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DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS

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Dedicated to the memory of
Prof. Alois Švec

PREFACE

The papers published in these Proceedings represent the final version of some lectures and communications presented at the International Conference on Differential Geometry and Its Applications. This Conference was organized by the Faculty of Science, J. E. Purkyně University, Brno, Czechoslovakia, from August 26 to September 2, 1989, and it became a part of the celebrations of the 70th anniversary of Brno University. The Organizing Committee consisted of Anton Dekrét, Josef Janyška, Ivan Kolář, Oldřich Kowalski, Demeter Krupka (chairman), Zbyněk Nádeník, and Alois Švec.

The scientific program covered most of the basic areas of differential geometry and its applications to physical sciences. We wish to thank the lecturers, as well as all the other participants for their substantial contributions to the high scientific level of the conference.

Our thanks should be expressed to the Dean of the Faculty of Science, J. E. Purkyně University, Professor Jan Knoz for his support, and to the Faculty of Electrical Engineering of the Technical University in Brno and the Faculty of Civil Engineering of the Czech Technical University in Prague for their financial help. Our thanks should also be expressed to World Scientific Publishing Co. for a very good collaboration.

Finally, we would like to thank all the members of the Faculty of Science, J. E. Purkyně University, especially to Jan Chrastina, Bořivoj Hertzlík, Ivana Horová, Ivana Konečná, Michal Marvan, Věra Mikolášová, Jaroslav Štefánek, Olga Vlašínová, and our students, for their effort in organizing and preparing this Proceedings for publication.

Having written the Preface and having finished the preparation of this Proceedings for print we were informed that on December 10, 1989 Professor A. Švec — a member of the Organizing Committee of our Conference — died. His sudden death surprised us as well as many of the geometers who knew him because we did not manage to tell him how we believed him as a mathematican and as a man.

Brno, December 15, 1989 The Editors

The days have come, the days have gone...
Our sessions are in the past light cone,
But we feel warmth of all these days,
And my heart's puls so to you says:

Let be all hearts both brave and kind,
Let prosper science and mankind!
Let blossom thee, the mighty tree
Of Differential Geometry!

N. Mitskievich

Then God Said
Let there be gauge theories
And there was light
clear and bright
Followed by Particles
fundamental or otherwise
They jostled along merrily
reacting strongly and sometimes weakly
They followed paths
as something as could be
And they called it
the miracle of gravity.

K. Marathe

September 1, 1989, Brno

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I. GEOMETRIC STRUCTURES

DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS
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World Scientific, Singapore, 1990, 3-8.

HOLOMORPHIC VECTOR BUNDLES WITH k-PINCHED RICCI CURVATURE

NOVICA BLAŽIĆ

Abstract. We study holomorphic vector bundles (E, h) of rank 2 over a compact Hermitian surface (M, g). Then the notion of a metric with a k-pinched Ricci curvature is introduced and it represents the generalization of the Einstein condition. Some necessary topological conditions for existence of a metric h with k-pinched$(0 \leq k \leq 1)$ Ricci curvature are obtained.

Keywords. Holomorphic vector bundle, Chern numbers
1985 Mathematics subject classifications 53C55, 53B35

1. INTRODUCTION

This is an exposition, in summary, of a work which will appear in detail elsewhere(see [**3**]).

Let (E, h) be a complex vector bundle over a compact Riemmannian manifold (M, g). Suppose that E admits complex connection D such that its curvature tensor R satisfies the Einstein conditions. Then the following question about the global structure of E can be asked: what can be said about the relations between the Chern numbers of E?

The result of this kind for a tangent bundle case are obtained, for example, in [**1**], [**4**] and [**5**] and for a general case in [**6**], [**8**] and [**2**]. This results can be generalized in the case when the Einstein condition is not satisfied, but the Ricci curvature is "nice", for example when the

This paper is in final form and no version of it will be submited for publication elsewhere.

Ricci tensor is k-pinched. For tangent bundle case is obtained in [**9**] the topological obstruction for existence of a metric with a k-pinched Ricci curvature.

The main result in this paper is Theorem 3.5.,the generalization of the result of Lübke(see [**8**]). We established the inequality

$$c_2(E) \geq \left\{ \frac{1}{4} - \frac{(1-k)^2}{16\delta} \right\} c_1^2(E)$$

for a holomorphic vector bundle of rank 2 with k-pinched Ricci curvature. Also, we study when the equality holds in the inequality.

I wish to thank N.Bokan for useful discussions and suggestions.

2. Preliminaries

In this section we will follow [**7**, Ch. IV]. Let (E,h) be a holomorphic Hermitian vector bundle of rank λ over an Hermitian manifold (M,g) of complex dimension n. Then (E,h) admits a uniquie Hermitian connection D and its curvature R is a $(1,1)$-form with values in the bundle $End(E)$. If $s = (s_1, \dots, s_\lambda)$ is a local frame field for E, the curvature form $\Omega = (\Omega^i_j)$ with respect to s is given by

$$R(s_j) = \sum \Omega^i_j s_i \ , \qquad \Omega^i_j = \sum R^i_{j\alpha\overline{\beta}} dz^\alpha \wedge d\overline{z}^\beta$$

in terms of a local coordinate system $(z^1, \dots, z^n)$ of M. We write

$$h_{ij} = h(s_i, s_j) \qquad \text{and} \qquad g = \sum g_{\alpha\overline{\beta}} dz^\alpha d\overline{z}^\beta .$$

Now we define ρ and $\widehat{\rho}$, the Ricci and $\widehat{\ }$-Ricci curvatures of (E,h) respectively, by

$$\rho^i_j = \sum g^{\alpha\overline{\beta}} R^i_{j\alpha\overline{\beta}} \ , \qquad \rho_{j\overline{k}} = \sum h_{i\overline{k}} \rho^i_j \tag{2.1}$$

and

$$\widehat{\rho}_{\alpha\overline{\beta}} = \sum h^j_i R^i_{j\alpha\overline{\beta}} \ , \qquad \widehat{\rho}^{\beta}_{\alpha} = \sum \widehat{\rho}_{\alpha\overline{\gamma}} g^{\overline{\gamma}\beta} . \tag{2.2}$$

Suppose now that $s = (s_1, \dots, s_\lambda)$, $e = (e_1, \dots, e_n)$ and $\theta = (\theta^1, \dots, \theta^n)$ are local unitary frame fields for (E,h), TM and T^*M respectively. Then we write

$$\Phi = \sqrt{-1} \sum \theta^\alpha \wedge \overline{\theta}^\alpha$$

$$\|R\|^2 = 4 \sum |R^i_{j\alpha\overline{\beta}}|^2$$

$$\|\rho\|^2 = 2\sum |\rho^i_j|^2 = 2\sum |\rho_{j\bar{i}}|^2$$
$$\|\widehat{\rho}\|^2 = 2\sum |\widehat{\rho}_{\alpha\bar{\beta}}|^2 = 2\sum |\widehat{\rho}^{\beta}_{\alpha}|^2$$
$$\tau = 2\sum \rho^i_i = 2\sum \widehat{\rho}_{\alpha\bar{\alpha}}$$

for the fundamental form of (M, g), the norms of the tensors R, ρ, $\widehat{\rho}$, and for the scalar curvature τ of (E, h) respectively. Scalar curvatures and $\widehat{\ }$-Ricci tensors of a holomorphic vector bundles (E', h') and (E'', h'') are denoted by τ', τ'', $\widehat{\rho}'$ and $\widehat{\rho}''$ respectively.

From now on we suppose $\lambda = n = 2$. Then let ϱ and $\widehat{\varrho}$ denote the sections of the bundles $End(E)$ and $End(TM)$ defined by

$$h(\varrho(s), t) = \rho(s, t) \qquad \text{and} \qquad g(\widehat{\varrho}(e), f) = \widehat{\rho}(e, f)$$

for $s, t \in E_p$ and $e, f \in TM_p, p \in M$. The endomorphisms ϱ and $\widehat{\varrho}$ are symmetric, so their corresponding eigenvalues r_1, r_2 and $\widehat{r}_1, \widehat{r}_2$ are real. We will use a local unitary frame field $s = (s_1, s_2)$ determined by the eigenvectors of ϱ corresponding to r_1 and r_2 . Also, a local unitary frame field $e = (e_1, e_2)$ is determined by the eigenvectors of $\widehat{\varrho}$.

Let $r = \max\{|r_1|, |r_2|\}$. Then, for $0 \leq k \leq 1$, we say that Ricci curvature ρ of (E, h) is *k-pinched* if

$$krh \leq \rho \leq rh \qquad \text{or} \qquad -rh \leq \rho \leq -krh \tag{2.3}$$

holds on M. Then, clearly, $r_1 r_2 \geq 0$ on M. When $k = 1$, (2.3) represents the weak Einstein condition as it was defined by Kobayashi.

The Gauss curvature $\widehat{\tau}$ of (E, h) can be defined by

$$\widehat{\tau} = \det \widehat{\varrho} = \widehat{r}_1 \cdot \widehat{r}_2.$$

The Gauss curvature $\widehat{\tau}$ is *δ-bounded from below* if

$$\widehat{\tau} \geq \delta r^2 \tag{2.4}$$

on M. The class of a holomorphic vector bundles which satisfy the conditions (2.3) and (2.4) is denoted by $\mathcal{E}_{k,\delta}$.

We now give the key technical lemmas of this note.

LEMMA 2.1. *Let (E, h) be a holomorphic vector bundle of rank 2 over an Hermitian surface (M, g). Then the following inequality holds*

$$\|R\|^2 \geq \|\widehat{\rho}\|^2 + \|\rho - \tfrac{\tau}{4} h\|^2 . \quad \blacksquare \tag{2.5}$$

When (E, h) satisfies the weak Einstein condition, i.e. ρ is a 1-pinched, the equality case was studied in [7]. In that case, the equality holds in (2.5) if and only if (E, h) is projectively flat, i.e. $R = \frac{1}{2}\widehat{\rho} \otimes h$. So it is natural to study the equality in the general case.

LEMMA 2.2. *Let (E, h) be a holomorphic vector bundle of rank 2 over an Hermitian surface (M, g) such that the Ricci curvature tensor ρ is parallel and $r_1 \neq r_2$ on M. Then the equality*

$$\|R\|^2 = \|\widehat{\rho}\|^2 + \|\rho - \tfrac{\tau}{4}h\|^2 \tag{2.6}$$

holds if and only if

$$E = E' \oplus E''$$

and

$$\widehat{\rho}\,' - \widehat{\rho}\,'' = \tfrac{1}{4}(\tau' - \tau'')g \ . \tag{2.7}$$

where (E', h') and (E'', h'') are holomorphic orthogonal line bundles. ∎

3. CHERN CLASSES $c_1^2(E)$ AND $c_2(E)$

We will mention now some consequences of the results from the previous section.

COROLLARY 3.1. *Let (E, h) be a holomorphic vector bundle of rank 2 over a compact Hermitin surface (M, g). If the Gauss curvature of E is nonnegative, then*

$$c_1^2(E) \geq 0 \quad . \ ∎$$

REMARK 3.2: This result is already known (see [**6**,Thm.4.1]) because $\widehat{\tau}$ is nonnegative if and only if $\widehat{\ }$-Ricci curvature is nonnegative or nonpositive.

COROLLARY 3.3. *Let (E, h) be a holomorphic line bundle over a compact Hermitian surface M and let E^* be the dual bundle of E. Then the holomorphic bundle $E \oplus E^*$ admits no metric with positive or negative Gauss curvature on M.* ∎

LEMA 3.4. *Let (E, h) be a holomorphic vector bundle of rank 2 over a compact Hermitian surface (M, g). If the Ricci tensor is k-pinched and the Gauss curvature is $\frac{1}{4}(1-k)^2$-bounded from below we have*

$$e(E) = c_2(E) \geq 0 \ .$$

where $e(E)$ is the Euler characteristic of E. If the Ricci curvature ρ is parallel and $k < 1$, the equality holds if and only if (E, h) admits a holomorphic orthogonal decomposition $(E, h) = (E', h') \oplus (E'', h'')$ with