

PSYCHOMETRIC METHODS

BY

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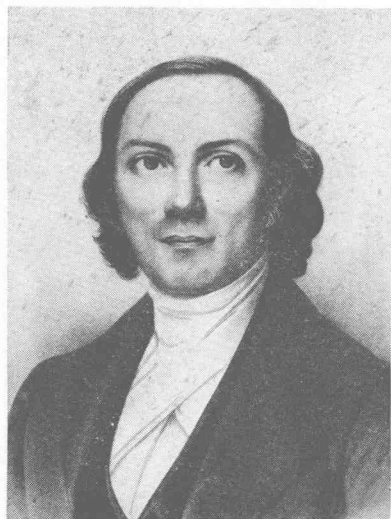
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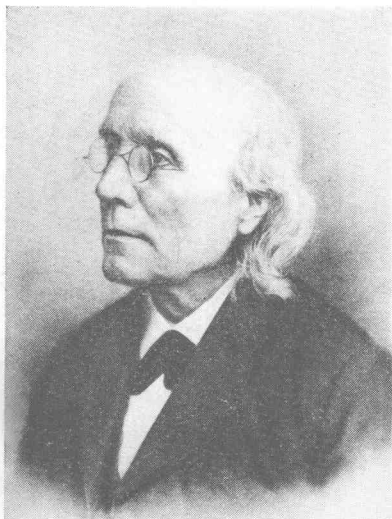
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PIONEERS IN PSYCHOMETRIC METHODS

It is a pleasure to be able to present the portraits of twelve of those who have led the way in the building of a quantitative psychology. Without their devotion to the ideal of objective exactness in psychology this volume could not have been written. To them the book is appropriately dedicated.



Ernst Heinrich Weber



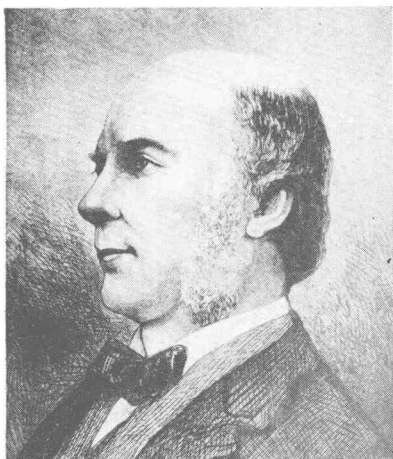
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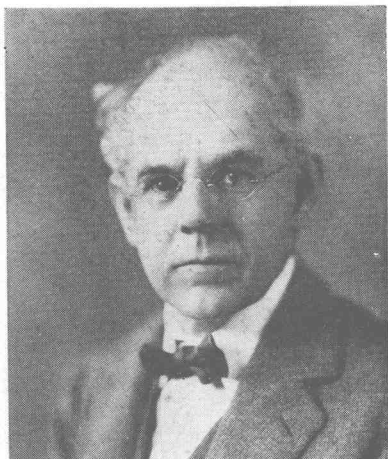
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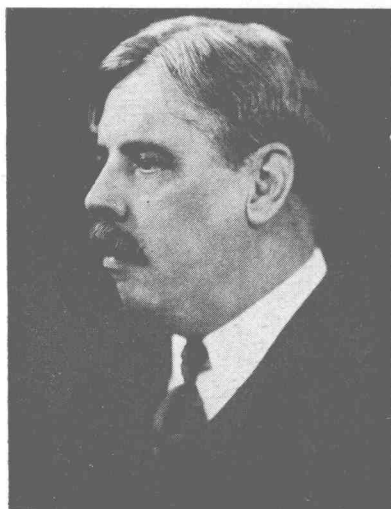
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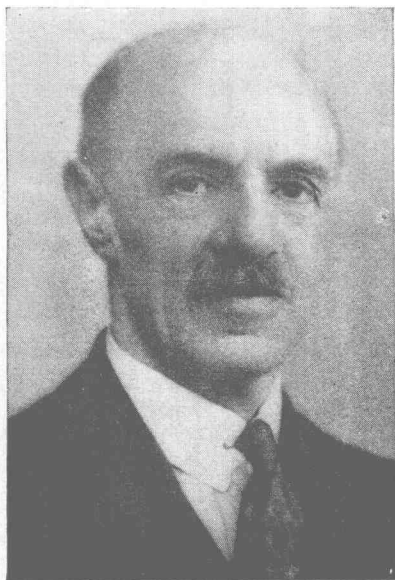
Alfred Binet



Truman L. Kelley



Lewis M. Terman



Charles E. Spearman

PREFACE

The primary aim of this volume is to teach the student of psychology how to deal effectively and intelligently with quantitative data. The typical student of statistical method is in his first year of graduate work and comes with the limited mathematical background of a single course in college algebra. Psychology has not yet reached the place where it can demand that all its graduate students shall have completed the equivalent of an undergraduate major in mathematics. The student must nevertheless become the master of certain statistical devices, must learn when to apply them and how to interpret the results. This can be accomplished, though with varying degrees of success, in a single intensive course. It is not possible to make a statistician of the average student, but he can be made to develop to some extent a sense of what is appropriate and what is not appropriate in the use of certain routine operations, and to grasp the meaning of what he does.

This volume is frankly concerned with applied statistics. There is a minimum of rationalizing of statistical processes and derivation of formulas. Statistical concepts are defined and their applications to concrete problems are discussed. References are made freely to secondary sources, since most primary sources in statistics are beyond the grasp of the average student.

A limited number of years of experience in assisting beginners in this field have led to certain convictions concerning the needs of the young investigator. It is one conviction of mine that experimental psychology is basic to all other fields, and that this is where the study of quantitative methods in psychology should naturally begin. In the past the student has typically been introduced to statistical methods in connection with problems in mental tests, and the student who is primarily concerned with experimental psychology rather than with educational psychology is left to apply what he can. On the other hand, if his only introduction to statistics has been in connection with purely experimental problems, his training

is too often deficient in some of the more refined procedures better known to students of mental tests.

For these reasons, the presentation in this volume begins with statistical methods as applied to experimental psychology, in particular to the psychophysical methods. From this point of departure the student is conducted ultimately to methods of correlation and their applications to mental-test procedures. Since Titchener's *Quantitative Manuals* appeared, some thirty years ago, no book in English has been devoted primarily or in great detail to the psychophysical methods. Contributions to that field have modestly though persistently made their appearances since that time, and in very recent years interest in the methods has shown a distinct revival. It is time that the gains in that field should be consolidated. Although the various methods for scaling stimuli, such as paired comparisons, equal-appearing intervals, and order of merit, have been used for many years, their relationships to the psychophysical methods have only recently been demonstrated. They now demand a systematic treatment in book form.

Another conviction of the author is that in the use of correlational methods too much attention has been paid to the amount of correlation between variables and far too little to the types of relationship between them. For this reason, methods of correlation are introduced in connection with a study of functional relationships, nonlinear as well as linear, and in connection with the method of least squares. Recent advances in correlational methods have been numerous, particularly within the past ten years, and we have gained much knowledge of these methods from their many applications in the field of psychological tests. Detailed and critical work with mental tests, which in these days are being studied item by item, has revealed that mental tests and psychophysical methods have much in common. And so these two great wings of psychological measurement, which historically have been so long independent, can be brought with justification within the pages of a single book.

We are rapidly approaching a time when a common rationalized basis can be elaborated for all the psychometric methods. The author has not attempted to attain the ideal here, but the common ground between the various methods is emphasized wherever possible. Last, but not least, the new subject of factor analysis has reached the stage of sufficient maturity that it can be given an established place in a book covering the field of mental measurement. The

name "Psychometric Methods," too long restricted to clinical tests and the like, is surely broad enough to encompass appropriately all the topics just mentioned.

The historical tone of the volume will be evident to the reader. In a book primarily for graduate students this aspect of mental measurement surely deserves mention. It helps to serve at the same time a unifying and a pedagogical function. It is doubtful whether any technique can be fully appreciated and understood without knowing something of its evolution. The device of presenting each new method in connection with a concrete experimental problem is a purely pedagogical one. In this I have kept in mind the psychological truism that the individual "learns by doing." At the end of each chapter, lists of problems present data which may be employed by the student in lieu of data obtained experimentally by him. Chapters II, III, X, and XI are fundamental. Any of the other chapters could be omitted from a course on statistics, if desired, without serious inconvenience.

My indebtedness must be expressed first of all to the many pioneers and investigators in statistical methods and in the applications of those methods to the problems of psychology. To my teachers I owe much, particularly to Professor K. M. Dallenbach for his introduction to psychophysics. From my students I have also gained much; among these I mention George R. Thornton, Harold Dahms, and Robert C. Hall, who have read and checked the manuscript for errors. Professor L. L. Thurstone has very kindly read in manuscript form the chapters on Paired Comparisons and Factor Analysis and Professor E. A. Culler the chapter on the Constant Methods. Professor Harry Helson has read and criticized most of the chapters. Dr. Quinn McNemar has kindly pointed out errors in fundamental statistical statements in a number of the chapters. Mrs. William W. Burke has improved the expression in many places. To all these I gladly render thanks and at the same time relieve them of any responsibility for errors that may still remain. To my wife, Ruth B. Guilford, I am most indebted for her continual help in the preparation of the manuscript and in its revision.

J. P. GUILFORD

EVANSTON, ILLINOIS,
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PSYCHOMETRIC METHODS

CHAPTER I

A GENERAL INTRODUCTION TO MENTAL MEASUREMENT

The great philosopher Kant once asserted that psychology could never rise to the dignity of a natural science because it is not possible to apply quantitative methods to its data. The *sine qua non* of a science, according to Kant, is measurement and the mathematical treatment of its data. Could Kant but glance at one of the contemporary journals of psychology, he would be amazed at the display of means, sigmas, critical ratios, coefficients of correlation, and other signs of statistical sophistication exhibited by those today called psychologists. If he were not impressed with the fact that psychology has at last become a science, he would at least be forced to conclude that psychologists as a group are expending an enormous amount of energy in maintaining a pretense that their work is science. Were his interests aroused sufficiently to examine some of the fruits of the many hours of labor at the calculating machine, he would be forced to realize that present-day methods, arduous as they are, carry us far beyond the armchair speculations of the psychologist of his day. Were he to question the motive that compels the modern investigator to phrase his conclusions in terms of probable errors and significance of differences, he would see in this urge an epitome of our struggle for objectivity. Objectivity is after all the touchstone of science, and quantitative methods are only a means to that end.

THE MEANING OF MEASUREMENT

Measurement Requires a Constant Unit.—We shall not attempt to formulate a short and comprehensive definition of measurement here. To state a formal definition invites controversy at the very outset. It should be sufficient for the sake of discussion to say that in making a measurement we assign numbers to phenomena and

those numbers may be added or subtracted. Our general conception of measurement dates from the most primitive of examples, that of measuring linear extents. This pencil, for example, is 15 cm. long and that one is 20 cm. The latter is 5 cm. longer than the former. When placed end to end, they extend over a space of 35 cm. Thus predictions can be made on the basis of this very elementary form of inference. Such inferences are valid only if the centimeter unit is universally constant. The constancy, and therefore the dependability of the unit, is the prime requirement of all measurement. Exact comparisons are impossible without constant units. The use of the centimeter, the gram, and the second have placed the physical sciences in a rather enviable position in the minds of those in the biological and social sciences. And yet it should be remembered that the greatest caution must be used in order to guarantee that those units shall be universally comparable. The lengths of measuring rods and other devices change with variations in temperature and other conditions. There is not a perfect centimeter, gram, or second to be found anywhere. All measuring devices are merely approximations, some better and some worse.

An Absolute Zero Desirable but Not Essential.—Many measuring scales, in addition to having a constant unit, start at an absolute zero point: zero length, zero weight, or zero time. Unfortunately, this cannot be true of many measuring scales, especially in psychology and the social sciences. It has not always been true of some physical scales. For many years the scale of temperatures lacked an absolute zero point. When the temperature of water rose from 30 to 45°C. it could merely be said that there was an increase of 15°. One could not also say that the increase was 50 per cent, nor could one say that 60° is twice as warm as 30°. Thus, although it is possible to make useful measurements with a scale that has no absolute zero point, the usefulness of those measurements is limited. Some psychological scales have been useful even when they lacked an absolute zero point and even when the units were probably not constant, for example, the mental-age scale for intelligence.

While a year is a year, to be sure, the increase of a year from age ten to eleven is now known not to be equivalent to an increase of a year from age four to five. Even assuming that the mental-age year were a constant unit, because there is no absolute zero point in the mental-age scale, we cannot say that a mental age of ten is twice as great as a mental age of five. Thanks to a rational solution of the