

←——— 经济学前沿影印丛书 ———→

金融衍生工具 定价、应用与数学

Jamil Baz George Chacko 著

FINANCIAL DERIVATIVES
PRICING, APPLICATIONS, AND MATHEMATICS

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A P P L I C A T I O N S , A N D M A T H E M A T I C S



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I N A N C I A L D E R I V A T I V E S P R I C I N G ,

丛书序言

应北京大学出版社之邀，为《经济学前沿影印丛书》写序。

中国现代主流经济学的教育开始于十多年前，那时，随着我国改革开放的深入，一批在海外卓有成就的学者回到国内，他们的归来对中国经济学教育与国际接轨具有重要意义。此间，各高校相继成立了现代经济学教育和研究的中心，如武汉大学高级研究中心（当时叫做武汉大学经济科学高级研究中心）完全按照国际著名大学经济学与金融学系的模式培养学生，取得了巨大的成就，大量的学生被送到世界知名的大学攻读博士学位，这在以前是不可想像的。北京大学中国经济研究中心也在北京大学开办了双学位班，对本科生教育起了巨大的推动作用。

通过这批海外归来的学者的介绍，国内的出版社开始引进当代经济学的主流教材，国内读者看到的有比较初级层次的 Samuelson 的《经济学原理》、Mankiw 的《宏观经济学》等和比较高层次的 Mas-Colell 的《高级微观经济学》、Varian 的《高级微观经济学》等，这些教材大多数以翻译的形式引进，他们为国内年轻的经济学者和学生的成长提供了重要的基础，很多青年学生在这一批引进教材的影响下，学习了现代经济学的主流理论。

十多年过去了，随着我国经济的高速增长，世界各国经济学者对中国经济的研究兴趣日益浓厚，中国经济发展的很多问题已经成为经济学界所关注的主流问题之一。因此，我们不仅需要大量的青年学生走出去，而且需要培养大量的既懂我国经济实际又对现代经济学研究具有较好掌握的学者。引进的翻译教材使得国内的青年学生掌握了现代经济学基本理论，但是要真正掌握现代经济学的研究方法还有很长的路。北京大学出版社引进的《经济学前沿影印丛书》就是在这个前提下产生的。

首先，这一系列引进的经典著作，可以使得大量的国内经济学者和青年学生在不走出国门的情况下就能了解相应领域的经典之作，从这些经典之作中探寻现代经济学的研究方法。

其次，北京大学出版社引进的这些经典著作可以为国内经济学者和

青年学生节省大量的时间和金钱,一方面这些经典的著作都是出版社邀请众多有一定成就的学者直接推荐的,这样就可以大量地节省青年学生的搜寻成本,避免他们走弯路;另一方面,这些原版的经典著作定价一般很高,通过出版社的版权引进后可以为国内的读者节省 60% 以上的成本。我以为,这套丛书的推出将对中国的经济学研究产生不可估量的影响,是一件功德无量的事。

龚六堂

2005 年 7 月于北京大学光华管理学院

Financial Derivatives

This book offers a succinct account of the principles of financial derivatives pricing. The first chapter provides readers with an intuitive exposition of basic random calculus. Concepts such as volatility and time, random walks, geometric Brownian motion, and Itô's lemma are discussed heuristically. The second chapter develops generic pricing techniques for assets and derivatives, determining the notion of a stochastic discount factor or pricing kernel, and then uses this concept to price conventional and exotic derivatives. The third chapter applies the pricing concepts to the special case of interest rate markets, namely, bonds and swaps, and discusses factor models and term-structure-consistent models. The fourth chapter deals with a variety of mathematical topics that underlie derivatives pricing and portfolio allocation decisions, such as mean-reverting processes and jump processes, and discusses related tools of stochastic calculus, such as Kolmogorov equations, martingales techniques, stochastic control, and partial differential equations.

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To Maurice and Elena J.B.

To my parents G.C.

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Introduction

This book is about risk and derivative securities. In our opinion, no one has described the issue more eloquently than Jorge Luis Borges, an intrepid Argentinian writer. He tells a fictional story of a lottery in ancient Babylonia. The lottery is peculiar because it is compulsory. All subjects are required to play and to accept the outcome. If they lose, they stand to lose their wealth, their lives, or their loved ones. If they win, they will get mountains of gold, the spouse of their choice, and other wonderful goodies.

It is easy to see how this story is a metaphor of our lives. We are shaped daily by doses of randomness. This is where the providential financial engineer intervenes. The engineer's thoughts are along the following lines: to confront all this randomness, one needs artificial randomness of opposite sign, called derivative securities. And the engineer calls the ratio of these two random quantities a hedge ratio.

Financial engineering is about combining the Tinker Toys of capital markets and financial institutions to create custom risk-return profiles for economic agents. An important element of the financial engineering process is the valuation of the Tinker Toys; this is the central ingredient this book provides.

We have written this book with a view to the following two objectives:

- to introduce readers with a modicum of mathematical background to the valuation of derivatives

- to give them the tools and intuition to expand upon these results when necessary

By and large, textbooks on derivatives fall into two categories: the first is targeted toward MBA students and advanced undergraduates, and the second aims at finance or mathematics PhD students. The former tend to score high on breadth of coverage but do not go in depth into any specific area of derivatives. The latter tend to be highly rigorous and therefore limit the audience. While this book is closer to the second category, it strives to simplify the mathematical presentation and make it accessible to a wider audience. Concepts such as measure, functional spaces, and Lebesgue integrals are avoided altogether in the interest of all those who have a good knowledge of mathematics but yet have not ventured into advanced mathematics.

The target audience includes advanced undergraduates in mathematics, economics, and finance; graduate students in quantitative finance master's programs as well as PhD students in the aforementioned disciplines; and practitioners afflicted with an interest in derivatives pricing and mathematical curiosity.

The book assumes elementary knowledge of finance at the level of the Brealey and Myers corporate finance textbook. Notions such as discounting, net present value, spot and forward rates, and basic option pricing in a binomial model should be familiar to the reader. However, very little knowledge of economics is assumed, as we develop the required utility theory from first principles.

The level of mathematical preparation required to get through this book successfully comprises knowledge of differential and integral calculus, probability, and statistics. In calculus, readers need to know basic differentiation and integration rules and Taylor series expansions, and should have some familiarity with differential equations. Readers should have had the standard year-long sequence in probability and statistics. This includes conventional, discrete, and continuous probability distributions and related notions, such as their moment generating functions and characteristic functions.

The outline runs as follows:

1. Chapter 1 provides readers with the mathematical background to understand the valuation concepts developed in Chapters 2 and 3. It provides an intuitive exposition of basic random

calculus. Concepts such as volatility and time, random walks, geometric Brownian motion, and Itô's lemma are exposed heuristically and given, where possible, an intuitive interpretation. This chapter also offers a few appetizers that we call paradoxes of finance: these paradoxes explain why forward exchange rates are biased predictors of future rates; why stock investing looks like a free lunch; and why success in portfolio management might have more to do with luck than with skill.

2. Chapter 2 develops generic pricing techniques for assets and derivatives. The chapter starts from basic concepts of utility theory and builds on these concepts to derive the notion of a stochastic discount factor, or pricing kernel. Pricing kernels are then used as the basis for the derivation of all subsequent pricing results, including the Black-Scholes/Merton model. We also show how pricing kernels relate to the hedging, or dynamic replication, approach that is the origin of all modern valuation principles. The chapter concludes with several applications to equity derivatives to demonstrate the power of the tools that are developed.
3. Chapter 3 specializes the pricing concepts of Chapter 3 to interest rate markets; namely bonds, swaps, and other interest rate derivatives. It starts with elementary concepts such as yield-to-maturity, zero-coupon rates, and forward rates; then moves on to naïve measures of interest rate risk such as duration and convexity and their underlying assumptions. An overview of interest rate derivatives precedes pricing models for interest rate instruments. These models fall into two conventional families: factor models, to which the notion of price of risk is central, and term-structure-consistent models, which are partial equilibrium models of derivatives pricing. The chapter ends with an interpretation of interest rates as options.
4. Chapter 4 is an expansion of the mathematical results in Chapter 1. It deals with a variety of mathematical topics that underlie derivatives pricing and portfolio allocation decisions. It describes in some detail random processes such as random walks, arithmetic and geometric Brownian motion, mean-reverting processes and jump processes. This chapter also includes an exposition of the rules of Itô calculus and contrasts it with the

competing Stratonovitch calculus. Related tools of stochastic calculus such as Kolmogorov equations and martingales are also discussed. The last two sections elaborate on techniques widely used to solve portfolio choice and option pricing problems: dynamic programming and partial differential equations.

We think that one virtue of the book is that the chapters are largely independent. Chapter 1 is essential to the understanding of the continuous-time sections in Chapters 2 and 3. Chapter 4 may be read independently, though previous chapters illuminate the concepts developed in each chapter much more completely.

Why Chapter 4 is at the end and not the beginning of this book is an almost aesthetic undertaking: Some finance experts think of mathematics as a way to learn finance. Our point of view is different. We feel that the joy of learning is in the process and not in the outcome. We also feel that finance can be a great way to learn mathematics.