compared to the cladding causes total internal reflection at the core cladding interface in step-index fibers. In graded-index fibers, the gradual decrease in the index of refraction with distance from the fiber axis causes light rays to bend back toward the axis as they propagate. Multimode guides are characterized by multiple propagation paths for rays. A modal description of multimode fibers shows different propagation velocities for different modes. Therefore, energy input into the fiber from a short pulse, coupled into a multiple of modes, will arrive at the receiving end of the fiber distributed over a time interval. The spreading out in time of the received pulse is due to the differing propagation delays of the different modes. This pulse spreading is referred to as modal dispersion. It reduces the information capacity of the fiber by limiting the number of distinct pulses that can be transmitted in a given time interval. Graded-index guides have less modal dispersion than step-index guides. Of course, modal dispersion does not occur in single-mode guides where only one mode propagates. Pulse broadening in single-mode fibers is due to material dispersion and the dispersion associated with the waveguide mode. Single-mode fibers can be designed so that these two sources of dispersion cancel at a particular wavelength.

B. Applications

Optical fibers have advantages that make them attractive in a variety of applications. They have extremely high bandwidth. Their small diameter and high tensile strength result in smaller, lighter weight cables and connectors. Since they are electrical insulators, optical fibers are immune to inductive interference and are not subject to ground loop problems. They can be used in high voltage environments without providing unwanted conduction paths. They do not radiate electromagnetic energy. In addition, they are tolerant to temperature extremes, resist corrosion, are reliable, and easily maintained. The raw material used to fabricate glass fibers is sand, an abundant resource.

First generation optical fiber communication systems operate at a wavelength of 0.82 μm. Second generation systems, operating at a longer wavelength of 1.3 μm where fiber losses and material dispersion are less, offer significantly less attenuation and greater bandwidth.

Optical fiber applications have been pioneered by telephone companies. General Telephone of California installed the first optical fiber link carrying regular telephone service in Long Beach, Calif., on April 22, 1977.[6] It was a 1.544 Mb/s link utilizing a graded-index fiber with a 6.2 dB/km mean loss, LED sources, and avalanche photodiode detectors. Only two repeaters were used with the 9.1 km link. An equivalent metallic link would have required five repeaters for the same data rate.[5] The current Bell System fiber optic program centers around digital trunk transmission at 44.7 Mb/s.[7] As many as 672 voice circuits are transmitted in a pair of fibers. Bell is planning a transatlantic cable operating at 1.3 μm. Each section of cable uses a laser and one or more standbys to achieve a mean time between failures of 8 years. Repeaters are at about 30 km intervals. Multiple single-mode fibers capable of carrying more than 4000 voice circuits are planned.[7]

CATV systems use fibers to transmit signals from earth stations to studio facilities.[9] Distribution of CATV signals from studios to subscribers is being tested.[10]

Optical fibers have promise for use in computer systems.[11] It is attractive to replace parallel interconnects with serial fiber optic links. Cable and connector bulk is significantly reduced. Reliability is improved. Fiber optic high data rates, noise immunity, and low loss make it possible to extend high data rate channel links beyond the confines of the computer room. The use of smart terminals increases the need for high bandwidth local networks. Smart terminals process and store information. Relatively large amounts of information are transmitted to the host computer in bursts. A large bandwidth is required if excessive response delays are to be avoided.[11] The interest of the military has stimulated a wide range of fiber applications, from rotation and sonar sensors to communication links. Fibers permit dramatic weight and bulk reduction and provide large bandwidth and high reliability. A 64-km fiber optic field link, used by the Army, transmitting 2.3 Mb/s requires seven repeaters and can be transported on one $2^1/_2$ ton truck. The equivalent coaxial link requires 39 repeaters and four $2^1/_2$ ton trucks for transportation.[12] Fiber optic sonar links have been developed to transmit information from external sensors through the submarine hull to inboard signal processors.[13] These links reduce the size of submarine hull penetrators in addition to improving system performance.

One of the original applications of fibers is image transmission. The flexible fiberscope has been widely used in medicine since the 1950s. Modern fiber optics technology has recently been applied to office copy machines. This is discussed further in Chapter 7.

The following three sections of this chapter discuss basic electromagnetic theory and optics useful in describing phenomena in fiber optic systems. Section II reviews electromagnetic plane waves. Section III illustrates some examples of matrix methods for the description of ray propagation. Section IV covers some properties of optical beams.

II. WAVE OPTICS

The electromagnetic description of optics forms a basis for the explanation of a number of phenomena occurring in optical fiber systems. Reflection at a dielectric interface occurs when light passes through a lens, is injected into a fiber, or is reflected at the fiber core cladding interface. Phenomena such as total internal reflection, penetration of the optical field into the cladding, and Snell's law follow from the description of the reflection of plane waves from a dielectric interface.

A. Maxwell's Equations

Maxwell's equations written in differential form are

$$\overrightarrow{\nabla \times \vec{E}} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(a)} \qquad (1)$$

$$\overrightarrow{\nabla \times \vec{H}} = J + \frac{\partial \vec{D}}{\partial t} \quad \text{(b)}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{(c)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{(d)}$$

where E,B,H,J,D and ρ are the electric field intensity, the magnetic flux density, the magnetic field intensity, the current density, the electric flux density, and the electric charge density, respectively. The del operator is ∇. In rectangular coordinates $\nabla = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are unit vectors in the x, y, and z directions.

In a source-free region such as air or glass with no free charge, $\vec{J} = 0$ and $\rho = 0$. Also, if the medium is time invariant, homogeneous, isotropic, and linear, then $\vec{B} = \mu\vec{H}$ and $\vec{D} = \epsilon E$, where the permeability, μ, and the permittivity, ε, are scalar quantities that do not vary in space or time. Under these conditions Maxwell's equations are

$$\overrightarrow{\nabla \times \vec{E}} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{(a)} \qquad (2)$$

$$\overrightarrow{\nabla \times \vec{H}} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{(b)}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{(c)}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \quad \text{(d)}$$

The wave equation is obtained by combining Equation 2a and 2b. The first step is to take the curl of Equation 2a.

$$\vec{\nabla} \times (\overrightarrow{\nabla \times \vec{E}}) = \vec{\nabla} \times \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) \qquad (3)$$

Interchanging the order of differentiation with respect to space and time, and moving the constant μ out of the derivative

$$\overrightarrow{\nabla \times (\nabla \times \vec{E})} = -\mu \frac{\partial}{\partial t} (\overrightarrow{\nabla \times \vec{H}}) \qquad (4)$$

Substituting Equation 2b for $\overrightarrow{\nabla \times H}$,

$$\overrightarrow{\nabla \times (\nabla \times E)} = -\mu\epsilon \frac{\overrightarrow{\partial^2 E}}{\partial t^2} \tag{5}$$

Recall the vector identity $\overrightarrow{\nabla \times \nabla \times E} = \overrightarrow{\nabla (\nabla \cdot E)} - \overrightarrow{\nabla^2 E}$. Applying this vector identity to Equations 5 and using Equation 2c, results in the vector wave equation,

$$\overrightarrow{\nabla^2 E} = \mu\epsilon \frac{\overrightarrow{\partial^2 E}}{\partial t^2} \tag{6}$$

When the field vectors have sinusoidally varying components it is convenient to use phasors to represent the components. Consider the electric field,

$$\vec{E} = \hat{x}\, A \cos(\omega t + \theta_x) + \hat{y}\, B \cos(\omega t + \theta_y) + \hat{z}\, C \cos(\omega t + \theta_z) \tag{7}$$

The x, y, and z components vary sinusoidally at the same frequency. Each component may have a different amplitude and phase. When phasors are used to represent the sinusoidally varying components the following phasor vector results

$$\vec{E} = E_x\, \hat{x} + E_y\, \hat{y}\, E_z\, \hat{z} \tag{8}$$

where the complex numbers $E_x = A < \Theta_x$, $E_y = B < \Theta_y$, and $E_z = C < \Theta_z$ are the phasor representations of the components of the vector.

Taking the derivative of a sinusoid with respect to time corresponds to multiplying its phasor representation by $j\omega$. Therefore, when field quantities are represented by phasors the wave equation is

$$\overrightarrow{\nabla^2 E} = -\omega^2 \mu\epsilon \frac{\overrightarrow{\partial^2 E}}{\partial t^2} \tag{9}$$

B. Plane Waves

Waves with planar constant phase surfaces are called plane waves. Although plane waves have a simple mathematical form, their behavior is of general interest. Complex waves can be represented as a sum of plane waves, and in the neighborhood of a point all waves appear to be plane waves. When the amplitude and phase of field quantities are constant on a plane, the wave is called a uniform plane wave. A simple solution to Equation 9 is a uniform plane wave, polarized in the x direction and propagating in the z direction. A wave polarized in the x direction has electric field components only in the x direction. Assume the phasor vector representation of the electric field is only a function of z

$$\vec{E} = \hat{x}\, E_x\, (z) \tag{10}$$

Since for this case there is no dependence on x and y, the Laplacian operator reduces to, $\nabla^2 = \partial^2/\partial z^2$. The wave equation becomes

$$\frac{\partial^2 E_x}{\partial z^2} = \omega^2 \mu\epsilon E_x \tag{11}$$

The solution to Equation 11 is

$$E_x = E_o e^{\pm jkz} \qquad (12)$$

where $k = \omega\sqrt{\mu\epsilon}$. $E_o = A < \Theta$ is a complex constant depending on the boundary conditions. E_x given by Equation 12 is a phasor quantity that corresponds to the following time function

$$E_x\,(t,z) = A \cos\,(\omega t \pm kz + \theta) \qquad (13)$$

A point of constant phase for the wave given by Equation 13 is

$$\text{const} = \omega t \pm kz \qquad (14)$$

The phase velocity v'_p, is obtained by differentiating Equation 14 with respect to time

$$0 = \omega \pm k\frac{dz}{dt} \qquad (15)$$

The velocity of a constant phase point is

$$v_p = \frac{dz}{dt} = \pm\omega/k = \pm\frac{1}{\sqrt{\mu\epsilon}} \qquad (16)$$

A plus sign before k in Equation 16 corresponds to a wave propagating in the negative z direction, and a minus sign corresponds to a wave propagating in the positive z direction. One wavelength is the distance for a 2π phase shift. Therefore, $2\pi = k\lambda$ or $k = 2\pi/\lambda$ where λ is the wavelength.

The index of refraction of a medium is defined as the ratio of the phase velocity in free space to the phase velocity in the medium. It follows from Equation 16 that the index of refraction is

$$n = \frac{v_\rho}{v_p} = \frac{k}{k_o} = \sqrt{\frac{\epsilon}{\epsilon_o}} = \sqrt{\epsilon_r} \qquad (17)$$

where c is the velocity of light in free space, $k_o = \omega\sqrt{\mu_o\epsilon_o}$, ϵ is the permittivity of free space, and ϵ_r is the dielectric constant.

1. Generalized Plane Waves

Constant phase surfaces for the plane waves given by Equation 12 are planes perpendicular to the z axis (x = constant). Consider the same wave propagating in some direction other than along the z axis. Such a wave would result if the coordinate system were rotated. The constant phase surface would be a plane described by $k_x\hat{x} + k_y\hat{y} + k_z\hat{z} = \phi =$ constants. A field quantity associated with this wave is

$$E = E_o\, e^{-j(k_x x + k_y y + k_z z)} \qquad (18)$$

where E_o is a constant phasor vector. Recall $\vec{r}$, the vector position of any point in space, $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$. Also define the propagation vector $k = \vec{k}_x\hat{x} + k_y\hat{y} + k_z\hat{z}$. The constant phase point, ϕ, may be written $\phi = \vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$

$$E = E_o e^{-jk\cdot r} \quad (19)$$

Taking derivatives with respect to x of plane wave field quantities with exponential spatial variation such as given in Equation 18 is equivalent to multiplying by $-jk_x$. Derivatives with respect to other coordinates may be treated similarly. This allows the del operator, which in rectangular coordinates is $\nabla = \hat{x}\,\partial/\partial x + \hat{y}\,\partial/\partial y + \hat{z}\,\partial/\partial z$, to be replaced by, $-jk$. Also, the Laplacian operator which in rectangular coordinates is $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, is replaced by

$$\nabla^2 = -k_x^2 - k_y^2 - k_z^2 = -k^2 \quad (20)$$

where k^2 is the magnitude of $\vec{k}$ squared.

Substituting Equation 19 into Maxwell's equations, (Equations 2) results in the following representation of Maxwell's equations for plane waves.

$$\vec{k} \times \vec{E} = \overrightarrow{\mu\omega H} \quad (a) \quad (21)$$

$$\vec{k} \times \vec{H} = -\epsilon\omega\vec{E} \quad (b)$$

$$\vec{k} \cdot \vec{E} = 0 \quad (c)$$

$$\vec{k} \cdot \vec{H} = 0 \quad (d)$$

The magnitude of k is found by applying Equation 20 to Equation 9:

$$k = \omega\sqrt{\mu\epsilon} \quad (22)$$

The magnetic field associated with the plane wave is obtained using Equation 21a

$$\vec{H} = \frac{1}{\omega\mu}\vec{k} \times \vec{E} \quad (23)$$

It follows from Equation 21b that $\vec{E}$ is perpendicular to both $\vec{k}$ and $\vec{H}$. Since $\vec{k}$ and $\vec{E}$ are perpendicular, the magnitude of $\vec{H}$ is the product of the magnitudes of $\vec{k}$ and $\vec{E}$ divided by $\omega\mu$. Using Equations 23 and 22 it follows that

$$E = \sqrt{\frac{\mu}{\epsilon}}\,H \quad (24)$$

where E and H are the magnitudes of $\vec{E}$ and $\vec{H}$ and $\sqrt{\mu/\epsilon}$ is the characteristic impedance of the medium. Note that

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \frac{Z_o}{n} \quad (25)$$

where n is the index of refraction and Z_c is the characteristic impedance of free space.

It follows from Equations 21a and 21b that $\vec{E}$, $\vec{H}$, and $\vec{k}$ form an orthogonal set of vectors. $E \times H$ is in the direction of $\vec{k}$. Since $\vec{E}$ and $\vec{H}$ are perpendicular, the magnitude of $\overrightarrow{E \times H}$ is the magnitude of $\vec{E}$ multiplied by the magnitude of $\vec{H}$. From this and Equation 24 it follows that

$$\overrightarrow{E \times H} = \frac{E^2}{Z}\hat{k} \tag{26}$$

where $\hat{k}$ is the unit vector in the direction of $\vec{k}$. $E \vec{\times} H$ is the Poynting vector. Its units are watts per square meter (W/m^2). The Poynting vector is in the direction of energy flow in isotropic media.

C. Reflection at a Dielectric Interface

The amount of light reflected at a dielectric interface depends on the polarization, the angle of incidence, and the indices of refraction of the two media. The reflection and transmission coefficients are found by satisfying the boundary conditions. At the boundary conditions, the tangential components of both the E and H vectors are continuous across the interface.

A representation of a plane wave reflection is shown in Figure 1. The plane wave shown is polarized parallel to the plane of incidence. The plane of incidence is defined as the plane containing the normal to the surface and the k vector of the incident plane wave.

The incident, reflected and transmitted plane waves are

$$\vec{\mathscr{E}}_i = \vec{E}_i \cdot e^{-j\vec{k_i}\cdot\vec{r}}$$

$$\vec{\mathscr{E}}_R = \vec{E}_R\, e^{-j\vec{k_R}\cdot\vec{r}}$$

$$\vec{\mathscr{E}}_t = \vec{E}_t\, e^{-j\vec{k_t}\cdot\vec{r}} \tag{27}$$

1. Snell's Law

Snell's law follows from the boundary conditions and states that the phase variation along the interface must be the same for the incident, reflected, and transmitted fields. That is, at the interface

$$\vec{k}_i \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_t \cdot \vec{r} \tag{28}$$

At the interface $z = 0$. Also in Figure 1, the coordinate system has been drawn with the incident k vector in the xz plane; therefore, the y component of $\vec{k}_i = 0$. In this situation, the phase variation along the interface depends only on x. Equating the phases of the incident, reflected, and transmitted waves at the interface.

$$n_1k_o \sin\theta_i = n_1k_o \sin\theta_R = n_2k_o \sin\theta_t \tag{29}$$

where $n_1k_o\sin\Theta_i$, $n_1k_o\sin\Theta_R$, and $n_2k_o\sin\Theta_t$ are the x components of the incident, reflected, and transmitted propagation vecors, respectively.

Equation 29 leads to two important observations. The first is that the angle of incidence equals the angle of reflection, $\Theta_i = \Theta_R$. The second is the familiar form of Snell's law

$$n_1 \sin\theta_i = n_2 \sin\theta_t \tag{30}$$

Total internal reflection occurs at the interface between two media where n_2 is less than n_1 and the angle of incidence is large. This happens in a step-index fiber at the core-cladding interface.

As Θ_i the angle of incidence is increased, Θ_t, the transmitted angle also increases. When $n_1 > n_2$, Θ_t is greater than Θ_i. There is a critical incidence angle for which $\Theta_t = 90°$. For incident angles greater than this critical angle no light propagates into medium 2.

The propagation vector in medium 2 is

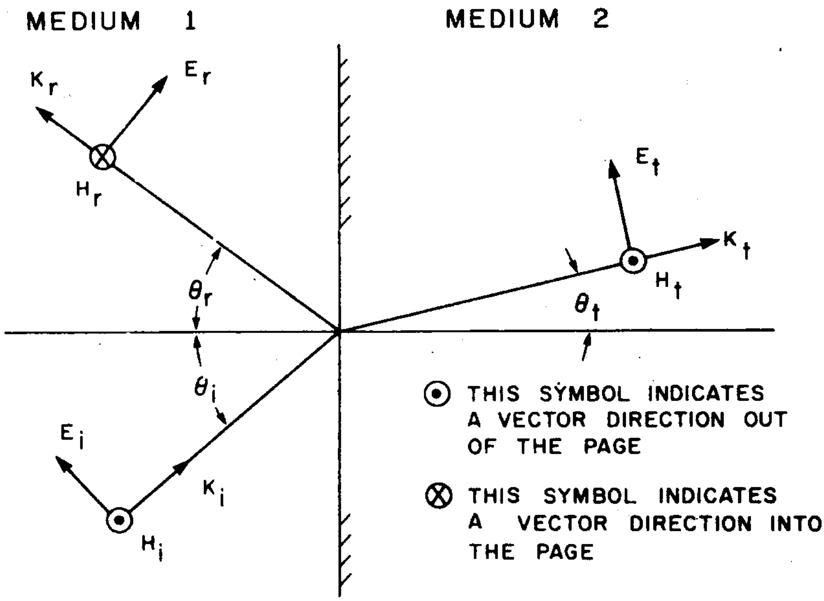

FIGURE 1. Plane wave reflection from a dielectric interface is represented in terms of the E, H, and K vectors of the incident reflected and transmitted waves. The plane of the paper is the plane of incidence (defined as the plane containing the surface normal and the incident ray). The polarization is parallel since the E field is parallel to the plane of incidence.

$$\vec{k}_t = n_2 k_o (\sin\theta_t\,\hat{x} + \cos\theta_t\,\hat{z}) \tag{31}$$

where by Snell's Law and a well-known trigonometric identity $\cos\Theta_t = [1 - (n_1/n_2)^2 \sin^2\Theta_i]^{1/2}$. For incident angles greater than the critical angle, the $\cos\Theta_t$ and therefore the z component of k_t are imaginary. This results in an evanescent transmitted wave. The transmitted wave does not propagate in the z direction, but decays exponentially with z. The expression for the electric field associated with the wave in medium 2 is

$$\vec{\mathscr{E}}_t = \vec{E}_t\, e^{-z/\delta}\, e^{-j(n_2 k_o \sin\theta_t)x} \tag{32}$$

where $\vec{E}_t$ is a constant vector, and $\delta = (1/k_o)(n_2^2 - n_1^2 \sin^2\Theta_1)^{-1/2}$ is the depth of penetration of the light into the second medium.

2. The Reflection and Transmission Coefficients

The reflected and transmitted waves are found in terms of the incident wave by applying boundary conditions to the interface. The tangential E and H fields are continuous across the interface. For parallel polarization shown in Figure 1, equating the tangential electric field in medium 1 to the tangential field in medium 2 results in the following equation,

$$E_i \cos\theta_i + E_R \cos\theta_R = E_t \cos\theta_t \tag{33}$$

Similarly, equating the tangential components of the magnetic fields results in the following equation

$$H_i - H_R = H_t \tag{34}$$

Equations 33 and 34 apply only to parallel polarization. A similar set of equations can be obtained for perpendicular polarization.

Since E and H for each of the three waves are related by the characteristic impedance of the medium, Equation 34 can be written in terms of the electric fields as follows,

$$n_1(E_i - E_R) = n_2E_t \tag{35}$$

Multiplying Equation 33 by n_2 and using Equation 35 to eliminate E_t from the right hand side of Equation 33, results in the following

$$(E_i + E_R)\, n_2 \cos\theta_i = (E_i - E_R)\, n_1 \cos\theta_t \tag{36}$$

Solving for the reflection coefficient, defined as the ratio of E_R to E_i,

$$\rho_{\parallel} = \frac{E_R}{E_i} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \tag{37}$$

Since by Snell's law and a well-known trigonometric identity, $\cos\Theta_t = [1 - (n_1/n_2)^2 \sin^2\Theta_i]^{1/2}$ it follows that

$$\rho_{\parallel} = \frac{n_1[1 - (n_1/n_2)^2 \sin^2\theta_i]^{1/2} - n_2 \cos\theta_i}{n_1[1 - (n_1/n_2)^2 \sin^2\theta_i]^{1/2} + n_2 \cos\theta_i} \tag{38}$$

This equation is good for polarization parallel to the plane of incidence. A similiar analysis for polarization perpendicular to the plane of incidence yields the following expression for the reflection coefficient

$$\rho_{\perp} = \frac{\cos\theta_i - [(n_2/n_1)^2 - \sin^2\theta_i]^{1/2}}{\cos\theta_i + [(n_2/n_1)^2 - \sin^2\theta]^{1/2}} \tag{39}$$

Brewster's angle is the incident angle at which no parallel polarized light is reflected at a dielectric interface. When this occurs, the numerator of Equation 38 equals zero. That is,

$$n_1[1 - (n_1/n_2)^2 \sin^2\theta_B]^{1/2} = n_2 \cos\theta_B \tag{40}$$

where Θ_i has been replaced by Θ_B, Brewster's angle. Squaring both sides of Equation 40, solving for the cos Θ_B using the trigonometric identity $\sin^2\Theta = 1 - \cos^2\Theta$ results in the following expression for the tangent of Θ_B

$$\text{Tan}\,\theta_B = n_2/n_1 \tag{41}$$

The transmission coefficient is the ratio of the phasor representing the transmitted wave to the phasor representing the incident wave, E_t/E_i. It can be found using the boundary conditions in a manner similar to that used to obtain the reflection coefficients. The transmission coefficient for parallel polarization is

$$T_{\parallel} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1\,[1 - (n_1/n_2)^2 \sin^2\theta_i]^{1/2}} \tag{42}$$

The transmission coefficient for perpendicular polarization is

$$T_{\perp} = \frac{2 \cos \theta_i}{\cos \theta_i + [(n_2/n_1)^2 - \sin^2 \theta_i]^{1/2}} \tag{43}$$

III. MATRIX RAY OPTICS

Ray propagation analysis provides a useful description in many situations. In isotropic materials, the ray direction is the direction of energy propagation. Rays are related to the wave description of optics in that rays are normal to constant phase surfaces. The matrix description of ray propagation allows complex optical structures to be described as combinations of simple elements. The matrix describing the structure is the product of the matrices of its elements. Below, the ray matrices for three optical elements that occur frequently in fiber optic systems are found.

A. Homogeneous Medium

A ray propagating through a homogeneous medium follows a straight line as shown in Figure 2. The displacement and slope of the line relative to the optic axis at reference plane 2 is described by the following set of linear equations

$$\begin{aligned} r_2 &= r_1 + dr_1' \\ r_2' &= r_1' \end{aligned} \tag{44}$$

where r_2 and r'_2 are the ray displacement and slope relative to the optic axis at reference plane 2. r_1 and r'_1 are the ray position and slope at reference plane 1. The distance between reference planes is d. When matrices are used, Equation 44 becomes

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix} \tag{45}$$

B. Thin Lens

The thin lens is an idealized model that provides an accurate approximation to actual lenses. Paraxial rays are those rays propagating in directions nearly parallel to the axis. A thin lens changes the slope of a paraxial ray propagating through it an amount proportional to the displacement of the ray from the optical center of the lens. Actual lenses not obeying this law are said to have aberrations. Ray displacement from the axis is unchanged by the thin lens. Ray propagation from reference plane 1 just before a thin lens to reference plane 2 just after the thin lens is depicted in Figure 3 and is described by the following equation,

$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix} \tag{46}$$

where f is the focal length of the lens.

C. Quadratic Index Medium

A medium whose index of refraction varies as the square of the distance from the optical axis has the ability to guide rays. Such a medium is an idealized model for graded-index fibers. The index of refraction for such a medium is

$$n = n_0 [1 - 1/2 (r/a)^2] \tag{47}$$

where n_0 is the index of refraction on the optical axis, r is the distance from the optical axis, and a is a constant. Ray trajectories in this type of medium can be determined by applying