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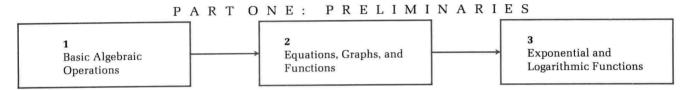
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Chapter Dependencies



Selected topics from Part One may be referred to as needed in Parts Two and Three or reviewed systematically before starting either part.

FINITE MATHEMATICS PART TWO: Mathematics of Systems of Linear Probability Equations; Matrices Finance Linear Inequalities and **Additional Topics** Linear Programming in Probability CALCULUS PART THREE: 10 11 Additional Integration The Derivative **Derivative Topics**

The second edition of Essentials of College Mathematics for Business, Economics, Life Sciences, and Social Sciences is designed for mathematics courses that include topics from algebra, finite mathematics, and calculus, and for students who have had 1 or 2 years of high school algebra, or the equivalent. It is one of six books in the authors' College Mathematics series (see page ii for a brief comparison of all six books). The choice and organization of topics make the book readily adaptable to a variety of courses, including a combined course in finite mathematics and calculus, or a separate course in either area with a strong algebra component (see the chapter dependency chart on page viii).

◆ PRINCIPAL CHANGES FROM THE SECOND EDITION

This edition has been divided into three parts, "Part One: Preliminaries," "Part Two: Finite Mathematics," and "Part Three: Calculus." Part One (Chapters 1–3) deals with algebraic concepts, including a review of basic algebraic operations, a thorough treatment of functions and their graphs, and a brief but comprehensive discussion of exponential and logarithmic functions. Depending on the background of a given class, this material can be totally omitted, referred to as needed, or selected topics can be covered in detail at the beginning or during the course. A few additional review topics are also included in Appendix A for convenient reference. Part Two (Chapters 4–8) covers the topics that have become standard in a finite mathematics course: mathematics of finance, linear systems, matrices, linear programming, and probability. And Part Three (Chapters 9–11) provides basic coverage of differential and integral calculus for functions of one variable, including the exponential and logarithmic functions.

The following general improvements are found throughout the text: an increased emphasis on and use of calculators; boxed definitions, results, and step-by-step processes; and schematic illustrations. Examples have been improved, and many new examples have been added. Exercise sets, including applications, have been improved and expanded. All exposition has been carefully reviewed and fine-tuned or rewritten.

Specific improvements are as follows:

- 1. The development of the simplex method (Sections 6-3 and 6-4) has been completely rewritten. Together, these two revised sections provide a well-motivated, clear, shorter, and less wordy introduction to the simplex process. The important terms "basic variable," "nonbasic variable," "basic solution," and "basic feasible solution," and the main idea of the simplex algorithm are introduced through examples in a geometric setting in Section 6-3. In the development of the simplex method in Section 6-4, the selection of basic and nonbasic variables in the simplex tableau has been made more precise, and the pivot operation now emphasizes entering basic variables and exiting basic variables. Even though the treatment of the simplex method in the last edition was very well-received, we believe that the new treatment is even better.
- 2. Other topics in linear algebra also received quite a bit of attention, and Sections 5-1, 5-2, 5-6, 6-1, and 6-2 were substantially rewritten. There is now a more uniform use of the technical terms "independent," "dependent," "consistent," and "inconsistent." The discussion of the use of parameters in representing infinite solution sets is clearer, as is the discussion of finding the inverse of a matrix and solving matrix equations. The discussion on solving linear inequalities and systems of linear inequalities has been simplified.
- 3. Important improvements have been made in the chapters on probability. Sections 7-1, 7-2, and 8-2 were substantially rewritten. Tree diagrams are used effectively to illustrate the difference between permutations and combinations in Section 7-1. The important section on experiments and events (Section 7-2) has been simplified, and is now less formal and more concise. The development of independent and dependent events in Section 8-2 is improved.
- 4. Chapter 4, on mathematics of finance, is an independent chapter and can be covered at any time. It is placed directly after Chapter 3, on exponential and logarithmic functions, for the convenience of those who wish to tie together the material in these two chapters. Exposition has been improved, applications made current, and some new applications added.
- 5. The first four sections in the introductory derivative chapter of the last edition have been completely rewritten and condensed into three sections. The treatment has been both simplified and expanded. The new first section provides an intuitive geometric introduction to limits and continuity. Because of numerous reviewer requests, left- and right-hand limits have been added. Because of its importance in solving optimization problems, continuity on a closed interval is now defined. Increment notation has been removed from this introductory chapter to keep the notation as simple as possible. Computation of difficult limits has been reduced, and more emphasis has been placed on the 0/0 indeterminate form because of its relevance to the definition of the derivative and its various interpretations (slope, instantaneous rate, marginal analysis, and so on). Infinite limits and

- limits at infinity receive more attention. Vertical and horizontal asymptotes are defined using these concepts.
- 6. Solving inequalities using continuity has become a subsection of the section on the first derivative and graphs (Section 10-1). This process of solving inequalities is fundamental to the production of **sign charts** throughout the chapter. A new section on **curve sketching** (Section 10-3) presents a simple and straightforward curve sketching strategy that is easy for students to remember and use. [Step 1: Use f(x); Step 2: Use f'(x); Step 3: Use f''(x); Step 4: Graph f.]
- 7. In Chapter 11, on integration, more attention is given to substitution techniques and area between the graphs of two functions.

◆ IMPORTANT FEATURES

Emphasis and Style

The text is **written for student comprehension.** Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Examples and Matched Problems

Over 300 completely worked examples are included. Each example is followed by a similar problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

The book contains over 3,500 problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

Applications

Enough applications are included to convince even the most skeptical student that mathematics is really useful. The majority of the applications are included at the end of exercise sets and are generally divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

♦ STUDENT AIDS

1. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-2 and 1-3).

- 2. Annotation of examples and developments, in color type, is found throughout the text to help students through critical stages (see Sections 1-2 and 1-3).
- 3. Functional use of color improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-2 and 1-3).
- 4. Boldface type is used to introduce new terms and highlight important comments.
- 5. Screened boxes are used to highlight important definitions, theorems, results, and step-by-step processes.
- 6. Answers to odd-numbered problems are included in the back of the book.
- 7. Chapter review sections include a review of all important terms and symbols and a comprehensive review exercise. Answers to all review exercises are included in the back of the book.
- 8. A student's solution manual is available at a nominal cost through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.
- 9. A manual for an Interactive Computer Applications Package (ICAP) by Carolyn L. Meitler is available at a nominal cost through a book store. The manual contains instructions, examples, and exercises that demonstrate the use of the programs on the ICAP for Essentials of College Mathematics disks. The disks containing the programs, for use on APPLE II® and IBM-PC® computers, are distributed free of charge to institutions using this book. No previous computer experience is necessary to use this package.
- 10. A Supplemental Applications and Topics manual by Jon E. Baum is available at a nominal cost through a book store. Part I of the manual expands the application exercises in the text and reinforces the important role of the mathematics presented in the text. These exercises provide the student with a richer and more varied experience in solving real-world problems. Part II of the manual presents some applications that are not covered in the text, including transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics. After completing the prerequisite material in the text, students interested in these more specialized topics will realize substantial benefits by studying this portion of the manual.

INSTRUCTOR AIDS

See page xvi for detailed information regarding examination copy requests and orders for the instructor aids described below.

1. A unique computer-generated random test system is available to instructors without cost. The test system utilizes an IBM-PC, XT, or AT Personal Computer® and will produce high-quality output on an IBM-compatible dot-matrix printer or on a Hewlett-Packard Laserjet II®-compatible laser printer. The test system has been greatly expanded and now contains over 300 different problem algorithms directly related to material in the text.

These carefully constructed algorithms use random number generators to produce different, yet equivalent, versions of each of these problems. The test system is available now in both free-response and multiple-choice editions. An almost unlimited number of quizzes, review exercises, chapter tests, mid-terms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys and student work sheets, if desired. Upon request, the publisher will supply institutions using this textbook with DellenTest III (IBM Free-Response Edition or Multiple-Choice Edition) on 5.25 inch floppy disks. IBM Edition User Notes and Annotated Problem Printouts are included with the disks. The notes provide step-by-step instructions for using the testing system and a complete description of the options in this menudriven program. The annotated printouts identify by chapter and number each question the system is capable of generating, and also correlate each question with the prerequisite section from the text. When used in conjunction with the user notes, the annotated printouts enable instructors to select any combination of questions for an examination.

- 2. An **instructor's test battery** is also available to instructors without cost. The battery, organized by chapter, contains three equivalent versions (with answers) of over 300 different problems.
- 3. An **instructor's resource manual** provides over 130 transparency and handout masters, a detailed discussion of chapter and topic dependencies, a comparison of this edition with the previous edition, and a detailed topic chart for comparing this book with other books in the authors' College Mathematics series.
- 4. An **instructor's answer manual** containing all the answers not included in the text is available to instructors without charge.
- 5. A **student's solution manual** (see Student Aids) is available to instructors without charge from the publisher.
- 6. An Interactive Computer Applications Package (ICAP) by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The programs in this package are available on diskettes for APPLE II® and IBM-PC® computers. Included on these disks are programs related to the mathematics of finance, row operations, matrix arithmetic, the simplex method, limit estimation, function graphing, and numerical integration. The publisher will supply these disks without charge to institutions using this book.
- 7. A Supplemental Applications and Topics manual by Jon E. Baum (see Student Aids) is available to instructors without charge from the publisher. Instructors can use Part I of this manual to supplement the exercise sets in the text, providing students with additional experience in solving applications utilizing the mathematics presented in the text. Part II of the manual can be used to provide coverage of applications not covered in the text, such as transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics, either as part of the syllabus for a course or as subjects for independent study.

8. Z-graph, a HyperCard[©] graphing stack for the APPLE Macintosh[®] computer, allows a user to graph most of the mathematical functions likely to be encountered, quickly, accurately, and with considerable control over axes, scales, graph size, and labeling. In addition to graphing functions, this program will perform a variety of mathematical operations related to numerical integration, root approximation, interpolating polynomials, least-square polynomials, and approximate solutions of differential equations. Instructors will find this program useful for preparing examination material, transparency masters, and handouts. The publisher will supply this program free of charge to instructors using this book, and the program may be freely distributed to students.

ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Dellen Publishing Company, 400 Pacific Avenue, San Francisco, CA 94133.

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> R. A. Barnett M. R. Ziegler

Ordering Information

When requesting examination copies or placing orders for this text or any of the related supplementary materials listed below, please refer to the corresponding ISBN numbers.

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Manual	0-02-380185-9
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Student's Solution Manual to accompany Essentials of College Mathematics, Second Edition	0-02-334385-0
Supplemental Applications and Topics to accompany the Barnett and Ziegler College Mathematics Series	0-02-306770-5
Z-graph Macintosh Disk	0-02-306255-X

ix
276

CONTENTS	Preface	1X
PART ONE	Preliminaries	1
◆ CHAPTER 1	Basic Algebraic Operations	3
	1-1 Algebra and Real Numbers	4
*	1-2 Basic Operations on Polynomials	13
	1-3 Factoring Polynomials	20
w	1-4 Basic Operations on Rational Expressions	26
	1-5 Integer Exponents and Square Root Radicals	33
	1-6 Rational Exponents and Radicals	41
	1-7 Chapter Review	48
◆ CHAPTER 2	Equations, Graphs, and Functions	53
	2-1 Linear Equations and Inequalities in One Variable	54
	2-2 Quadratic Equations	64
	2-3 Cartesian Coordinate System and Straight Lines	69
	2-4 Functions	83
	2-5 Linear and Quadratic Functions	97
	2-6 Chapter Review	110
◆ CHAPTER 3	Exponential and Logarithmic Functions	115
	3-1 Exponential Functions	116
	3-2 The Exponential Function with Base e	125
	3-3 Logarithmic Functions	132
	3-4 Chapter Review	144
D 4 D 77 77 77 77	Finite Methematics	
PART TWO	Finite Mathematics	147
◆ CHAPTER 4	Mathematics of Finance	149
	4-1 Simple Interest	150
	4-2 Compound Interest	156
	4-3 Future Value of an Annuity; Sinking Funds	166

Present Value of an Annuity; Amortization

4-4

4-5

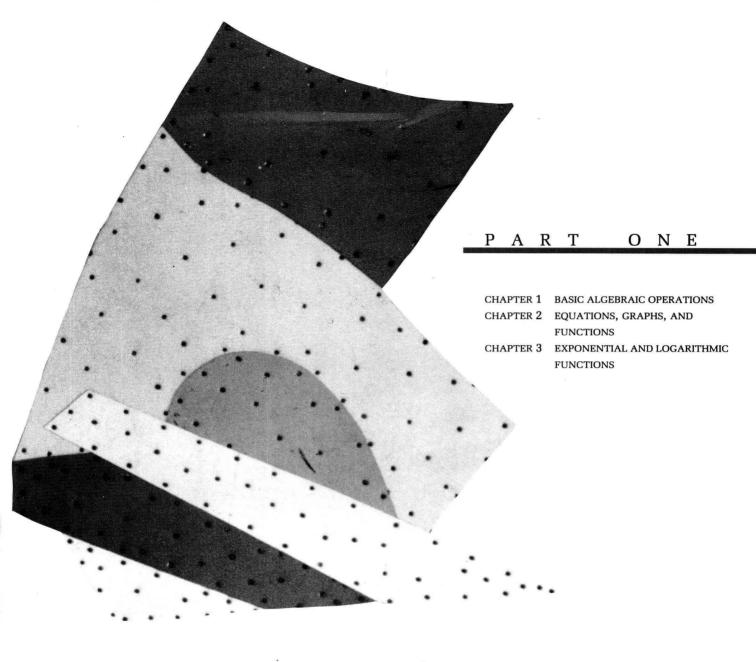
Chapter Review

173

183

•	CHAPTER 5	Sys	tems of Linear Equations; Matrices	189
		5-1 5-2 5-3 5-4 5-5 5-6 5-7	Review: Systems of Linear Equations Systems of Linear Equations and Augmented Matrices Gauss – Jordan Elimination Matrices — Addition and Multiplication by a Number Matrix Multiplication Inverse of a Square Matrix; Matrix Equations Chapter Review	190 209 218 231 238 247 261
•	CHAPTER 6	Lin	ear Inequalities and Linear Programming	265
		6-1 6-2	Systems of Linear Inequalities in Two Variables Linear Programming in Two Dimensions—	266
			A Geometric Approach	279
		6-3 6-4	A Geometric Introduction to the Simplex Method The Simplex Method: Maximization with Problem	298
		0.1	Constraints of the Form ≤	308
		6-5	Chapter Review	330
	_	_		
•	CHAPTER 7	Pro	bability	335
		7-1	Multiplication Principle, Permutations, and Combinations	337
		7-2	Sample Spaces and Events	355
		7-3	Empirical Probability	371
		7-4 7-5	Random Variable, Probability Distribution, and Expectation Chapter Review	379 390
•	CHAPTER 8	Add	litional Topics in Probability	205
*	CHAITER O		•	395
		8-1 8-2	Union, Intersection, and Complement of Events; Odds Conditional Probability, Intersection, and Independence	396
		8-3	Bayes' Formula	413 432
		8-4	Chapter Review	442
	DAD	C-1	1	
	PART THREE	Cai	culus	447
*	CHAPTER 9	The	Derivative	449
		9-1	Limits and Continuity—A Geometric Introduction	451
		9-2	Computation of Limits	470
		9-3	The Derivative	486

			 9-4 Derivatives of Constants, Power Forms, and Sur 9-5 Derivatives of Products and Quotients 9-6 Chain Rule: Power Form 9-7 Marginal Analysis in Business and Economics 9-8 Chapter Review 	ms 500 513 522 530 540
	CHAPTER	10	Additional Derivative Topics	547
			10-1 First Derivative and Graphs	548
			10-2 Second Derivative and Graphs	563
			10-3 Curve Sketching Techniques: Unified and Exte	nded 575
			10-4 Optimization; Absolute Maxima and Minima	590
			10-5 The Constant e and Continuous Compound Int	
			10-6 Derivatives of Logarithmic and Exponential Fu	
			10-7 Chain Rule: General Form	629
			10-8 Chapter Review	640
\	СНАРТЕК	11	Integration	647
			11-1 Antiderivatives and Indefinite Integrals	648
			11-2 Integration by Substitution	662
			11-3 Definite Integrals	674
			11-4 Area and the Definite Integral	686
			11-5 Definite Integral as a Limit of a Sum	698
			11-6 Consumers' and Producers' Surplus	713
			11-7 Chapter Review	720
*	APPENDIX	A	Special Topics	A1
			A-1 Sets	A2
			A-2 Sequences, Series, and Summation Notation	A9
			A-3 Arithmetic Progressions	A17
			A-4 Geometric Progressions	A21
			A-5 The Binomial Theorem	A27
•	APPENDIX	В	Tables	A33
			Table I Basic Geometric Formulas	A35
			Table II Mathematics of Finance	A37
			and a manage	1107
			Answers	A53
			Index	I1
			Applications Index	Inside front cover



Preliminaries