Wendy MacCaull
Michael Winter
Ivo Düntsch (Eds.)

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Relational Methods in Computer Science

8th International Seminar on Relational Methods in Computer Science
3rd International Workshop on Applications of Kleene Algebra
and Workshop of COST Action 274: TARSKI
St. Catharines, ON, Canada, February 2005
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Selected Revised Papers

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Preface

This volume is the post conference proceedings of the 8th International Seminar on Relational Methods in Computer Science (RelMiCS 8), held in conjunction with the 3rd International Workshop on Applications of Kleene Algebra and a COST Action 274 (TARSKI) Workshop. This combined meeting took place in St. Catharines, Ontario, Canada, from February 22 to February 26, 2005. The purpose of this meeting was to bring together researchers from various subdisciplines of computer science and mathematics who use the calculus of relations and/or Kleene algebra as methodological and conceptual tools in their work.

The meeting was a continuation of three different series of meetings. Previous RelMiCS meetings were held in:

- Schloß Dagstuhl, Germany, January 1994
- Parati (near Rio de Janeiro), Brazil, September 1995
- Hammamet, Tunisia, January 1997
- Stefan Banach Center, Warsaw, Poland, September 1998
- Valcartier (near Québec City), Canada, January 2000
- Osterwijk (near Tilburg), The Netherlands, October 2001
- Malente (near Kiel), Germany, May 2003

The first two workshops on applications of Kleene algebra were held Schloß Dagstuhl, Germany, February 2001 and Malente (near Kiel), Germany, May 2003. COST Workshops were held four times per year in various locations in the European Union since 2001. The substantial common interests and overlap of these communities motivated the joint meeting. Proceedings, edited by Düntsch and Winter and containing extended abstracts of the 34 accepted papers and abstracts of the 4 invited talks, were available at the conference. After the conference, a Call for Papers was issued.

This volume contains the 17 (full) papers accepted from the submissions at that time and three invited papers, "Topological Representation of Precontact Algebras" by Georgi Dimov and Dimiter Vakarelov, "Relational Semantics through Duality" by Ewa Orłowska, Ingrid Rewitzky and Ivo Düntsch and "Duality Theory for Projective Algebras" by Alasdair Urquhart. The contributed papers underwent a thorough refereeing process, in which each paper was sent to several referees. Contributed papers include such topics as static analysis of programs, representation theory, theories of programming, evolutionary algorithms, verification and quantifier elimination.

We are grateful to the members of the Program Committee and to the many people who acted as external referees and who must remain anonymous. Their efforts have ensured the high quality of the papers in this volume.

January 2006

Wendy MacCaull
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Topological Representation of Precontact Algebras*

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Abstract. The notions of *2-precontact* and *2-contact spaces* as well as of *extensional* (and other kinds) *3-precontact* and *3-contact spaces* are introduced. Using them, new representation theorems for precontact and contact algebras (satisfying some additional axioms) are proved. They incorporate and strengthen both the discrete and topological representation theorems from [3, 1, 2, 4, 10]. It is shown that there are bijective correspondences between such kinds of algebras and such kinds of spaces. In particular, such a bijective correspondence for the RCC systems of [8] is obtained, strengthening in this way the previous representation theorems from [4, 1].

1 Introduction

In this paper we present a common approach both to the discrete and to the non-discrete region-based theory of space. It is a continuation of the line of investigation started in [10] and continued in [1, 2].

Standard models of non-discrete theories of space are the contact algebras of regular closed subsets of some topological spaces ([10, 1, 2, 4]). In a sense these topological models reflect the continuous nature of the space. However, in the "real-world" applications, where digital methods of modeling are used, the continuous models of space are not so much suitable. This motivates a search for good "discrete" versions of the theory of space. One kind of discrete models are the so called *adjacency spaces*, introduced by Galton [6] and generalized by Düntsch and Vakarelov in [3]. Based on the Galton's approach, Li and Ying [7] presented a "discrete" generalization of the Region Connection Calculus (RCC). The latter, introduced in [8], is one of the main systems in the non-discrete region-based theory of space. A natural class of Boolean algebras related to adjacency spaces are the *precontact algebras*, introduced in [3] under the name of *proximity algebras*. The notion of precontact algebra is a generalization of the notion of contact algebra. Each adjacency space generates canonically a precontact algebra. It is proved in [3] (using another terminology) that each precontact algebra can be embedded in the precontact algebra of an adjacency space. In [1]

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we prove that each contact algebra can be embedded in the standard contact algebra of a semiregular T_0-space, answering the question of Düntsch and Winter, posed in [4], whether the contact algebras have a topological representation. This shows that contact algebras possess both a discrete and a non-discrete (topological) representation. In this paper we extend the representation techniques developed in [1, 2] to precontact algebras, proving that each precontact algebra can be embedded in a special topological object, called a *2-precontact space*. We also establish a bijective correspondence between all precontact algebras and all 2-precontact spaces. This result is new even in the special case of contact algebras: introducing the notion of *2-contact space* as a specialization of 2-precontact space, we show that there is a bijective correspondence between all contact algebras and all 2-contact spaces. Similar representation theorems hold also for precontact and contact algebras satisfying some additional axioms, namely, for *extensional* (resp., *N-regular; regular; normal*) *precontact* and *contact algebras*. The topological objects that correspond to these algebras are introduced here under the names of *extensional* (resp., *N-regular; regular; normal*) *3-precontact* and *3-contact spaces*.

The paper is organized as follows. In Section 1 we introduce the notions of precontact and contact algebra and give the two main examples of them: the precontact algebras on adjacency spaces, and the contact algebras on topological spaces. In Section 2 we introduce different kinds of points in precontact algebras: ultrafilters, clans, maximal clans, clusters, co-ends. The notions of *topological adjacency space* and *Stone adjacency space* are introduced and our first representation theorem for precontact algebras is proved there. In Section 3 we introduce the notions of *2-precontact space* and *canonical precontact algebra of a 2-precontact space*. In Section 4 we associate with each precontact algebra $\mathbf{B}$ a 2-precontact space, called the *canonical 2-precontact space of* $\mathbf{B}$. In Section 5 we present the main theorem of the paper, the representation theorem for precontact algebras. In Section 6 we introduce the notion of *2-contact space* and we prove that there exists a bijective correspondence between all (up to isomorphism) contact algebras and all (up to isomorphism) 2-contact spaces. This is a generalization of the similar result about complete contact algebras obtained in [1]. In Section 7 we introduce the axiom of extensionality for precontact algebras which generalizes the well-known axiom of extensionality for contact algebras. In this way we obtain a class of precontact algebras containing as a subclass some well-known systems as, for example, RCC systems from [8]. We modify the representation constructions in order to obtain topological representation theorems for extensional precontact algebras. The notions of *extensional 3-precontact space* and *extensional 3-contact space* are introduced there and it is proved that there exists a bijective correspondence between all (up to isomorphism) extensional precontact algebras (resp., extensional contact algebras) and all (up to isomorphism) extensional 3-precontact spaces (resp., extensional 3-contact spaces). This is a generalization of a similar result about complete extensional contact algebras obtained in [1]. In Section 8 we extend the results for the extensional precontact and contact algebras to many other kinds of precontact and contact algebras.

The main reference book for all undefined in the paper topological notions is [5]. In this paper we present only the scheme of the proofs (i.e., all lemmas and propositions which are used in the proofs). The detailed proofs will be given in the full version of this paper.

2 Precontact Algebras

Definition 1. *An algebraic system* $\mathbf{B} = (B, C)$ *is called a* **precontact algebra** *([3]) (abbreviated as PCA) if the following holds:*

- $B = (B, 0, 1, +, ., *)$ *is a Boolean algebra (where the complement is denoted by "$*$");*
- C *is a binary relation on* B*, called the* **precontact relation**, *which satisfies the following axioms:*

$(C0)$ *If* aCb *then* $a, b \neq 0$;
$(C+)$ $aC(b+c)$ *iff* aCb *or* aCc; $(a+b)Cc$ *iff* aCc *or* bCc.

A precontact algebra (B, C) *is said to be* **complete** *if the Boolean algebra* B *is complete. Two precontact algebras* $\mathbf{B} = (B, C)$ *and* $\mathbf{B_1} = (B_1, C_1)$ *are said to be* **PCA-isomorphic** *(or, simply,* **isomorphic***) if there exists a Boolean isomorphism* $\varphi : B \longrightarrow B_1$ *such that, for every* $a, b \in B$, aCb *iff* $\varphi(a) C_1 \varphi(b)$.

The negation of the relation C *is denoted by* $(-C)$.

Let us define the **non-tangential inclusion** "$\ll_C$" *by* $a \ll_C b$ *iff* $a(-C)b^*$.

We will also consider precontact algebras satisfying some additional axioms:

$(Cref)$ *If* $a \neq 0$ *then* aCa *(reflexivity axiom);*
$(Csym)$ *If* aCb *then* bCa *(symmetry axiom);*
(Ctr) *If* $a \ll_C c$ *then* $(\exists b)(a \ll_C b \ll_C c)$ *(transitivity axiom);*
$(Ccon)$ *If* $a \neq 0, 1$ *then* aCa^* *or* a^*Ca *(connectedness axiom).*

A precontact algebra (B, C) *is called a* **contact algebra** *([1]) (and* C *is called a* **contact relation***) if it satisfies the axioms* $(Cref)$ *and* $(Csym)$. *A precontact algebra* (B, C) *is called* **connected** *if it satisfies the axiom* $(Ccon)$.

The following lemma says that in every precontact algebra we can define a contact relation.

Lemma 1. *Let* (B, C) *be a precontact algebra and define* $aC^{\#}b$ *iff* aCb *or* bCa *or* $a.b \neq 0$. *Then* $C^{\#}$ *is a contact relation on* B *and hence* $(B, C^{\#})$ *is a contact algebra.*

Remark 1. We will also consider precontact algebras satisfying the following variant of the transitivity axiom (Ctr):
$(Ctr\#)$ If $a \ll_{C^{\#}} c$ then $(\exists b)(a \ll_{C^{\#}} b \ll_{C^{\#}} c)$.

Examples of Precontact and Contact Algebras

1. Precontact algebras on adjacency spaces. (Galton [6], Düntsch and Vakarelov [3])

By an **adjacency space** we mean a relational system (W, R) where W is a non-empty set whose elements are called **cells**, and R is a binary relation on W called the **adjacency relation**; the subsets of W are called **regions**.

The reflexive and symmetric closure $R^\flat$ of R is defined as follows: $xR^\flat y$ iff xRy or yRx or $x = y$.

A precontact relation C_R between the regions of an adjacency space (W, R) is defined as follows: for every $a, b \subseteq W$,

$$aC_Rb \text{ iff } (\exists x \in a)(\exists y \in b)(xRy). \tag{1}$$

Proposition 1. ([3]) *Let (W, R) be an adjacency space and let 2^W be the Boolean algebra of all subsets of W. Then:*

(a) $(2^W, C_R)$ is a precontact algebra;
(b) $(2^W, C_R)$ is a contact algebra iff R is a reflexive and symmetric relation on W. If R is a reflexive and symmetric relation on W then C_R coincides with $(C_R)^\#$ and $C_{R^\flat}$;
(c) C_R satisfies the axiom (Ctr) iff R is a transitive relation on W;
(d) C_R satisfies the axiom $(Ccon)$ iff R is a connected relation on W (which means that if $x, y \in W$ and $x \neq y$ then there is an R-path from x to y or from y to x).

Theorem 1. ([3]) *Each precontact algebra (B, C) can be isomorphically embedded in the precontact algebra $(2^W, C_R)$ of some adjacency space (W, R). Moreover if (B, C) satisfies some of the axioms $(Cref)$, $(Csym)$, (Ctr) then the relation R is, respectively, reflexive, symmetric, transitive.*

2. Contact algebras on topological spaces. Let X be a topological space and let $RC(X)$ be the set of all regular closed subsets of X (recall that a subset F of X is said to be *regular closed* if $F = cl(int(F))$). Let us equip $RC(X)$ with the following Boolean operations and contact relation C_X:

- $a + b = a \cup b$;
- $a^* = cl(X \setminus a)$;
- $a.b = (a^* \cup b^*)^*$;
- $0 = \emptyset$, $1 = X$;
- aC_Xb iff $a \cap b \neq \emptyset$.

The following lemma is a well-known fact.

Lemma 2. $(RC(X), C_X) = (RC(X), 0, 1, +, ., *, C_X)$ *is a contact algebra.*

Recall that a space X is said to be **semiregular** if $RC(X)$ is a closed base for X.

The following theorem answers the question, posed by Düntsch and Winter in [4], whether contact algebras have a topological representation:

Theorem 2. ([1]) *Each contact algebra (B, C) can be isomorphically embedded in the contact algebra $(RC(X), C_X)$ of some semiregular T_0-space X. The algebra (B, C) is connected iff the space X is connected.*

The aim of this work is to generalize Theorem 1 and Theorem 2 in several ways: to find a topological representation of precontact algebras which incorporates both the "discrete" and the "continuous" nature of the space; to find representation theorems in the style of the Stone representation of Boolean algebras instead of embedding theorems; to establish, again as in the Stone theory, a bijective correspondence between precontact algebras and the corresponding topological objects; to extend this new representation theory to different classes of precontact algebras, satisfying some natural additional axioms.

3 Points in Precontact Algebras

In this section we will introduce different kinds of abstract points in precontact algebras: ultrafilters, clans, maximal clans, clusters and co-ends. This is done by analogy with the case of contact algebras (see, e.g., [1, 10]). We assume that the notions of a filter and ultrafilter in a Boolean algebra are familiar. Clans were introduced by Thron [9] in proximity theory. Our definition is a lattice generalization of Thron's definition.

The set of all ultrafilters of a Boolean algebra B is denoted by $Ult(B)$.

Definition 2. *Let $\mathbf{B} = (B, C)$ be a precontact algebra. A non-empty subset Γ of B is called a* **clan** *if it satisfies the following conditions:*

$(Clan1)$ $0 \notin \Gamma$;
$(Clan2)$ *If* $a \in \Gamma$ *and* $a \leq b$ *then* $b \in \Gamma$;
$(Clan3)$ *If* $a + b \in \Gamma$ *then* $a \in \Gamma$ *or* $b \in \Gamma$;
$(Clan4)$ *If* $a, b \in \Gamma$ *then* $aC^{\#}b$.

A clan Γ in $\mathbf{B}$ is called a **maximal clan** *in $\mathbf{B}$ if it is maximal among all clans in $\mathbf{B}$ with respect to set-inclusion.*

The set of all clans (resp., maximal clans) of a precontact algebra $\mathbf{B}$ is denoted by $Clans(\mathbf{B})$ (resp., $MClans(\mathbf{B})$).

The following lemma is obvious:

Lemma 3. *Each ultrafilter is a clan and hence $Ult(B) \subseteq Clans(\mathbf{B})$.*

We will define a binary relation R between ultrafilters in B which will make the set $Ult(B)$ an adjacency space.

Definition 3. *Let $\mathbf{B} = (B, C)$ be a precontact algebra and let U_1, U_2 be ultrafilters. We set*

$$U_1 R U_2 \text{ iff } (\forall a \in U_1)(\forall b \in U_2)(aCb) (\text{ i.e., iff } U_1 \times U_2 \subseteq C). \tag{2}$$

The relational system $(Ult(B), R)$ is called the **canonical adjacency space of $\mathbf{B}$**.

We say that U_1, U_2 are **connected** *iff $U_1 R^b U_2$, where R^b is the reflexive and symmetric closure of R.*

Lemma 4. *Let I be a set of connected ultrafilters. Then the union $\bigcup\{U \mid U \in I\}$ is a clan and every clan can be obtained in this way.*

Lemma 5. ([1,3]) **Ultrafilter and clan characterizations of precontact and contact relations.**

Let $\mathbf{B} = (B, C)$ be a precontact algebra and $(Ult(B), R)$ be the canonical adjacency space on $\mathbf{B}$. Then the following is true for any $a, b \in B$:

(a) aCb *iff* $(\exists U_1, U_2 \in Ult(B))(U_1 R U_2)$;
(b) $aC^{\#}b$ *iff* $(\exists U_1, U_2 \in Ult(B))(U_1 R^b U_2)$;
(c) $aC^{\#}b$ *iff* $(\exists \Gamma \in Clans(\mathbf{B})(a, b \in \Gamma)$ *iff* $(\exists \Gamma \in MClans(\mathbf{B})(a, b \in \Gamma)$;
(d) R *is a reflexive relation iff* $\mathbf{B}$ *satisfies the axiom* $(Cref)$.;
(e) R *is a symmetric relation iff* $\mathbf{B}$ *satisfies the axiom* $(Csym)$;
(f) R *is a transitive relation iff* $\mathbf{B}$ *satisfies the axiom* (Ctr).

Definition 4. Clusters and co-ends. *Let $\mathbf{B} = (B, C)$ be a precontact algebra.*
• *A clan Γ in $\mathbf{B}$ is called a* **cluster** *in $\mathbf{B}$ if it satisfies the following condition:*
(Clust) *If for every $x \in \Gamma$ we have $xC^{\#}y$ then $y \in \Gamma$.*
• *A clan Γ in $\mathbf{B}$ is called a* **co-end** *in $\mathbf{B}$ if it satisfies the following condition:*
(Coend) *If $x \notin \Gamma$ then there exists a $y \notin \Gamma$ such that $x(-C^{\#})y^*$.*

The set of all clusters (resp., co-ends) in $\mathbf{B}$ is denoted by $Clust(\mathbf{B})$ (resp., $Coend(\mathbf{B})$).

Stone Adjacency Spaces and Representation of Precontact Algebras

Definition 5. *An adjacency space (X, R) is called a* **topological adjacency space** *(abbreviated as TAS) if X is a topological space and R is a closed relation on X. When X is a compact Hausdorff zero-dimensional space (i.e. X is a* Stone space*), we say that the topological adjacency space (X, R) is a* **Stone adjacency space**.

Two topological adjacency spaces (X, R) and (X_1, R_1) are said to be **TAS-isomorphic** *if there exists a homeomorphism $f : X \longrightarrow X_1$ such that, for every $x, y \in X$, xRy iff $f(x)R_1 f(y)$.*

If X is a topological space, we denote by $Clopen(X)$ the set of all clopen subsets of X.

Now we can obtain the following strengthening of Theorem 1:

Theorem 3. *(a) Each precontact algebra (B, C) is isomorphic to the precontact algebra $(Clopen(X), C_R)$ of a Stone adjacency space (X, R), where X is the Stone space of the Boolean algebra B and the isomorphism is just the Stone map $s : B \longrightarrow 2^{Ult(B)}$, $a \longrightarrow \{U \in Ult(B) \mid a \in U\}$. Moreover, the relation C satisfies the axiom (Cref) (resp., (Csym); (Ctr)) iff the relation R is reflexive (resp., symmetric; transitive).*

(b) There exists a bijective correspondence between the class of all (up to PCA-isomorphism) precontact algebras and the class of all (up to TAS-isomorphism) Stone adjacency spaces (X, R).

As it is shown in [3], there is no bijective correspondence between precontact algebras and adjacency spaces (up to isomorphisms). Hence, the role of the topology in Theorem 3 is essential. However, Theorem 3 is not completely satisfactory because the representation of the precontact algebras (B, C) obtained here does not give a topological representation of the contact algebras $(B, C^{\#})$ generated by (B, C): we would like to have an isomorphism f such that, for every $a, b \in B$, aCb iff $f(a)C_R f(b)$, and $aC^{\#}b$ iff $f(a) \cap f(b) \neq \emptyset$ (see (1) for C_R). The isomorphism s in Theorem 3 is not of this type. Indeed, there are many examples of contact algebras (B, C) where $a.b = 0$ (and hence $s(a) \cap s(b) = \emptyset$) but aCb (note that C and $C^{\#}$ coincide for contact algebras). We now construct some natural topological objects which correspond bijectively to the precontact algebras and satisfy the above requirement. In the case when (B, C) is a contact algebra (satisfying some additional axioms), we will show that these topological objects are just topological pairs (resp., topological triples) satisfying some natural conditions. In such a way we will obtain new representation theorems for the contact algebras (satisfying some additional axioms), completely different from those given in [10, 1, 2, 4]. In particular, we will describe the topological triples which correspond bijectively to the connected extensional contact algebras, i.e. to the RCC systems of Randel, Cui and Cohn [8], strengthening in such a way the representation theorems for RCC systems given in [4, 1].

4 2-Precontact Spaces

Definition 6. *(a) Let X be a topological space and X_0, X_1 be dense subspaces of X. Then the pair (X, X_0) is called a* **topological pair** *and the triple (X, X_0, X_1) is called a* **topological triple**.

(b) Let (X, X_0) be a topological pair. Then we set

$$RC(X, X_0) = \{cl_X(A) \mid A \in Clopen(X_0)\}. \tag{3}$$

(c) Let (X, X_0, X_1) be a topological triple. Then we set

$$RC(X, X_0, X_1) = \{X_1 \cap cl_X(A) \mid A \in Clopen(X_0)\}. \tag{4}$$

Lemma 6. *(a) Let (X, X_0) be a topological pair. Then $RC(X, X_0) \subseteq RC(X)$; the set $RC(X, X_0)$ with the standard Boolean operations on the regular closed subsets of X is a Boolean subalgebra of $RC(X)$; $RC(X, X_0)$ is isomorphic to the Boolean algebra $Clopen(X_0)$; the sets $RC(X)$ and $RC(X, X_0)$ coincide iff X_0 is an extremally disconnected space. If $C_{(X,X_0)}$ is the restriction of the contact relation C_X (see Lemma 2) to $RC(X, X_0)$ then $(RC(X, X_0), C_{(X,X_0)})$ is a contact subalgebra of $(RC(X), C_X)$.*

(b) Let (X, X_0, X_1) be a topological triple. Then $RC(X, X_0, X_1) \subseteq RC(X_1)$; the set $RC(X, X_0, X_1)$ with the standard Boolean operations on the regular closed subsets of X_1 is a Boolean subalgebra of the Boolean algebra $RC(X_1)$; the Boolean algebra $Clopen(X_0)$ is isomorphic to $RC(X, X_0, X_1)$; the sets $RC(X, X_0, X_1)$ and $RC(X_1)$ coincide iff X_0 is an extremally disconnected space. Let us denote

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