

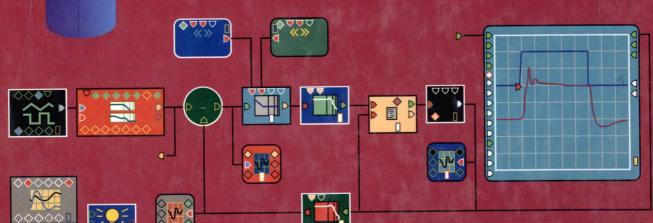


# GEORGE ELLIS

# CONTROL SYSTEM DESIGN GUIDE

USING YOUR COMPUTER TO UNDERSTAND AND DIAGNOSE FEEDBACK CONTROLLERS

THIRD EDITION



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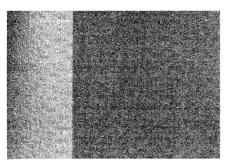
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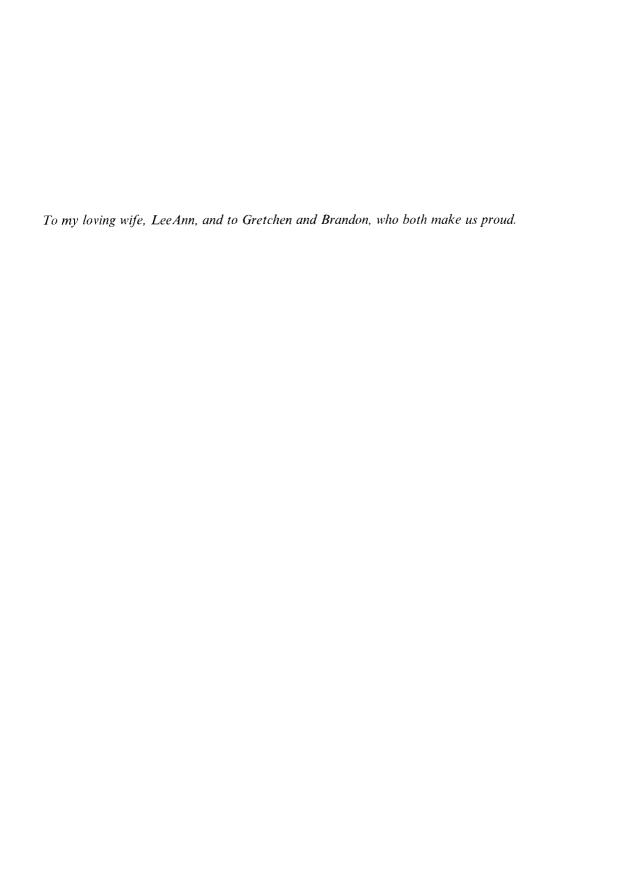
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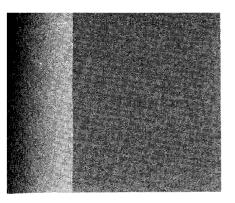
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# **Control System Design Guide**

Third Edition





### **Preface**

The basics of control systems were developed in the first half of the 20th century. Our predecessors aimed a cannon or warmed a bath using many of the same concepts we use. Of course, time and technology have generated many refinements. Digital processors have changed the way we implement a control law, but in many cases they haven't changed the law itself. Proportional integral differential (PID) control works about the same today as it did four or five decades ago.

Control systems are broadly used and are thus well integrated into our educational system. Courses are offered at most engineering universities, and a few disciplines even require students to undergo modest training in the subject. Given the longevity of the principles and the number of trained engineers engaged in their use, one might expect most of the trade's practitioners to be comfortable with the basics. Unfortunately, that does not seem to be the case.

Over the past several years, I've had the opportunity to teach a total of about 1500 engineers through a daylong seminar entitled "How to Improve Servo Systems." These are motivated people, willing to spend time listening to someone who might provide insight into the problems they face. Most are degreed engineers who work in industry; roughly half have taken at least one controls course. A few minutes into the seminar, I usually ask, "How many of you regularly apply principles of controls you learned at school?" Normally, fewer than one in ten raises a hand. It's clear there is a gap between what is taught and what is used.

So why the gap? It might be because the subject of controls is so often taught with an undue emphasis on mathematics. Intuition is abandoned as students learn how to calculate and plot one effect after another, often only vaguely understanding the significance of the exercise. I was one of those students years ago. I enjoyed controls and did well in all my controls classes, but I graduated unable to design or even tune a simple PI control system.

It doesn't have to be that way. You can develop a feel for controls! This book endeavors to help you do just that. Principles are presented along with practical methods of analysis. Dozens of models are used to help you practice the material,

for practice is the most reliable way to gain fluency. A goal of every chapter is to foster intuition.

#### What's New in This Edition?

This third edition of *Control System Design Guide* includes several improvements over the previous edition. First, *ModelQ*, the modeling environment from the second edition, has been rewritten to create *Visual ModelQ*; the preprogrammed models have been replaced with a fully graphical modeling environment. You should find it easier to follow what is being modeled. Second, two chapters have been added, both concerning observers: Chapter 10 is a general presentation of observers; Chapter 18 focuses on observers in motion-control systems. I hope these presentations will convey the power of these remarkable software mechanisms as well as the ease with which they can be implemented. Also, a question set has been added to the end of almost every chapter, with answers provided in Appendix G.

#### Organization of the Book

The book is organized into three sections. Section I, Applied Principles of Controls, consists of ten chapters. Chapter 1, Introduction to Controls, discusses the role of controls and controls engineers in industry. Chapter 2, The Frequency Domain, reviews the *s*-domain, the basis of control systems. Chapter 3, Tuning a Control System, gives you an opportunity to practice tuning; for many, this is the most difficult part of commissioning control systems.

Chapter 4, Delay in Digital Controllers, culls out the fundamental difference in the application of digital and analog controllers, the contribution of instability from sample delay. Chapter 5, The z-Domain, discusses z-transforms, the technique that extends the s-domain to digital control. Chapter 6, Six Types of Controllers, covers practical issues in the selection and use of six variations of PID control. Chapter 7, Disturbance Response, provides a detailed discussion of how control systems react to inputs other than the command. Chapter 8. Feed-Forward, presents techniques that can substantially improve command response. Chapter 9, Filters in Control Systems, discusses the use of filters in both analog and digital controllers. Chapter 10, Introduction to Observers in Control Systems, is a general presentation of observers.

Section II, Modeling, has three chapters. Chapter 11, Introduction to Modeling, provides overviews of time- and frequency-domain modeling methods. Chapter 12, Nonlinear Behavior and Time Variation, addresses how to deal with nonlinear operation when using linear control techniques. Unfortunately, this subject is missing from most texts on controls, although significant nonlinear effects are common in industrial applications. Chapter 13, Seven Steps to Developing a Model, gives a step-by-step procedure for developing models.

Section III, Motion Control, concentrates entirely on motion control using electric servomotors. Chapter 14, Encoders and Resolvers, discusses the most common feedback sensors used with electric servomotors. Chapter 15, Basics of the Electric Servomotor and Drive, reviews the operation of these motors. Chapter 16, Compliance and Resonance, is dedicated to one of the most widely felt problems in motion control, instability resulting from mechanical resonance. Chapter 17, Position-Control Loops, addresses the control of position, since the great majority of applications control position rather than velocity or torque. Chapter 18, Using the Luenberger Observer in Motion Control, focuses on observers in motion-control systems.

#### Reader Feedback

I have endeavored to right the errors of the second edition; for those errata that slip through into this edition, corrections will be posted at qxdesign.com. Please feel free to contact me at george.ellis@DanaherMotion.com or gellis@qxdesign.com.

#### Acknowledgments

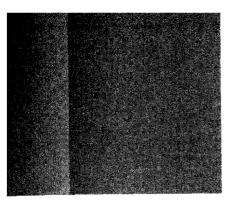
Writing a book is a large task and requires support from numerous people, and those people deserve thanks. First, I thank LeeAnn, my devoted wife of more than 25 years. She has been an unflagging fan, a counselor, and a demanding editor. She taught me much of what I have managed to learn about how to express a thought in writing. Thanks also to my mother, who, when facts should have dissuaded her, was sure I would grow into someone of whom she would be proud. And thanks to my father, for his unending insistence that I obtain a college education, a privilege that was denied to him, an intelligent man born into a family of modest means.

I am grateful for the education provided by Virginia Tech. Go Hokies. The basics of electrical engineering imparted to me over my years at school allowed me to grasp the concepts I apply regularly today. I am grateful to Mr. Emory Pace, a tough professor who led me through numerous calculus courses and who, in doing so, gave me the confidence on which I relied throughout my college career and beyond. I am especially grateful to Dr. Charles Nunnally; having arrived at university from a successful career in industry, he provided my earliest exposure to the practical application of the material I strove to learn. I found him then, as now, an admirable combination of analytical skill and practical application.

I also thank Dr. Robert Lorenz of the University of Wisconsin at Madison, the man most influential in my education on controls since I left college. His instruction has been well founded, enlightening, and thoroughly practical. Several of his university courses are available in video format and are recommended for those who would like to extend their knowledge of controls. I took the video version of ME 746 and found it quite useful; much of the material of Chapter 7, Disturbance Response, is derived from that class.

#### xxiv ▶ PREFACE

Thanks to the people of Danaher (manufacturer of Kollmorgen products), my long-time employer, for their continuing support in the writing of this book. My gratitude to each of you is sincere.



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