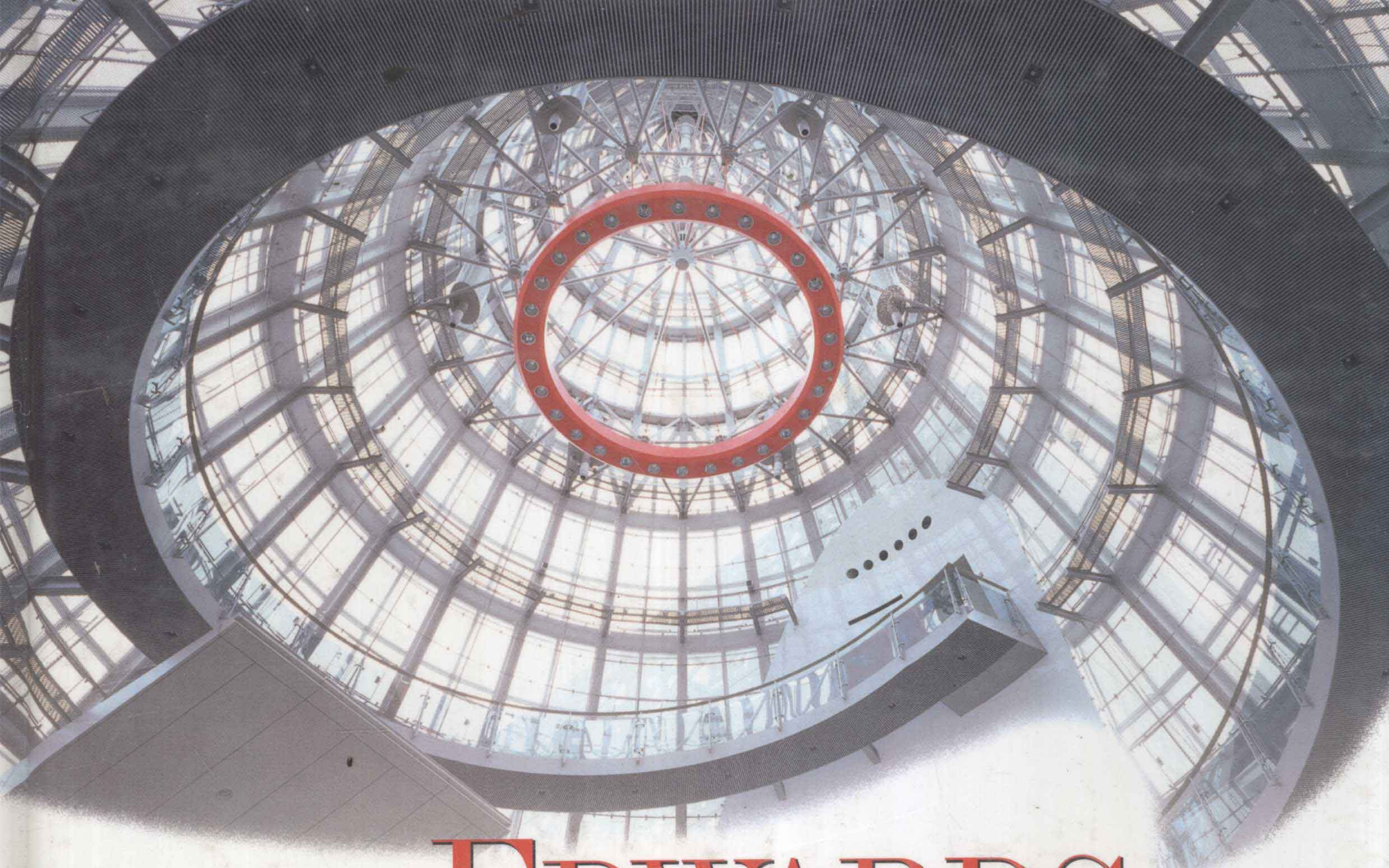

# EDWARDS & PENNEY

FIFTH EDITION

# CALCULUS

## WITH ANALYTIC GEOMETRY

INSTRUCTOR'S EDITION

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FIFTH EDITION

# CALCULUS

## WITH ANALYTIC GEOMETRY

C. HENRY EDWARDS

*The University of Georgia, Athens*

DAVID E. PENNEY

*The University of Georgia, Athens*

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# ABOUT THE AUTHORS

***C. Henry Edwards,*** University of Georgia, received his Ph.D. from the University of Tennesee in 1960. He then taught at the University of Wisconsin for three years and spent a year at the Institute for Advanced Study (Princeton) as an Alfred P. Sloan Research Fellow. Professor Edwards has just completed his thirty-third year of teaching at Georgia (including teaching calculus almost every year) and has received numerous University-wide teaching awards (including his recent selection as the single 1997 recipient of the state-wide Georgia Regents award for research university faculty teaching excellence). His scholarly career has ranged from research and dissertation direction in topology to the history of mathematics to computing and technology in mathematics ( his focus in recent years). In addition to his calculus, advanced calculus, linear algebra, and differential equations textbooks, he is also well-known to calculus instructors as author of *The Historical Development of the Calculus* (Springer-Verlag, 1979). He has served as a principal investigator on three recent NSF-supported projects: (1) A project to introduce technology throughout the mathematics curricula in two northeast Georgia public school systems (including *Maple* for beginning algebra students), (2) A *Calculus-with-Mathematica* pilot program at the University of Georgia, (3) A *Matlab*-based computer lab project for upper division numerical analysis and applied mathematics students. Currently he is leading the development of a technology-intensive web-based freshman mathematics course for non-science majors.

***David E. Penney,*** University of Georgia, completed his Ph.D. at Tulane University in 1965 while teaching at the University of New Orleans. Earlier he had worked in experimental biophysics at Tulane University and the Veteran's Administration Hospital in New Orleans. He began teaching calculus in 1957 and has taught the course almost every term since then. He joined the mathematics department at the University of Georgia in 1966 and has since received numerous university-wide teaching awards as well as directing several doctoral dissertations and undergraduate research projects. He is the author of research papers in number theory and topology and is author or co-author of books on calculus, differential equations, linear algebra, and liberal arts mathematics.

# PREFACE

The role and practice of mathematics in the world at large is now undergoing a revolution that is driven largely by computational technology. Calculators and computer systems provide students and teachers with mathematical power that no previous generation could have imagined. We read even in daily newspapers of stunning mathematical events like the proof of Fermat's last theorem, finally completed since the fourth edition of this text appeared. Surely *today* is the most exciting time in all history to be mathematically alive! So in preparing this new edition of ***Calculus with Analytic Geometry,*** we wanted first of all to bring a sense of this excitement to the students who will use it.

We also realize that the calculus course is a principal gateway to technical and professional careers for a still increasing number of students in an ever widening range of curricula. Wherever we look—in business and government, in science and technology—almost every aspect of professional work in the world involves mathematics. We therefore have re-thought once again the goal of providing calculus students the solid foundation for their subsequent work that they deserve to get from their calculus textbook.

The text for this edition has been reworked from start to finish. Discussions and explanations have been rewritten throughout in language that (we hope) today's students will find lively and accessible. Seldom-covered topics have been trimmed to accommodate a leaner calculus course. Historical and biographical notes have been added to show students the human face of calculus. Graphics calculator and computer lab projects (with *Derive, Maple,* and *Mathematica* options) for key sections throughout the text have been added. Indeed, a new spirit and flavor reflecting the prevalent interest in graphics calculators and computer systems will be discernible throughout this edition. Consistent with the graphical emphasis of the current calculus reform movement, several hundred new computer-generated figures have been added. Many of these additional figures serve to illustrate a more deliberative and exploratory approach to problem-solving. Our own teaching experience suggests that the use of contemporary technology can make calculus more concrete and accessible to many students.

## FIFTH EDITION FEATURES

In preparing this edition, we have benefitted from many valuable comments and suggestions from users of the first four editions. This revision was so pervasive that the individual changes are too numerous to be detailed in a preface, but the following paragraphs summarize those that may be of widest interest.

***Additional Problems*** This revision incorporates the most substantial additional of new problems since the first edition was published in 1982. Over 1250 of the fifth edition's approximately 6700 problems are new for this edition. Almost all of these new problems lie in the intermediate range of difficulty, neither highly theoretical nor computationally routine. Many of them have a new technology flavor, suggesting (if not requiring) the use of technology ranging from a graphing calculator to a computer algebra system.

***New Examples and Computational Details*** Throughout we have rewritten discussions and explanations in language that today's students will find more lively and accessible. The extent of this revision in text content is illustrated by the fact that approximately 20% of the fifth edition's over 700 in-text examples are new. Moreover, we have inserted an additional line or two of computational detail in many of the worked-out examples to make them easier for student readers to follow. The purpose of these computational changes is to make the computations themselves less of a barrier to conceptual understanding.

***Project Material*** Each chapter now contains several supplementary projects—a total of more than 50, many of them new for this edition. Each project typically employs some aspect of modern computational technology to illustrate the principal ideas of the preceding section, and typically contains additional problems intended for solution with the use of a graphics calculator or computer. Figures and data illustrate the use of graphics calculators and computer systems such as *Derive, Maple,* and *Mathematica.* This project material is suitable for use in a computer or calculator lab conducted in association with a standard calculus course, perhaps meeting weekly. It can also be used as a basis for graphics calculator or computer assignments that students will complete outside of class, or for individual study.

***Computer Graphics*** An increased emphasis on graphical visualization along with numeric and symbolic understanding is provided by the computer-generated artwork, about 25% of which is new for this edition. Over 550 MATLAB-generated figures (half of them new for this edition) illustrate the kind of figures that students using graphics calculators can produce for themselves. Many of these are included with new graphical problem material. *Mathematica*-generated color graphics are included to highlight all sections involving three-dimensional material.

***Historical Material*** Historical and biographical chapter openings offer students a sense of the development of our subject by real, live human beings. Both authors are fond of the history of mathematics and believe that it can favorably influence both our teaching and students' learning of mathematics. For this reason numerous historical comments appear in the text itself.

***Introductory Chapters*** Chapters 1 and 2 have been streamlined for a leaner and quicker start on calculus. Chapter 1 concentrates on functions and graphs. It includes two sections cataloging the elementary functions of calculus and provides a foundation for an early emphasis on transcendental functions. Chapter 1 concludes with a section addressing the question "What *is* calculus?" Chapter 2, on limits, begins with a section on tangent lines to motivate the official introduction of limits in Section 2.2. Trigonometric limits are treated throughout Chapter 2 in order to encourage a richer and more visual introduction to the limit concept.

***Differentiation Chapters*** The sequence of topics in Chapters 3 and 4 varies a bit from the most traditional order. We attempt to build student confidence by introducing topics more nearly in order of increasing difficulty. The chain rule appears quite early (in Section 3.3) and we cover the basic techniques for differentiating algebraic functions before discussing maxima and minima in Sections 3.5 and 3.6. The appearance of inverse functions is delayed until Chapter 7. Section 3.7 treats the derivatives of all six trigonometric functions. Implicit differentiation and related rates are combined in a single section (Section 3.8). The mean value theorem and its applications are deferred to Chapter 4. Sections 4.4 on the first derivative test and 4.6 on higher derivatives and concavity have been simplified and streamlined. A great deal of new graphic material has been added in the curve-sketching sections that conclude Chapter 4.

***Integration Chapters*** New and simpler examples have been inserted throughout Chapters 5 and 6. Antiderivatives (formerly at the end of Chapter 4) now begin Chapter 5. Section 5.4 (Riemann sums) has been simplified greatly, with upper and lower sums eliminated and endpoint and midpoint sums emphasized instead. Many instructors now believe that the first applications of integration ought not be confined to the standard area and volume computations; Section 6.5 is an optional section that introduces separable differential equations. To eliminate redundancy, the material on centroids and the theorems of Pappus is delayed to Chapter 14 (Multiple Integrals), where it can be treated in a more natural context.

***Early Transcendentals Functions Options*** An "early transcendental functions" version of this book is also available. In the present version, the flexible organization of Chapter 7 offers a variety of options to those instructors who favor an earlier treatment of transcendental functions. Section 7.1 begins with the "high school" approach to exponential functions, followed by the idea of a logarithm as "the power to which the base $a$ must be raised to get the number $x$." On this basis, Section 7.1 carries out a low-key review of the laws of exponents and of logarithms, and investigates informally the differentiation of exponential and logarithmic functions. This section on the elementary differential calculus of exponentials and logarithms can be covered any time after Section 3.3 (on the chain rule). If this is done, then Section 7.2—based on the definition of the logarithm as an integral—can be covered any time after the integral has been defined in Chapter 5 (along with as much of the remainder of Chapter 7 as the instructor desires). The remaining transcendental functions—inverse trigonometric and hyperbolic—are now treated in Chapter 8, which includes also indeterminate forms and l'Hôpital's rule (much earlier than in the third edition).

Thus the text offers a variety of ways to accommodate a course syllabus that includes exponential functions early in differential calculus, and/or logarithmic functions early in integral calculus.

***Streamlining Techniques of Integration*** Chapter 9 is organized to accommodate those instructors who feel that methods of formal integration now require less emphasis, in view of modern techniques for both numerical and symbolic integration. Integration by parts (Section 9.3) now precedes trigonometric integrals (Section 9.4). The method of partial fractions appears in Section 9.5, and trigonometric substitutions and integrals involving quadratic polynomials follow in Sections 9.6 and 9.7. Improper integrals appear in Section 9.8, and the more specialized rationalizing substitutions have been relegated to the Chapter 9 Miscellaneous Problems. This rearrangement of Chapter 9 makes it more convenient to stop wherever the instructor desires.

***Vectors*** The major reorganization for the fifth edition is a response to numerous user suggestions to combine the treatments of two-dimensional vectors and three-dimensional vectors, which appeared in separate chapters of the fourth edition. In this reorganization we have also amalgamated the treatments of polar curves and parametric curves, which also appeared in separate chapters in the fourth edition. As a consequence, the contents of three chapters in the fourth edition have been efficiently combined in two chapters of this revision—Chapter 10 on Polar Coordinates and Plane Curves, and Chapter 12 on Vectors, Curves, and Surfaces in Space.

***Infinite Series*** After the usual introduction to convergence of infinite sequences and series in Sections 11.2 and 11.3, a combined treatment of Taylor polynomials and Taylor series appears in Section 11.4. This makes it possible for the instructor to experiment with a much briefer treatment of infinite series, but still offer exposure to the Taylor series that are so important for applications.

***Differential Equations*** Many calculus instructors now believe that differential equations should be seen as early and as often as possible. The very simplest differential equations (of the form $dy/dx = f(x)$) appear in a subsection at the end of Section 5.2 (Antiderivatives). Section 6.5 illustrates applications of integration to the solution of separable differential equations. Section 9.5 includes applications of the method of partial fractions to population problems and the logistic equation. In such ways we have distributed enough of the spirit and flavor of differential equations throughout the text that it seemed expeditious to eliminate the (former) final chapter devoted solely to differential equations. But those who so desire can arrange with the publisher to obtain for supplemental use appropriate sections of Edwards and Penney, *Differential Equations: Computing and Modeling* (Englewood Cliffs, N.J.: Prentice Hall, 1996).

***Linear Algebra Notation and Terminology*** An innovation for the fifth edition is the inclusion (for optional coverage) of matrix terminology and notation in the multivariable portion of the text—for example, in the treatment of quadric surfaces in Chapter 12 and of directional derivatives and the multivariable chain rule in Chapter 13. These subsections will enhance the understanding of multivariable concepts for those students who are familiar with matrix notation at the level of the definition of the product of two matrices.

## MAINTAINING TRADITIONAL STRENGTHS

While many new features have been added, five related objectives remained in constant view: **concreteness**, **readability**, **motivation**, **applicability**, and **accuracy**.

- *CONCRETENESS* The power of calculus is impressive in its precise answers to realistic questions and problems. In the necessary conceptual development of the subject, we keep in sight the central question: How does one actually *compute* it? We place special emphasis on concrete examples, applications, and problems that serve both to highlight the development of the theory and to demonstrate the remarkable versatility of calculus in the investigation of important scientific questions.

- *READABILITY* Difficulties in learning mathematics often are complicated by language difficulties. Our writing style stems from the belief that crisp exposition, both intuitive and precise, makes mathematics more accessible—and hence more readily learned—with no loss of rigor. We hope our language is clear and attractive to students and that they can and actually will read it, thereby enabling the instructor to concentrate class time on the less routine aspects of teaching calculus.

- *MOTIVATION* Our exposition is centered around examples of the use of calculus to solve real problems of interest to real people. In selecting such problems for examples and exercises, we took the view that stimulating interest and motivating effective study go hand in hand. We attempt to make it clear to students how the knowledge gained with each new concept or technique will be worth the effort expended. In theoretical discussions, especially, we try to provide an intuitive picture of the goal before we set off in pursuit of it.

▼ *APPLICATIONS* Its diverse applications are what attract many students to calculus, and realistic applications provide valuable motivation and reinforcement for all students. This book is well-known for the broad range of applications that we include, but it is neither necessary nor desirable that the course cover all of the applications in the book. Each section or subsection that may be omitted without loss of continuity is marked with an asterisk. This provides flexibility for each instructor to determine his or her own flavor and emphasis.

▼ *ACCURACY* Our coverage of calculus is complete (although we hope it is somewhat less than encyclopedic). Still more than its predecessors, this edition was subjected to a comprehensive reviewing process to help ensure accuracy. For example, essentially every problem answer appearing in the Answers section at the back of the book in this edition has been verified using *Mathematica.* With regard to the selection and sequence of mathematical topics, our approach is traditional. But close examination of the treatment of standard topics may betray our own participation in the current movement to revitalize the teaching of calculus. We continue to favor an intuitive approach that emphasizes both conceptual understanding and care in the formulation of definitions and key concepts of calculus. Some proofs that may be omitted at the discretion of the instructor are placed at the ends of sections and others are deferred to the book's appendices. In this way we leave ample room for variation in seeking the proper balance between rigor and intuition.

## SUPPLEMENTARY MATERIAL

A variety of electronic and printed supplements are provided by the publisher, including a WWW site that consitutes an on-line calculator/computer guide for calculus. This web site at `www.prenhall.com/edwards` is designed to assist calculus students as they work on the book's projects using graphing calculators and computer algebra systems such as *Derive, Maple, Mathematica,* and MATLAB. The authors will maintain and expand this site to provide calculus students with new and evolving supplementary materials on a continuing basis, and to explore the use of emerging technology for new channels of communication and more active learning experiences.

Answers to most of the odd-numbered problems appear in the back of the book. Solutions to most problems (other than those odd-numbered ones for which an answer alone is sufficient) are available in the *Instructor's Solutions Manual.* A subset of that manual, containing solutions to problems numbered 1, 4, 7, 10, . . . is available as a *Student Solutions Manual.* A collection of some 1700 additional problems suitable for use as test questions, the *Calculus Test Item File,* is available (in both electronic and hard-copy form) for use by instructors. Finally, an *Instructor's Edition* including section-by-section teaching outlines and suggestions is available to those who are using this book to teach calculus.

## ACKNOWLEDGMENTS

All experienced textbook authors know the value of critical reviewing during the preparation and revision of a manuscript. In our work on this edition of the book we have benefitted greatly from the advice of the following exceptionally able reviewers:

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Many of the best improvements that have been made must be credited to colleagues and users of the first four editions throughout the United States, Canada, Europe, and South America. We are grateful to all those, especially students, who have written to us, and hope that they will continue to do so. We thank Mary and Nancy Toscano, who checked the accuracy of every example and odd-numbered answer. We also believe that the quality of the finished book itself is adequate testimony to the skill, diligence, and talent of an exceptional staff at Prentice Hall; we owe special thanks to George Lobell, our mathematics editor; Jack Casteel, production editor; Tony Palermino, developmental editor; Lorraine Castellano, designer; and Network Graphics, illustrator. Finally, we again are unable to thank Alice Fitzgerald Edwards and Carol Wilson Penney for their continued assistance, encouragement, support, and patience.

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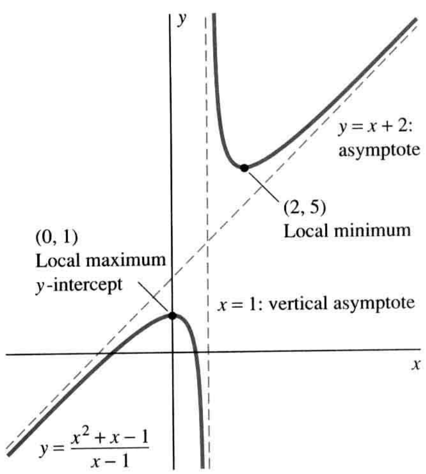
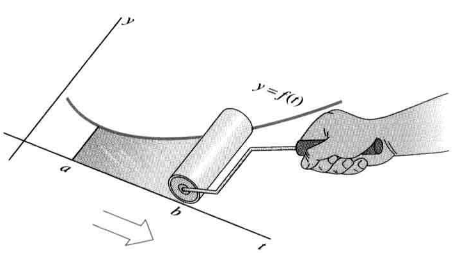
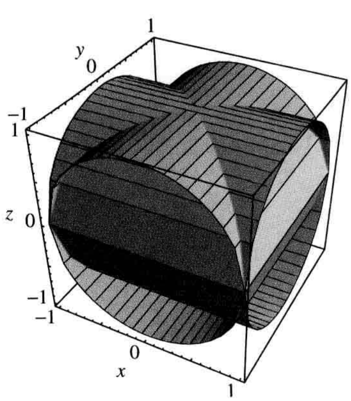
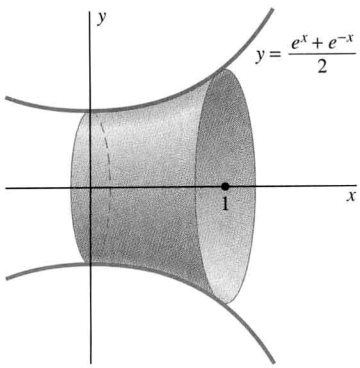

y
y = x + 2: asymptote
(2, 5)
Local minimum
(0, 1)
Local maximum
y-intercept
x = 1: vertical asymptote
x
$y = \frac{x^2 + x - 1}{x - 1}$



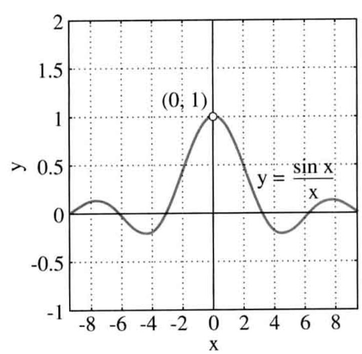

y = f(t)
a
b



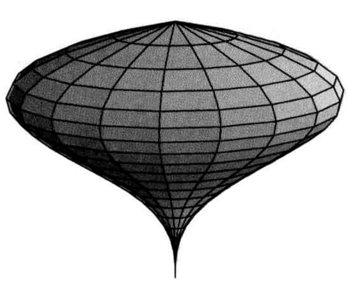

y
z
x

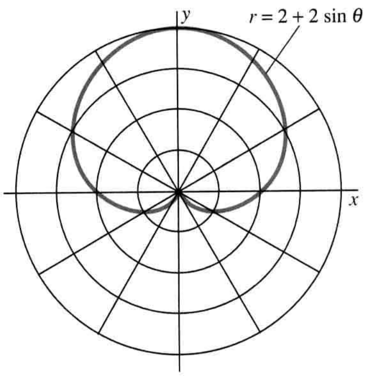

y
$y = \frac{e^x + e^{-x}}{2}$
1
x




2
1.5
1
0.5
0
-0.5
-1
y
(0, 1)
$y = \frac{\sin x}{x}$
-8 -6 -4 -2 0 2 4 6 8
x





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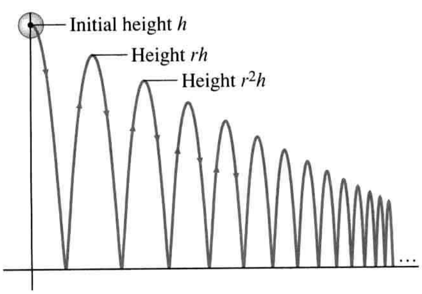


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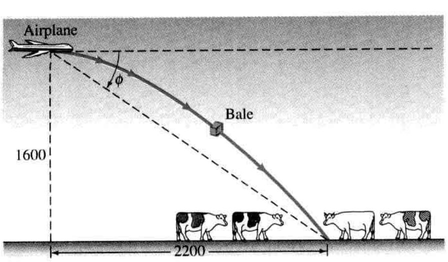


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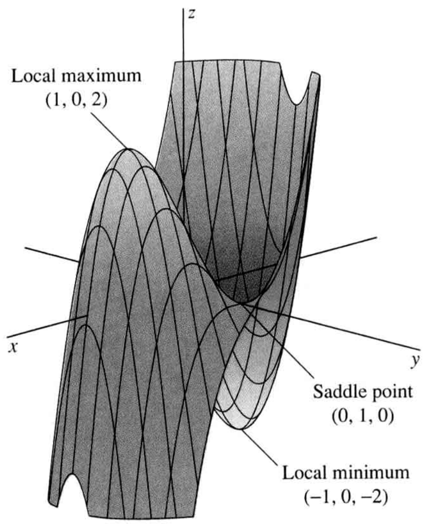
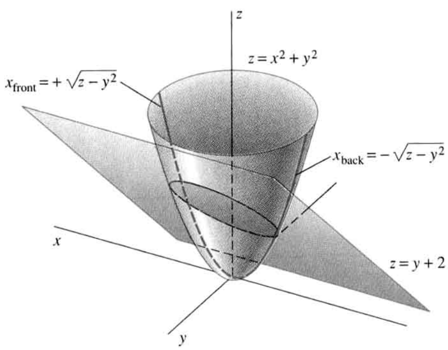
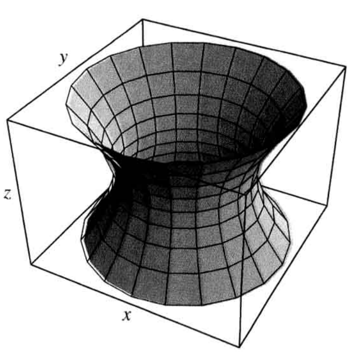

z
Local maximum
(1, 0, 2)
x
y
Saddle point
(0, 1, 0)
Local minimum
(−1, 0, −2)




z
$z = x^2 + y^2$
$x_{front} = +\sqrt{z - y^2}$
$x_{back} = -\sqrt{z - y^2}$
x
$z = y + 2$
y




y
z
x

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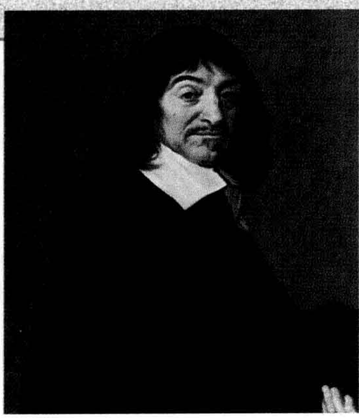
C H A P T E R 1

# FUNCTIONS AND GRAPHS



*René Descartes (1596–1650)*

The seventeenth-century French scholar René Descartes is perhaps better remembered today as a philosopher than as a mathematician. But most of us are familiar with the "Cartesian plane" in which the location of a point $P$ is specified by its coordinates $(x, y)$.

As a schoolboy Descartes was often permitted to sleep late because of allegedly poor health. He claimed that he always thought most clearly about philosophy, science, and mathematics while lying comfortably in bed on cold mornings. After graduating from college, where he studied law (apparently with little enthusiasm), Descartes traveled with various armies for a number of years, but more as a gentleman soldier than as a professional military man.

After finally settling down (in Holland), Descartes published, in 1637, his famous philosophical treatise *Discourse on the Method* (of Reasoning Well and Seeking Truth in the Sciences). One of three appendices to this work sets forth his new "analytic" approach to geometry. His principal idea (published almost simultaneously by his countryman Pierre de Fermat) was the correspondence between an *equation* and its *graph,* generally a curve in the plane. The equation could be used to study the curve and vice versa.

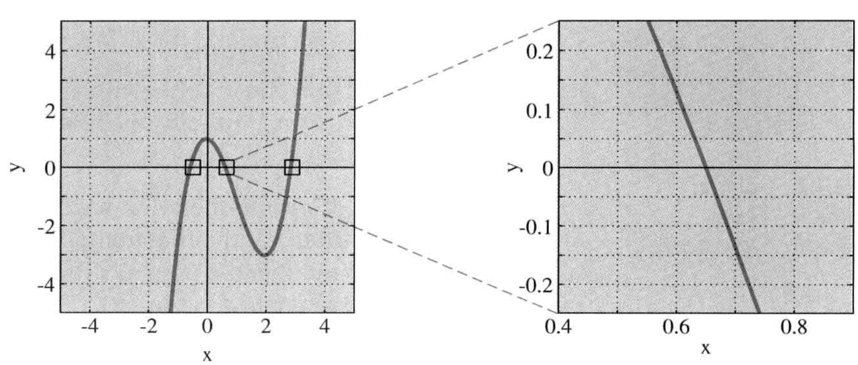
Suppose that we want to solve the equation $f(x) = 0$. Its solutions are the intersection points of the graph of $y = f(x)$ with the $x$-axis, so an accurate picture of the curve shows the number and approximate locations of the solutions of the equation. For instance, the graph

$$y = x^3 - 3x^2 + 1$$

has three $x$-intercepts, showing that the equation

$$x^3 - 3x^2 + 1 = 0$$

has three real solutions—one between $-1$ and 0, one between 0 and 1, and one between 2 and 3. A modern graphics calculator or computer graphing program can approximate these solutions more accurately by magnifying the regions in which they are located. For instance, the magnified center region shows that the corresponding solution is $x \approx 0.65$.




*The graph $y = x^3 - 3x^2 + 1$*