Noise and Vibration Control

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Preface

This book is written for the engineer who wishes to solve a noise control problem. The material is graded in technical level, starting with the fundamentals of sound in air, scales that are used to measure sound, and the measurement and analysis of noise and vibration; and followed by methods of controlling noise and vibration and the so-called criteria for noise control. The text is a successor to *Noise Reduction* (McGraw-Hill, 1960), though only three of its eighteen chapters have the same authors as in the 1960 book.

Much of the material included in the present volume was developed for Special Summer Programs on Noise and Vibration Reduction offered at the Massachusetts Institute of Technology through the years since 1953. Other material was taken from the general acoustical literature and from the research and consultation practice of the staff of Bolt Beranek and Newman Inc.

I wish to express my warm appreciation to my co-authors both outside and inside Bolt Beranek and Newman Inc. for their unstinting effort toward producing authoritative and broadly applicable chapters. Parenthetically, those authors whose affiliations are not designated on

the chapters are members of the BBN staff. I wish especially to thank Tony F. W. Embleton who assisted me at two recent Special Summer Programs at M.I.T., and Robert W. Young who made detailed comments on four chapters. I am indebted to O. L. Angevine, Jr., Peter K. Baade, James H. Botsford, Robert A. Heath, Ralph Huntley, Alan H. Marsh, Helmut A. Mueller, T. D. Northwood, Douglas W. Robinson, Hale J. Sabine, S. S. Stevens, Dean G. Thomas, Clayton H. Allen, Erich K. Bender, Warren E. Blazier, James J. Coles, Charles W. Dietrich, Ira Dyer, Parker W. Hirtle, Robert M. Hoover, George W. Kamperman, David H. Kaye, Edward M. Kerwin, Jr., Ronald L. McKay, Laymon N. Miller, Robert B. Newman, Denis U. Noiseux, and Bill G. Watters for their help and criticism during the preparation of the book.

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Leo L. Beranek

Introduction

Virtually every problem in noise and vibration control involves a system composed of three basic elements: a source, a path, and a receiver. Before a solution to a complex noise problem can be designed, the dominant source of the noise must be known, the characteristics of the significant transmission paths must be understood, and a criterion for the level of noise considered permissible or desirable in this situation must be available.

These three elements of the noise problem do not necessarily act independently. The sound power that is radiated depends on the environment surrounding the source. For example, a machine may radiate more sound if it is placed in the corner of a room rather than in some other location. A speaker raises or lowers his voice depending on the size and reverberation characteristics of the room in which he is talking. The path of the sound may be affected by the acoustical details of the source and the receiver, as well as by their heights above the ground. A listener's judgement of noisiness may depend on whether he is working with his hands, concentrating on a creative task, conversing, listening to music, or trying to sleep.

His attitude toward noise may be influenced not only by the nature of the path and the spectrum of the noise, but also by economic or psychological factors such as a bonus for working hours spent in noise, or the fear of audiological or financial consequences. All these considerations emphasize that each noise problem involves a complex system of interacting elements.

Noise "control" does not always involve reduction of the unwanted sound. In the modern open-plan office building, a background "noise" with carefully controlled tonal quality and loudness may be introduced into the space through concealed sources in order to "mask" or cover unwanted noises made by the occupants and their typewriters and calculators, the elevators, and the street traffic. Thus noisiness, the annoyance caused by noise, can sometimes be reduced by adding more "noise."

Solving a noise control problem usually involves a tradeoff. The cost of protecting equally the hearing of every worker in a manufacturing plant may be so prohibitive that a higher risk of damage would have to be accepted for some of the personnel. The solution might consist of a combination of partial noise-reduction measures, the institution of an audiological testing program to select those with sensitive ears who should be transferred to other jobs, and a plan to compensate the remaining few who might suffer some hearing loss.

Noise Control at the Source. A noise source is created by the motion of a solid, liquid, or gas. A solid source may be quieted if its mode of operation is changed so that it moves less, for example, by reducing the forces that cause motions and by strengthening, damping, or isolating all or parts of the structure. Liquid and gaseous sources may be quieted by eliminating turbulence, reducing flow velocity, smoothing flow, and attenuating pressure pulsations. Control at the source by planning while the product is in the design stage is often the most effective and least expensive of control measures.

Noise Control in the Path. Most corrective measures for an existing noise control problem utilize changes in the path. Consequently, a large part of the present book is devoted to that subject. Involved are control of sound propagation out of doors, in rooms, in structures, and in ducts. Solutions include barriers, porous materials, plugs, caulking, bracing, mufflers, enclosures, vibration damping, and vibration isolation.

The Demands of the Receiver. The level to which a noise should be reduced to be acceptable to human receivers often requires judgment on the part of the engineer and the owner of a building or machine. A criterion for noise control for listeners depends on whether the goal is to preserve hearing, to create space where conversation is easy, or to provide comfort in the home, at work, or in transportation vehicles. The first two goals for noise control are fairly accurately quantifiable, but comfort may depend on mental attitudes, which may be elusive or which may change on short notice.

Contents

Chapter One

The Behavior of Sound Waves

PETER A. FRANKEN

1.1 The Nature of Sound

Sound is a disturbance that propagates through an elastic material at a speed characteristic of that medium. This brief sentence, when converted into quantitative terms, contains a large amount of scientific information that constitutes the basis of the science of acoustics. When applying the science of sound to the practical task of noise and vibration control, to which this book is restricted, we do not need to reveal a detailed physical understanding of sound waves. Rather we shall draw directly on the results of fundamental studies as necessary, and refer the interested reader to basic literature for further details.

The simple definition of sound above suggests that sound can be sensed by the measurement of some physical quantity in the medium that is disturbed from its equilibrium value. The physical quantity that is generally of interest is *sound pressure*, the incremental variation in pressure above and below atmospheric pressure, which, in turn, is normally about 14.7 $lb/in.^2$ or about 1.013×10^5 N/m^2 in metric units at sea level.*

* See Appendix C for conversion factors.

Sound pressures are extremely small. For normal speech, they average about 0.1 N/m^2 (1 dyn/cm^2) above and below atmospheric pressure at a distance of a meter from a talker.

In order for there to be wave motion in a material medium, the medium must exhibit two properties, inertia and elasticity. Inertia is the property that permits one element of the medium to transfer momentum to adjacent elements. It is related to the density of the medium, that is to say, the mass of an element. Elasticity is the property that produces a force on a displaced element, tending to return it to its equilibrium position. In this chapter, we will be concerned only with sound waves in the air. In later chapters, we shall study sound in structural elements such as beams, plates, and walls.

How do we know that air has the properties of inertia (mass) and elasticity? Atmospheric pressure results from the weight of an atmospheric column of air. This is an indirect indication that air has weight and therefore mass. We could get a more direct indication by weighing a box containing air and then evacuating the box and weighing it again. We would find that it would take about 13.6 ft^3 of air at sea level (0.751 m Hg) and a temperature of 22°C (71.6°F) to weigh 1 lb. This corresponds to a density of about 1.18 kg/m^3 in metric units.

We can show by a number of means that air has elasticity. For example, if we take a basketball in its uninflated state and drop it on the floor, it thuds to a halt without rebounding. Inflating the ball with air imparts a resiliency to it which causes it to rebound when dropped. Even more simply, suppose while preparing to inflate the ball we had held a finger over the air outlet of our hand pump so that no air could come out. Pushing on the handle to compress the trapped air in the pump produces the same sensation as pushing on a spring. By knowing the physical dimensions of the pump and the amount of force we apply to the handle, we can determine the "spring constant" or compressibility of the gas, thus giving us the elasticity of the air quantitatively.

1.2 Basic Properties of Waves

We can gain insight into the nature of wave motion by looking at a very simple case, that of a single pulse of sound traveling down a tube, which constrains it from spreading. Figure 1.1*a* is a "spatial snapshot" of such a pulse. This figure presents the value of the disturbance, in this case, of the pressure deviation from atmospheric pressure, plotted as a function of the spatial coordinate x, at a time $t = 0$. The waveshape is a rectangular pulse of value A between the spatial coordinates $x = a$ and $x = b$. Elsewhere the value is zero. Let us assume that this wave pulse propagates in the positive x direction at a speed of c units per second. What will the spatial snapshot look like after d seconds have elapsed? The wavefront at coordinate b in Fig. 1.1a will move cd units in the positive x direction and will then be located at $b + cd$. Similarly, the wavetail at coordinate a in Fig. 1.1a will move to $a + cd$ in d seconds. The snapshot for d seconds is shown in Fig. 1.1b. Clearly, the pulse has the same width as before. Also, since there is no spreading of the wave, nor any energy loss in the medium, it must have the same value A as before.

It is not difficult to find an analytic form that expresses this idea of propagation without distortion in the positive x direction. The discussion in the previous paragraph shows that this form must permit the value at $x = a + cd$, $t = d$, to be exactly equal to the value at $x = a$, $t = 0$. Any function F that combines the spatial and temporal variables in the form $F(x - ct)$ satisfies this requirement. For the values $x = a$, $t = 0$, such a function would have the value $F(a)$, while for the values $x = a + cd$, $t = d$, the function takes the form $F(a + cd - cd) = F(a)$, exactly as required. We may emphasize the generality of this result. ***Any*** function $F(x - ct)$ which combines the spatial and temporal variables in the form $x - ct$ represents a wave propagating in the positive x direction at speed c. Examples of such functions are $A \sin k(x - ct)$, $A[(x - ct)^3 - 3(x - ct)^2]$, and $Ae^{jk(x-ct)}$.

We may use a pair of spatial snapshots similar to those in Fig. 1.1 to examine the case of a wave moving in the *negative* x direction. For this case we will find that the general analytic form must be any function $G(x + ct)$ which combines the spatial and temporal variables in the form $x + ct$.

We now know the general functional form that a wave traveling in the positive or negative x direction must assume. We must now examine the important physical properties of a sound wave, so that we may incorporate these properties in our functional form.

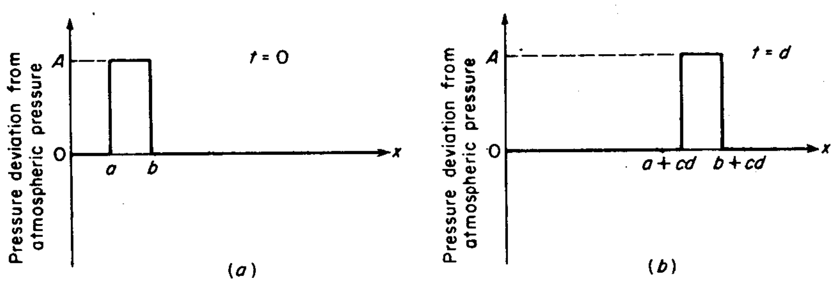

Fig. 1.1 Craphs showing the sound pressure for a single pulse of sound traveling down a tube, which constrains it from spreading. The waveshape is a rectangular pulse of amplitude A which propagates in the positive x direction (to the right) at a speed of c units per second: (a) the waveshape at $t = 0$ sec, and (b) the waveshape at $t = d$ sec.

Frequency. The subjective pitch of a simple sound is determined primarily by the number of times per second at which the sound-pressure disturbance oscillates between positive and negative values. The physical measure of this oscillation rate is called *frequency* and is denoted by the symbol f. The unit of frequency is the cycle per second (cps), which by international agreement is called hertz (Hz), after the man who first studied electromagnetic waves. The range of normal adult hearing extends approximately from 20 to 16,000 Hz. The ear is most sensitive, that is, the threshold of audibility is lowest, for sounds around 3,000 Hz.

Period. The reciprocal of the frequency f is the *period* T. It is the time required for one complete cycle. Thus, the period of a 1,000-Hz wave is 0.001 sec.

Wavelength. The *wavelength* of sound is the distance between analogous points on two successive waves. It is denoted by the Greek letter λ and is equal to the ratio of the speed of sound to the frequency of the sound, so that

$$\lambda = \frac{c}{f} = cT \tag{1.1}$$

In a very general way, we may distinguish between noises that consist of periodic sounds, that repeat regularly, and aperiodic sounds, that fluctuate randomly. The simplest periodic sound is a pure tone. A pure-tone sound is a pressure disturbance that fluctuates sinusoidally at a fixed frequency. Rotating machines, such as turbines and compressors, usually produce noise that is predominantly a set of pure-tone sounds. The set is often harmonically related: each of the *harmonic* sounds has a frequency that is an integral multiple of the lowest or fundamental frequency. For example, the fundamental frequency (first harmonic) of the noise from a turbine under certain operation conditions might be 3,000 Hz. The frequencies of the higher *harmonics* would then be 6,000 Hz, 9,000 Hz, and so on. There is a basic mathematical tool, called Fourier analysis, that can be used to subdivide any periodic signal into a series of pure-tone signals, harmonically related. Thus, any sound that repeats regularly can be subdivided into a series of harmonics, each with a particular amplitude.

The noises of a dishwasher, an air diffuser, and a rocket are examples of *aperiodic sounds*. An aperiodic sound cannot be subdivided into a set of harmonically related pure-tone sounds. It can, however, be described in terms of an infinitely large number of pure tones, of different frequencies, spaced an infinitesimal distance apart, and with different amplitudes.

It is mathematically convenient for us to study the behavior of pure-tone sounds first. We can then infer the behavior of more complex sounds.

1.3 Free Progressive Waves

Let us visualize a simple situation that will permit us to generate and study a pure-tone (sinusoidal) sound wave. In Fig. 1.2 a long tube is shown containing air, with a movable piston at one end. When the piston is moved back and forth by the mechanical arrangement shown, a plot of its position as a function of time is a sinusoidal function. This to-and-fro motion of the piston causes the air molecules adjacent to it alternately to be crowded together, or compressed, and a little later to be moved apart, or rarefied. This action of alternate compression and rarefaction moves down the tube owing to the elasticity and inertia of the medium. The wave thus generated is sinusoidal and has a frequency equal to the number of times per second at which the piston moves back and forth. The strength of the wave is determined by the magnitude of the displacement of the piston.

The three waves at the top of Fig. 1.2 show that the amplitude of the traveling wave is unchanged as the wave propagates to the right, and that the time delays between the same part of the wave at the plane at $x = 0$ and at the planes at $x = x_1$ and $x = 2x_1 = x_2$ are x_1/c sec and $2x_1/c$ sec respectively.

Speed of Sound. If we varied the properties of the gas through which the sound was traveling, we would find that the square of the speed of sound varied directly with the equilibrium gas pressure p_s and inversely with the equilibrium gas density ρ. The constant involved in the expres-

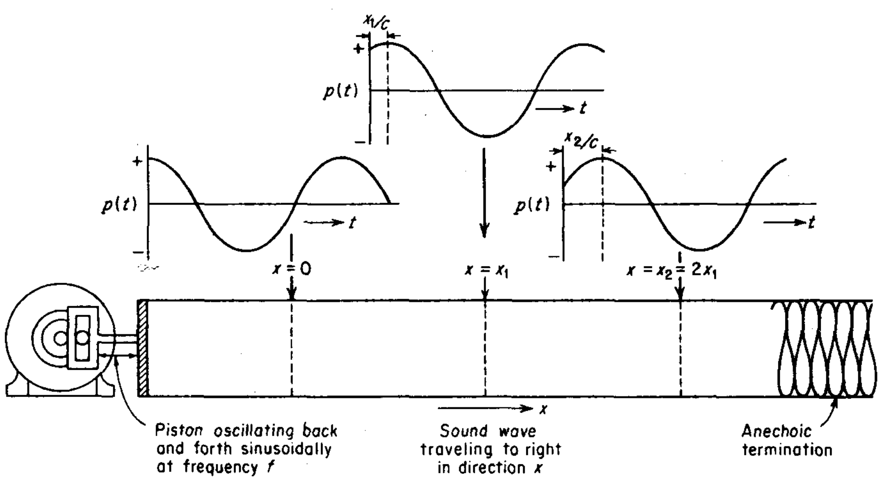

Fig. 1.2 Plane-wave propagation. A plane wave generated by the piston at the left of the tube travels to the right and is absorbed by the anechoic termination. The three waves at the top give the variation in sound pressure with time at the three points indicated, $x = 0$, $x = x_1$, and $x = x_2 = 2x_1$.

sion for the speed of sound is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume. For air at most temperatures with which we deal, this ratio is 1.4. Thus the speed of sound in air is given by the equation

$$c = \sqrt{\frac{1.4p_s}{\rho}} \qquad \text{m/sec or ft/sec} \tag{1.2}$$

If we assume that air behaves as an ideal gas, we can show that the speed of sound is dependent only on the absolute temperature of the air. This assumption is quite reasonable for most temperatures and densities with which we deal. The equations for the speed of sound become

$$c = 20.05\sqrt{T} \qquad \text{m/sec} \tag{1.3}$$

$$c = 49.03\sqrt{R} \qquad \text{ft/sec} \tag{1.4}$$

where R is the absolute temperature in degrees Rankine, that is, 459.7° plus the temperature in degrees Fahrenheit, and T is the absolute temperature in degrees Kelvin, that is, 273.2° plus the temperature in degrees centigrade.

Example 1.1 Determine the speed of sound at 70°F (21.1°C) in both English and metric units.

SOLUTION: The Rankine temperature is 459.7 + 70 = 529.7°R and the Kelvin temperature is 273.2 + 21.1 = 294.3°K. The speed of sound is then about

$$c = 49\sqrt{R} = 49\sqrt{530} = 1{,}128 \text{ ft/sec}$$

$$c = 20.05\sqrt{T} = 20.05\sqrt{294.3} = 344 \text{ m/sec}$$

In discussing the piston-tube experiment of Fig. 1.2, we must assume either that the tube is infinitely long or else that it has a nonreflecting (anechoic) termination so that no part of the energy in the sound wave will be reflected from the far end of the tube. The sound wave started by the piston then progresses down the tube without interacting with the side walls or the far end of the tube. All properties of the wave can then be described in terms of the distance down the tube from the piston and the action of the piston itself. This form of wave is known as a *one-dimensional, plane, free progressive* wave. The one-dimensional aspect relates to the specification of the parameters in terms of a single distance, the planar aspect to the fact that the wavefronts are parallel to each other, and the free progressive aspect to the advancement of the wave without interference from other objects or changes in the medium. In actuality, the wave is not truly free in that it is bounded by the walls of the tube and there will be some frictional losses at the boundaries.

For practical purposes, however, the motion is essentially the same as if the tube and the piston were infinite in cross section.

Sound Pressure and Particle Velocity. We have noted that at any point along the tube we can measure the varying disturbance from equilibrium pressure, the *sound pressure* p. In the mks system, its units are newtons per square meter, or N/m^2. Also, we can measure a varying *particle velocity* u, associated with the to-and-fro motions of the air molecules, which always occur along a line parallel to the direction of propagation. Its units are meters per second, m/sec.

Intensity. A free progressive sound wave transmits energy. The usual way in which the sound-energy propagation is described is in terms of *intensity* I, defined as the energy that flows through a unit area in a unit time. The unit for intensity is watts per square meter. In terms of the parameters describing a *free progressive sound wave* in which p and u are in phase, the *average* intensity in the direction of the wave propagation is the time average of the product of the sound pressure and the particle velocity measured *in the direction of the wave propagation,* expressed as

$$I = \overline{pu} \qquad \text{watts/m}^2 \tag{1.5}$$

1.3.1 One-dimensional Plane Wave We can now examine the quantitative behavior of a one-dimensional sound wave. Consider the piston-tube experiment (see Fig. 1.2) with an anechoic (echo-free) termination at the right of the tube, so that the tube contains a sinusoidal sound wave traveling in the positive x direction (toward the right). We have some freedom in our choice of time and space origins, and so let us select these origins so that the sound pressure at the plane $x = 0$ and at the time $t = 0$ has its maximum value of P_R. Figure 1.3 is a series of spatial snapshots representing the sound pressure as a function of distance, at four different instants of time. Each of these snapshots shows the sound pressure over a spatial extent of one wavelength λ. The vertical lines that separate the distance between $x = 0$ and $x = \lambda$ into 20 intervals help us to observe the sound-pressure behavior at any fixed point.

First, let us look at the uppermost plot of Fig. 1.3. Plot *a* represents the sound-pressure snapshot at the time $t = 0$. Because we are dealing with a periodic wave, it also represents the snapshots for times $t = T, 2T, 3T, \ldots, nT$. The value of the sound pressure at each position is represented by the location of the dots. As we required earlier, the maximum value $+P_R$ exists at the location $x = 0$ for this first snapshot. Again, because the wave is periodic in space, the same maximum value must occur at values of $x = \lambda, 2\lambda, 3\lambda, \ldots$

Plot *b* is the snapshot that describes the sound pressure a quarter of a period ($T/4$ sec) after *a*. In other words, the wave in *a* has moved to the right a distance equal to $\lambda/4$ to become the wave in *b*. Plots *c* and *d* represent the sound-pressure snapshots after two more successive intervals of $T/4$ sec. After T sec has elapsed, the wave has traveled a full wavelength λ to the right, and the sound-pressure distribution has returned to the snapshot of *a*. Then the entire sequence of snapshots repeats itself as the wave continues to travel to the right. To convince yourself that the wave is traveling to the right, allow your eyes to jump successively from *a* to *b* to *c* to *d*.

We can now combine our knowledge of the behavior of sound waves with the snapshots of Fig. 1.3 to obtain an analytical expression for a one-dimensional sound wave. Our earlier discussion of Fig. 1.1 showed that our expression must have the functional form $F(x - ct)$. Figure 1.3 shows us that this function must be a sinusoid. From trigonometry we know that a general expression must then have the form $C \cos [k(x - ct)]$

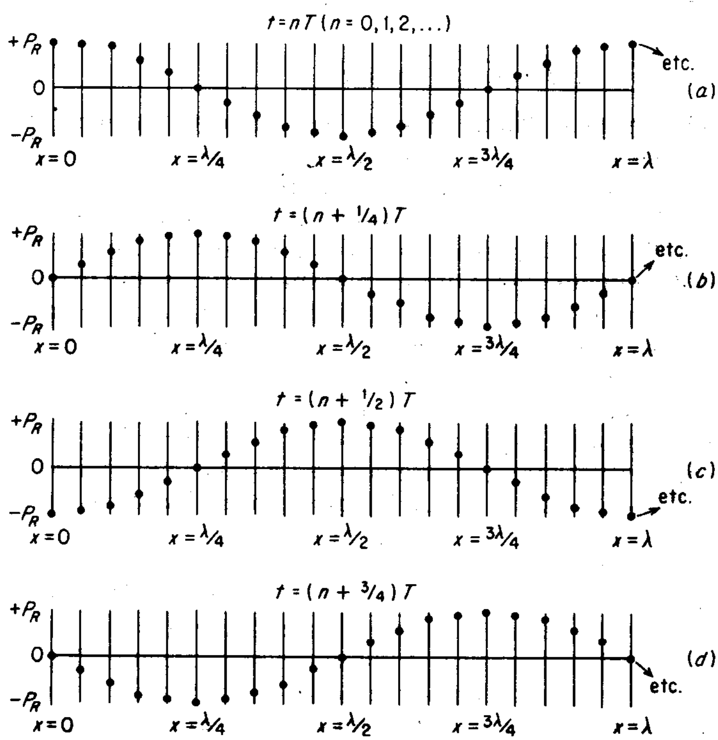

Fig. 1.3 Graphs showing the sound pressure in a plane free-progressive forward-traveling wave at 20 places in space at four instants of time t. The wave is produced by a source at the left and travels to the right with a speed c. The length of time it takes a wave to travel a distance equal to a wavelength is called the period T. Forward-traveling wave: $p(x,t) = P_R \cos [k(x - ct)]$; $k = 2\pi/\lambda = 2\pi/(cT) = \omega/c$.

$+ S \sin [k(x - ct)]$ where C and S are unknown amplitudes and k is an unknown parameter whose meaning will become clear shortly. We have already required that the sound pressure have its maximum value P_R for $x = 0$, $t = 0$. Our expression will satisfy this requirement if we set $C = P_R$ and $S = 0$, so that

$$p(x,t) = P_R \cos [k(x - ct)] \qquad \text{N/m}^2 \tag{1.6}$$

Wavenumber. The cosine function is periodic and repeats its value every time the argument increases 2π radians (360°). From the definition of wavelength λ, we can write this periodicity condition as

$$\cos [k(x + \lambda - ct)] = \cos [k(x - ct) + 2\pi] \tag{1.7}$$

so that

$$k\lambda = 2\pi \qquad k = \frac{2\pi}{\lambda} \qquad \text{radians/m} \tag{1.8}$$

We thus see the meaning of the parameter k. It is called the *wavenumber*.

Combining Eqs. (1.1) and (1.8), we can obtain an alternate expression for k

$$k = \frac{2\pi}{c/f} = \frac{2\pi f}{c} = \frac{\omega}{c} \qquad \text{radians/m} \tag{1.9}$$

where the quantity $2\pi f$ is defined as the *circular frequency* ω, so that

$$\omega = 2\pi f = kc \qquad \text{radians/sec} \tag{1.10}$$

Thus the argument in the trigonometric function may also be written in any one of the following ways

$$k(x - ct) = \frac{2\pi}{\lambda}(x - ct) = 2\pi f\left(\frac{x}{c} - t\right) = 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) = \frac{2\pi x}{\lambda} - 2\pi ft$$
$$= kx - \omega t \tag{1.11}$$

In Fig. 1.3 we have looked at the spatial behavior of a traveling sound wave, at certain fixed instants of time. From Eq. (1.7) the equations for the snapshots in Fig. 1.3 are given by

(*a*) $t = nT$: $\quad p = P_R \cos \frac{2\pi x}{\lambda}$

(*b*) $t = (n + \frac{1}{4})T$: $\quad p = P_R \cos \left(\frac{2\pi x}{\lambda} - \frac{\pi}{2}\right)$

(*c*) $t = (n + \frac{1}{2})T$: $\quad p = P_R \cos \left(\frac{2\pi x}{\lambda} - \pi\right)$

(*d*) $t = (n + \frac{3}{4})T$: $\quad p = P_R \cos \left(\frac{2\pi x}{\lambda} - \frac{3\pi}{2}\right)$

where $n = 0, 1, 2, 3, \ldots$

We could equally well look at the temporal behavior at fixed points, for any particular time t.

$$x = 0: \quad p = P_R \cos 2\pi ft$$

$$x = \lambda/4: \quad p = P_R \cos \left[2\pi f\left(t - \frac{\lambda}{4c}\right)\right] = \cos\left[2\pi f\left(t - \frac{T}{4}\right)\right]$$

$$= \cos\left(2\pi ft - \frac{\pi}{2}\right)$$

$$x = \lambda/2: \quad p = P_R \cos \left[2\pi f\left(t - \frac{\lambda}{2c}\right)\right] = \cos\left(2\pi ft - \pi\right)$$

$$x = 3\lambda/4: \quad p = P_R \cos \left[2\pi f\left(t - \frac{3\lambda}{4c}\right)\right] = \cos\left(2\pi ft - \frac{3\pi}{2}\right)$$

where we have used the trigonometric identity $\cos(-A) = \cos A$. We see that the temporal behavior at $x = \lambda/4$ is the same as the behavior at $x = 0$, except that it is delayed by a time of $T/4$ sec. Another way to express this time delay is to say that a shift in phase (the argument of the trigonometric function) of $\pi/2$ radians (90°) has occurred relative to $t = 0$. Similarly, at $x = \lambda/2$ the behavior is the same as at $x = 0$, except for a time delay of $T/2$ sec. This corresponds to a phase shift of π radians (180°), relative to $t = 0$.

Consider the piston-tube experiment with the sound source and anechoic termination interchanged, so that the tube contains a sinusoidal sound wave traveling in the negative x direction (toward the left). The corresponding sequence of spatial snapshots for this backward-traveling wave is shown in Fig. 1.4. Earlier in the chapter we stated that the general form for a wave moving in the negative x direction is any function $G(x + ct)$ which combines the spatial and temporal variables in the form $x + ct$. We can use this functional form to describe the wave shown in Fig. 1.4, and we obtain the result for the backward-traveling wave

$$p(x,t) = P_L \cos\left[k(x + ct)\right] \qquad \text{N/m}^2 \tag{1.12}$$

where P_L is the pressure amplitude.

A comparison of Eqs. (1.6) and (1.12) emphasizes that the sign relation between the space and time variables determines the direction in which the wave propagates. If the two variables are of different sign, the wave propagates in the positive spatial direction; if the two variables are of the same sign, the wave propagates in the negative spatial direction. This fact is confirmed by scanning Fig. 1.4 from *a* to *d*. The wave can only be made to "travel" from right to left as one's eyes jump from one plot to the next.