Special Issue on

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Scanning the Issue

SPECIAL ISSUE ON ATOMIC AND MOLECULAR PLASMAS

In the decade since the last PROCEEDINGS special issue on plasmas,[1] much progress has been made in understanding and learning how to use gaseous conductors—plasmas—in a variety of ways. Some uses are new but interest in plasmas can be traced back to the beginnings of electrical technology. As an example, the arcs and sparks occurring when a circuit is opened or closed by switchgear have long been studied, and recent developments have resulted in important improvements to high-power circuit breakers. A new field of interest is the gas laser, either with a flowing or nonflowing gas, usually excited and/or ionized by an electrical discharge.

This special issue of the PROCEEDINGS is an attempt to bring together under one cover review and original papers on basic properties of atomic and molecular plasmas and the use of these properties in predicting and analyzing the performance of engineering devices. Since the emphasis is on applications in which atomic and molecular properties play a key role in governing the behavior of the plasma, papers on thermonuclear phenomena, on ionospheric plasmas, and on waves and instabilities were excluded—perhaps somewhat arbitrarily. On the other hand, papers in such diverse fields as shock-generated plasmas and vacuum arcs were encouraged. It is hoped that workers in these various fields will profit from exposure to the different applications of plasmas.

The last ten years have also seen a considerable increase in our knowledge of atomic and molecular properties basic to understanding of the many phenomena occurring in a plasma. It is worth noting, for example, that the now classical paper on collisional-radiative recombinations between electron and atomic ions by Bates, Kingston, and McWhirter[2] did not appear until 1962. In this regard it is appropriate that the present issue includes an extensive review of the progress in this field since that time.

Among the many techniques contributing to the advances in our knowledge of the plasma state, perhaps the most important is the widespread use of the digital computer. Nearly all of the authors made some use of the computer in the research they report. Probably more important, some of the research could never have been undertaken if the authors could not have carried out their computations automatically.

Space does not allow the mention here of all the excellent contributions to this issue. We should like to note some of the subjects covered which had not been investigated at the

[1] *Proc. IRE*, vol. 49, Dec. 1961, pp. 1741–1994.

[2] D. R. Bates, A. E. Kingston, and R. W. P. McWhirter, "Recombination between electrons and atomic ions I, II," *Proc. Roy. Soc., Ser. A*, vol. 267, 1962, pp. 297–310, and vol. 270, 1962, pp. 155–168.

time of the last plasma issue. Already mentioned is the study of electron–ion recombination. The measurements of transport properties of ionized gases reported here are the result of techniques developed and perfected in the last decade. The frequent mention of the electronegative gas sulfur hexafluoride reflects its expanding use as an insulating and arc-quenching medium in circuit breakers. It has been increasingly recognized of late that nonequilibrium behavior can significantly alter the characteristics of electric arcs near and below atmospheric pressure. Application of non-local thermal equilibrium (non-LTE) theory to two situations is reported here. Perhaps the next few years will see further application of these techniques to the arcs present in practical devices such as discharge lamps. Although lasing action had been obtained about a decade ago, each year sees new developments in this field, and some are reported here. Due to page limitations, a paper for this special issue on ion laser plasmas will appear in the May 1971 issue of the PROCEEDINGS.

In closing we wish to thank the authors for the many fine contributions to this issue, and also to express our appreciation to the reviewers for their helpful comments on the manuscripts. We also thank our associates and our secretaries for their efforts on this issue.

The Editorial Committee
T. E. Browne, Jr.
Charles H. Church, *Chairman*
R. Stephen Devoto
James P. Wade, Jr.

NOTE ON ADDITIONAL COVERAGE

Because of the variety of applications treated, the reader wishing to follow developments in the areas covered by this special issue will find continuing coverage in a number of publications, as evidenced by the lists of references in the papers that follow. Among the most relevant of the IEEE journals are the IEEE TRANSACTIONS ON POWER APPARATUS AND SYSTEMS, the IEEE JOURNAL OF QUANTUM ELECTRONICS, and the TRANSACTIONS of the Aerospace and Electronic Systems Group and the Electron Devices Group.

The Editor

T. E. Browne, Jr., *was born on June 30, 1908. He graduated from the North Carolina State University in 1928. He received the M.S. degree in 1933 from the University of Pittsburgh and the Ph.D. degree from the California Institute of Technology in 1936.*

Since completing the Westinghouse graduate student course in 1929, he has been engaged in studies of electric arc phenomena in stationary and flowing liquids and gases. His studies, which began under the direction of Dr. J. Slepian, have aided the fundamental understanding of circuit interrupting processes in switches, circuit breakers, and fuses, and have contributed to the design and development of these devices. He has over 40 years of experience working on arc problems. He is the holder of 20 patents in circuit interruption and related fields and has been author or co-author of 26 technical papers. He has also served as a Westinghouse lecturer in the Department of Electrical Engineering, University of Pittsburgh.

Dr. Browne has served as a member of the AIEE Electrical Insulation Committee and as a Vice-Chairman of the Subcommittee on Gaseous Insulation. He is a member of Tau Beta Pi, Sigma Xi, and Phi Kappa Phi, and is a Registered Professional Engineer in the State of Pennsylvania.

Charles H. Church (*S'56–M'59*) *was born in Phoenix, Ariz., on May 15, 1929. He received the B.S. degree in physics from the University of Missouri–Rolla, in 1950, the M.S. degree in physics from the Pennsylvania State University, University Park, in 1951, and the Ph.D. degree in physics from the University of Michigan, Ann Arbor, in 1959.*

In 1951 he joined the Research Laboratories Division, General Motors Corporation. In 1952 he was called to active duty, serving as an Armament Systems Officer in the U. S. Air Force. Upon release from active duty in 1954, he received a Dow Chemical Company Fellowship in elasticity at the University of Michigan. He was also a Research Associate in the Engineering Research Institute and a Teaching Fellow in physics at the University of Michigan. In 1958 he joined the Central Research Laboratory, Crucible Steel Corporation of America, Pittsburgh, Pa. In 1959 he became a Member of the Staff of Westinghouse Research Laboratories, Pittsburgh, Pa. In 1968 he was appointed Staff Specialist—Laser Systems and Technology in the Advanced Research Projects Agency, Arlington, Va. His professional interests include lasers, optical instrumentation, arc discharges and weapon technology.

Dr. Church is a fellow of the Optical Society of America, and a member of the American Institute of Aeronautics and Astronautics, the American Physical Society, and the American Ordnance Association. He served as Chairman of the Laser Technical Group of the Optical Society of America for 1968 and 1969.

R. Stephen Devoto *was born in San Francisco, Calif., on October 23, 1934. He received the S.B. and S.M. degrees in mechanical engineering from the Massachusetts Institute of Technology, Cambridge, in 1958 and 1960, respectively, and the Ph.D. degree in aeronautics and astronautics from Stanford University, Stanford, Calif., in 1965, partly under a Ford Foundation Plasma Fellowship.*

From 1965 to 1967 he was a Research Associate and then an Assistant Professor at Stanford University, working on the properties of partially ionized gases. In 1967–1968 he was at the Institut für Plasmaphysik in Garching, Germany, on an NSF postdoctoral fellowship, and in 1969 returned to Stanford University as an Assistant Professor. He is currently engaged in research on very high pressure electric arcs.

Dr. Devoto is a member of the American Physical Society, the American Institute of Aeronautics and Astronautics, Sigma Xi, and the Sierra Club.

James P. Wade, Jr., *was born in Richmond Heights, Mo., on December 26, 1930. He received the B.S. degree in engineering from the United States Military Academy, West Point, N. Y., in 1953, and the M.S. and Ph.D. degrees in physics from the University of Virginia, Charlottesville, Va., in 1959 and 1961, respectively.*

In 1961, he was assigned by the Department of the Army to the physics staff of the Lawrence Radiation Laboratory, University of California, Livermore, Calif. In 1967, he was appointed a Staff Specialist in the Strategic Technology Office of the Advanced Research Projects Agency, Arlington, Va. In 1970, he joined the staff in the Office of the Director, Defense Research and Engineering (Strategic and Space Systems), Department of Defense. He recently participated as a member of the United States Delegation to the Strategic Arms Limitation Talks (SALT) with the USSR.

Dr. Wade is a member of the American Physical Society and Sigma Xi.

Principles of Arc Motion and Displacement

HEINZ H. MAECKER

Abstract—It is shown that the motion of an arc being described as a temperature cloud can be divided into two parts, one being the relative velocity of the arc phenomenon with respect to the mass flow and the other the mass motion itself. The first relative motion is determined by the equation for the change of internal energy with time, the second mass flow has to be calculated by means of the magnetohydrodynamic and continuity equations. Three groups of examples are given in each of which one of the three velocities disappears. In the first group, no mass flow exists and the motion of the arc is caused by various types of inhomogeneous heating. In the second group, the arc itself does not move due to the opposing effects of mass motion and relative arc motion. In the last group, the arc follows the mass flow without relative motion. As long as the boundary conditions do not change from the standpoint of the arc the motion continues steadily. If, however, the boundary conditions form any type of obstacles, the motion of the arc ends up in a new equilibrium position, a displacement occurring.

I. Introduction

If an observer looks upon a moving arc he is inclined intuitively to combine the arc motion with the motion of the hot matter. This may be correct in the case where an arc is moving between two rail electrodes driven by a crossed magnetic field. If, however, a transverse magnetic field is applied to a wall stabilized arc the Lorentz forces displace the arc in an excentric position and continue to act in this new equilibrium situation as a pump producing a double whirl of mass flow within the chamber. In this case the arc by no means follows the mass motion. The reason for this behavior is that due to the boundary conditions, given by the chamber, a relative motion of the arc occurs, with respect to the matter, in opposite direction but of the same amount. The "force" of this relative backward motion is the asymmetry of the heat conduction to the wall caused by the excentric position. If, finally, a stable arc is exposed to a high energy radiation from one direction, a motion of the arc towards the radiation source can be observed. In this case the motion of matter is of no significant importance for the processes. Because of these different types of arc motion it is desirable to have a comprehensive theory describing all cited phenomena in a uniform manner.

II. Definition of Arc Motion V_A

For an external observer the electric arc appears as a light phenomenon. The physicist knows that the origin of this light distribution is a temperature cloud. Talking about the position and the motion of such a cloud of a state quantity one has, first of all, to define these notions, for example, by formation of an effective center of gravity. But in so doing, serious difficulties arise from the fact that in a temperature cloud, in contrast to a mass cloud, new amounts of state quantities can be created in each maximum by temperature elevation. Furthermore, it is impossible to define the position of a certain temperature without additional assumptions, because all points of same temperature form mostly closed areas. Finally, it is hard to determine a proper weight for each volume element to calculate a meaningful center of gravity. Regarding these difficulties let us agree for sake of simplicity in defining the position of the cloud as the position of the temperature maximum (subscript m) and the motion of the arc (subscript A) as that of the temperature maximum

$$r_A = r_m; \qquad V_A = V_m. \tag{1}$$

This definition may hold even if the temperature in the maximum changes with time: $(dT/dt)_m \neq 0$.

Since the temperature maximum is defined as the point with vanishing temperature gradient the following equation holds:

$$\nabla_m T = 0. \tag{2}$$

So if we want to follow the temperature maximum this is equivalent to following a certain temperature gradient in the general case. This can be done by expanding the temperature distribution depending on space and time in a Taylor series around a chosen point r_0, t_0, and taking the gradient of this series which is cut off after the linear terms

$$\nabla T(r,t) = \nabla_{00} T + (r \cdot \nabla_{00}) \nabla T + t \nabla_{00} \frac{\partial T}{\partial t} \cdots. \tag{3}$$

The subscript 00 refers to the chosen point of expansion at time zero and the curved ∂'s mean the local derivative. If we ask now for the variation of the temperature gradient with time for an observer moving with the velocity V regarding the alteration of temperature in space and time, we have to form the total derivative with respect to time

$$\frac{d\nabla T}{dt} = (V \cdot \nabla_{00}) \nabla T + \nabla_{00} \frac{\partial T}{\partial t}. \tag{4}$$

So the moving observer notices a change of the temperature gradient both by the motion in the fixed temperature field and by the change of the local temperature gradient with time. This is the well-known substantial differentiation. If we demand now that the observer moves in such direction and with such speed that he does not notice any change of the temperature gradient, then he has taken on the velocity of the temperature gradient originally existing in the point of expansion. In other words, the substantial derivative has to disappear if the velocity V shall mean the velocity of the special temperature gradient ($d\nabla T/dt = 0$). Applying the equation to the temperature maximum with the vanishing temperature gradient the defining equation for the motion

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of the arc follows

$$\nabla_{m0} \frac{\partial T}{\partial t} = -(V_A \cdot \nabla_{m0})\nabla T. \tag{5}$$

This equation can be read as a phenomenological equation according to which the gradient of the temperature variation with time, i.e., the inhomogeneity of heating in the maximum, causes the arc motion and where the operator $-\{(\nabla_{m0})\nabla T\}^{-1}$ plays the role of a coefficient. The simplest example explaining this equation is a stretched wire with ohmic heating having its temperature maximum in the middle between the holders. If this wire experiences an inhomogeneous heating around its temperature maximum, so that one side of the maximum is heated and the other is cooled, the temperature maximum will move in that direction where the heating occurs. A uniform heating of both sides of the maximum would raise the temperature but never move the maximum. For this simple linear example the velocity of the temperature maximum can be determined according to (5) resulting in

$$V_A = \left(\frac{-\partial^2 T}{\partial x \partial t}\right)_m \bigg/ \left(\frac{\partial^2 T}{\partial x^2}\right)_m \tag{6}$$

where the inhomogeneous heating in the numerator is the cause, the negative denominator the coefficient, and the velocity V_A the effect.

III. The Relative Arc Velocity with Respect to Matter V_{AM}

For the general case of the motion of a temperature cloud with just one temperature maximum the inhomogeneity of heating in the maximum has to be determined by means of the equation for the internal energy. This equation describes the variation of the internal energy with time for a certain mass element (subscript M) even if it is moving. The last statement is of essential importance. Assuming constant pressure in the whole space, the change of energy can be transformed into the change of temperature with the aid of the specific heat c_p. The variation of temperature with time for a certain mass element moving with the velocity V_M can be split again into two parts with the substantial derivative

$$\left(\frac{dT}{dt}\right)_M = \frac{\partial T}{\partial t} + V_M \cdot \nabla T. \tag{7}$$

Forming the gradient of this equation in the maximum with $\nabla_m T = 0$ and applying the well known equation of the vector analysis

$$\nabla(A \cdot B) = (A \cdot \nabla)B + A \times (\nabla \times B) + (B \cdot \nabla)A + B \times (\nabla \times A) \tag{8}$$

we obtain

$$\nabla_m \frac{\partial T}{\partial t} = \nabla_m \frac{dT}{dt} M - (V_M \cdot \nabla_m)\nabla T. \tag{9}$$

Identification of (9) with the defining equation (5) for the arc velocity yields

$$\nabla_m \left(\frac{dT}{dt}\right)_M = -\{(V_A - V_M) \cdot \nabla_m\}\nabla T \tag{10}$$

where the term $(dT/dt)_M$ is determined by the internal energy equation. From (10) it turns out that the equation for the internal energy determines only the relative velocity between the arc and the mass and not the absolute velocity of the arc.

Defining the relative velocity between arc and mass by the abbreviation

$$V_{AM} = V_A - V_M$$

and solving this equation for the absolute arc velocity

$$V_A = \underset{\text{thermo.}}{V_{AM}} + \underset{\text{hydro.}}{V_M} \tag{11}$$

it proves to be reasonable to compose the arc motion V_A by the relative velocity of the arc with respect to matter V_{AM} determined by the equation for internal energy, i.e., by thermodynamics on the one hand and by the mass velocity V_M to be derived from the magneto-hydrodynamic equations including the continuity equation for mass conservation on the other hand.

The temperature rise with time of a moving mass element is given by the equation of internal energy at constant pressure

$$\left(\frac{dT}{dt}\right)_M = \frac{1}{\rho c_p}\left(\frac{j^2}{\sigma} - \operatorname{div} W + a \cdots\right). \tag{12}$$

ρ is the mass density, j^2/σ is the ohmic heating, div W is the heat conduction loss, and the a denotes the radiative absorption per unit volume. These terms serve as a representation of all the processes that are able to change the internal energy. For the current density j the generalized Ohm's law has to be inserted with the mean forces, the applied and the induced electric field strengths

$$j = \sigma(E + V_M \times B \cdots). \tag{13}$$

Introducing the gradient of (12) into (10) we get the defining equation for the relative velocity of the arc with respect to matter V_{AM}:

$$(V_{AM} \cdot \nabla_m)\nabla T = -\nabla_m\left\{\frac{1}{\rho c_p}\left(\frac{j^2}{\sigma} - \operatorname{div} W + a \cdots\right)\right\}. \tag{14}$$

The splitting of the absolute arc velocity V_A into the relative velocity of the arc with respect to the mass flow V_{AM} and into the mass velocity V_M itself as well as the prescription for the calculation of the relative velocity V_{AM} with help of the internal energy equation are the most important results of the preceding considerations.

IV. Discussion of Various Types of Arc Motion

The motion of arcs shall be explained by a few typical examples in the following. The classification of these examples can be carried out by setting one of the three velocities equal to zero. In the first case, no flow of matter may exist and the arc moves in the laboratory system with the relative velocity between arc and matter itself. Here the motion of the arc can be caused only by the inhomogeneity of heating in the vicinity of the temperature maximum while a heating in the maximum itself would only raise the maximum tem-

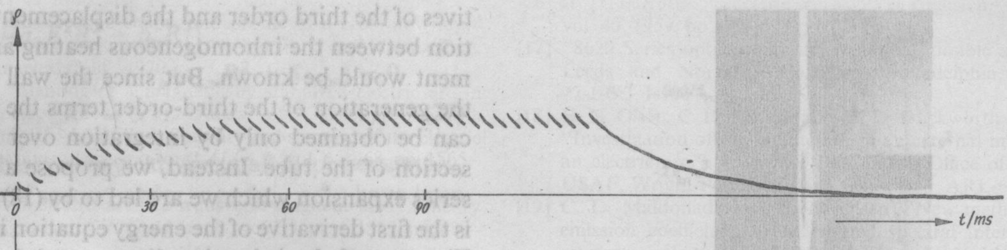


Fig. 1. The deflection of the arc from the tube axis by directed HF irradiation after switch-on and off. The sawtooth-like steps are due to the pulsed irradiation (Schwertl [1]).

perature. In the second case, the velocity of the arc in the laboratory system shall vanish which means that both the velocity of matter and the relative velocity of the arc with respect to matter are equal in amount but opposite in direction thereby compensating each other. In the last case, no relative motion of the arc with respect to matter shall occur so that the motion of the arc in the laboratory system coincides with that of the matter. Assumed in the last phenomenon is that no inhomogeneous heating takes place. If the boundary conditions are such that they do not change from the standpoint of the arc during the motion this will continue steadily. If, however, the boundary conditions change for the arc with the motion, a final state will be reached and a discrete displacement can be stated. An example for the first case of steady motion is a running arc between rail electrodes driven by a crossed magnetic field. For the second case of a new fixed position the wall stabilized arc exposed to a radiation field is an example because the arc reaches a new excentric situation closer to the wall and experiences a displacement. The mathematical condition for the displacement according to (14) is that the gradient of the supply of internal energy disappears in the displaced position.

V. Arc Motion Caused by Inhomogeneous Absorption a

According to (14) the gradient of the heating or cooling around the temperature maximum results in a motion of the arc without motion of matter as long as we disregard the equilization flow due to the expansion and contraction of the gas. The simplest example for this case is the irradiation from one side onto the arc with a diverging beam. Due to this divergence and to the absorption of radiation in front of the arc this side directed to the source will be heated more than the back whereby the temperature maximum i.e., the arc, is moved towards the source. This experiment has been carried out by Schwertl [1] who exposed a wall-stabilized argon arc of a few amperes in a quartz tube of 10 mm in diameter at atmospheric pressure to a high-frequency radiation of $3 \cdot 10^{10}$ cm/s leaving the end of the hollow waveguide in a diverging field. As may be seen from Fig. 1 the arc actually moves from the axial position according to an exponential time function to a new excentric place. The saw-tooth like steps in the curve are the consequence of the fact that the emitter is pulsed with a few microsecond emission time and a few millisecond interval. Mathematically, the cause of the initial motion is the term gradient a only while the field strength E is constant all over the tube and the temperature depending coefficients ρ, c_p, κ and σ do not vary in the temperature maximum. The heat conduction term $-\operatorname{div} W$ contains the heat flux density explained by

$$W = -\kappa \nabla T = -\nabla S \tag{15}$$

where κ is the heat conductivity and

$$S = \int_0^T \kappa dT$$

the heat flux potential. The heat conduction term div W does not vary either, initially, because of symmetry reasons. So the calculation of the initial arc velocity by means of (14) yields for this two-dimensional problem

$$-\frac{1}{\rho_m c_{p,m}} \nabla_m a = (V_{AM} \cdot \nabla_m) \nabla T$$

$$-\frac{1}{\rho_m c_{p,m}} \left(\frac{da}{dx}\right)_m e_x = V_{AM,x} e_x \left(\frac{\partial^2 T}{\partial x^2}\right)_m$$

$$V_{AM,x} = -\frac{1}{\rho_m c_{p,m}} \left(\frac{da}{dx}\right)_m \Bigg/ \left(\frac{\partial^2 T}{\partial x^2}\right)_m . \tag{16}$$

Here x is the direction from the T maximum in the tube axis to the radiation source. We recognize the proportionality between the initial arc velocity $V_{AM,x}$ and the causing inhomogeneity of absorption $(da/dx)_m$ in the maximum while the most important term in the factor of proportionality is the negative reciprocal of the curvature of the temperature distribution around the maximum $-1/(\partial^2 T/\partial x^2)_m$ which is a measure for the thickness of the arc. Therefore thin arcs start with a small velocity, thick arcs with a higher one. The negative sign in (16) cancels with the negative magnitude of $(\partial^2 T/\partial x^2)_m$, so that the motion is actually directed toward the source. If dT is replaced by dS/κ the factor of proportionality consists of the product of the thermal diffusivity $\kappa_m/\rho_m c_{p,m}$ in the maximum and, roughly speaking of the thickness of the arc $-1/(\partial^2 S/\partial x^2)_m$:

$$V_{AM,x} = -\frac{\kappa_m}{\rho_m c_{p,m}} \frac{1}{(\partial^2 S/\partial x^2)_m} \cdot \left(\frac{da}{dx}\right)_m . \tag{16'}$$

The factor of proportionality between $V_{AM,x}$ and $(da/dx)_m$ can be interpreted as the mobility of the arc, i.e., its velocity per unit of inhomogeneous heating. For a crude estimation of the effective arc radius squared may be taken

$$r_0^2 = -1/2 S_m (\partial^2 S/\partial r^2)_m . \tag{16''}$$

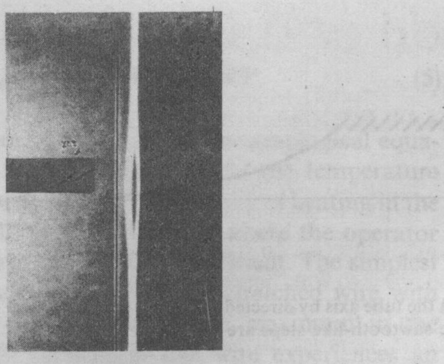


Fig. 2. The displacement of an Ar arc at 1 A and 1 atm with 10-mm tube in diameter caused by pulsed irradiation of 3×10^{10} cm/s from the end of the waveguide. The weak bow at the right is due to reflection on the back of the tube (Schwertl [1]).

The mass flow V_M is not absent completely, but there arises a small equalizing flow originating from the heated and thereby expanding regions which enters the cooled contracting regions on the other side of the temperature maximum. The direction of this compensation flow is obviously opposite to the motion of the arc and its amount has been calculated by Seeger [2] under the assumptions that this mass flow is a potential flow with zero rotation and that the walls do not hinder the motion, i.e., small arcs in wide tubes. Assuming, furthermore, that the region in which the inhomogeneous heating is effective covers about that paraboloid described by

$$T/T_m = 1 + \frac{1}{2T_m}(\partial^2 T/\partial r^2)_m r^2$$

down to $r_0(T=0)$, it turns out that the equalization mass flow is just $\frac{1}{7}$ of the relative arc velocity. So for the absolute arc velocity results

$$V_A = V_{AM} + V_M = V_{AM} - \tfrac{1}{7}V_{AM} = \tfrac{6}{7}V_{AM}. \tag{17}$$

VI. The Displacement of an Inhomogeneously Heated Arc

During the motion of a wall stabilized arc heated inhomogeneously, for example by irradiation, the initially vanishing term for the gradient of heat conduction $-\mathrm{div}\, \boldsymbol{W}$ increases more and more due to the growing excentric position of the arc. This gradient can be interpreted roughly as the change of curvature of the temperature distribution along the diameter in the x direction. This curvature is enhanced at that side of the maximum approaching the wall and is diminished at the other side producing thereby an increasing gradient of div $\boldsymbol{W}$ in the maximum which retards the motion. The new steady excentric position, i.e., the displacement, is reached when in (14) the gradient of inhomogeneous heating is compensated by the gradient of the heat conduction term in the maximum (Fig. 2). In the case of irradiation there is

$$\nabla_m a + \nabla_m(\nabla^2 S) = 0$$

$$\left(\frac{da}{\partial x}\right)_m + \left(\frac{\partial^3 S}{\partial x^3}\right)_m + \left(\frac{\partial^3 S}{\partial x \partial y^2}\right)_m = 0. \tag{18}$$

If it would be possible to find a relation between the derivatives of the third order and the displacement x_m the connection between the inhomogeneous heating and the displacement would be known. But since the wall is the cause for the generation of the third-order terms the desired relation can be obtained only by integration over the entire cross section of the tube. Instead, we propose a procedure with series expansion which we are led to by (18) which evidently is the first derivative of the energy equation in the maximum. The term of ohmic heating disappeared due to the absence of a gradient in the maximum. For the calculation of the displacement we expand the S distribution depending on the Cartesian coordinates x and y already normalized on the tube radius R in a Taylor series around the maximum, so that the center of the tube lies on the x axis at $-x_m$

$$\begin{aligned} S(x, y) = S_m &+ \tfrac{1}{2}S_{2x,m}x^2 + \tfrac{1}{2}S_{2y,m}y^2 + \tfrac{1}{6}S_{3x,m}x^3 \\ &+ \tfrac{1}{2}S_{x2y,m}xy^2 + \tfrac{1}{24}S_{4x,m}x^4 + \tfrac{1}{4}S_{2x2y,m}x^2y^2 \\ &+ \tfrac{1}{24}S_{4y,m}y^4 \cdots . \end{aligned} \tag{19}$$

Here the symmetry in y direction has been used already by omitting all terms with an odd power of y. The partial derivatives have been shortened by using suffixes indicating the order and the direction of derivation. The coefficients in the Taylor series have to be determined on the one hand by the energy equation of the arc differentiated sufficient times and employed at the maximum and on the other hand by satisfaction of the boundary conditions which require vanishing temperature at the wall. For the energy equation and its derivatives at the maximum we get

$$\begin{aligned} \sigma_m E^2 R^2 + a_m R^2 + S_{2x,m} + S_{2y,m} &= 0 \\ a_{x,m}R^2 + S_{3x,m} + S_{x2y,m} &= 0 \\ \left(\frac{d\sigma}{dS}\right)_m S_{2x,m}E^2R^2 + a_{2x,m}R^2 + S_{4x,m} + S_{2x2y,m} &= 0 \\ \left(\frac{d\sigma}{dS}\right)_m S_{2y,m}E^2R^2 + a_{2y,m}R^2 + S_{2x2y,m} + S_{4y,m} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \tag{20}$$

The boundary is described by the cycle

$$y_w^2 + (x_w + x_m)^2 = 1. \tag{21}$$

The energy equations allow the elimination of all coefficients containing a derivation with respect to y while the cycle equation permits the replacement of all factors containing y_w by those with x_w in the Taylor expansion applied at the wall. Because the left-hand side of these expansion vanishes, the disappearance of the right-hand side can be achieved only if each of the total coefficients attached to the x_w terms of same power are made equal to zero individually. This procedure supplies us with a sufficient number of equations for all coefficients in the Taylor series and the displacement x_m.

In order to explain this procedure let us include the first terms up to the third order only, for the coefficients of which the following four equations result

$$x_w^0) \quad S_m - \tfrac{1}{2}(1 - x_m^2)(\sigma_m E^2 R^2 + a_m R^2 + S_{2x,m}) = 0$$

$$\begin{aligned} x_w^1) \quad & x_m(\sigma_m E^2R^2 + a_mR^2 + S_{2x,m}) \\ & - \tfrac{1}{2}(1 - x_m^2)(a_{x,m}R^2 + S_{3x,m}) = 0 \end{aligned}$$

$$x_w^2)\quad S_{2x,m} + \tfrac{1}{2}(\sigma_m E^2 R^2 + a_m R^2) + x_m(a_{x,m}R^2 + S_{3x,m}) = 0$$

$$x_w^3)\quad \tfrac{2}{3}S_{3x,m} + \tfrac{1}{2}a_{x,m}R^2 = 0. \qquad (22)$$

The solution of this system yields for small displacements x_m where quantities of second order, e.g., $(x_m \cdot a_{x,m})$ have been dropped

$$S(x,y) = \frac{R^2}{4}(\sigma_m E^2 + a_m)\{1 - (1 + 2x_m x)(x^2 + y^2)\}$$

$$S_m = \frac{R^2}{4}(\sigma_m E^2 + a_m);$$

$$x_m = \frac{a_{x,m}}{4(\sigma_m E^2 + a_m)} = a_{x,m}\frac{R^2}{16 S_m}$$

$$S_{3x,m} + S_{x2y,m} = -16 S_m x_m. \qquad (23)$$

Notice that all lengths are related to R.

The last line gives the desired relation between the third-order terms of S and the displacement x_m. In the third line there is the relationship between the inhomogeneous absorption $a_{x,m}$ and the resultant displacement x_m while the factor of proportionality is again a sort of mobility.

This solution is exact only for a constant electric conductivity over the entire cross section and describes a parabola on which a curve of third order is superposed in the case of finite displacement (Fig. 3). Within this very rough approximation neither the maximum temperature nor the breadth changes with displacement. For a better description of the arc behavior the terms of fourth order in the series expansion can be included yielding satisfactory results for those cases in which the material function $\sigma(S)$ has a negative curvature in the mean. For weak absorption there results:

$$S_m = \frac{R^2}{4}(\sigma_m E^2 + a_m)\left(1 - \frac{1}{16}\left(\frac{d\sigma}{dS}\right)_m E^2 R^2\right)$$

$$x_m = a_{x,m}/4(\sigma_m E^2 + a_m)\left(1 - \frac{1}{8}\left(\frac{d\sigma}{dS}\right)_m E^2 R^2\right). \qquad (24)$$

A very good approximation for practical purposes is achieved if also the fifth and sixth orders are included in the expansion. But because these calculations become very complicated and tedious even for small displacements they have not yet been completed.

So we see in this typical example that a directed irradiation causes a movement and displacement of a wall-stabilized arc and that the ohmic heating is not involved in this process except that it is taken along with the core of the arc.

VII. Time Constants for the Displacement and Return Process

If an originally steady arc is exposed to a directed irradiation in the beginning the only cause for motion of the arc is, as we saw, the gradient of absorption a. During the motion a gradient of the heat conduction term arises whereby the arc approaches its final displacement position with an exponential time function, if small displacements and no changes of gradient a are allowed. The final position is determined by the equality of the gradient a and gradient

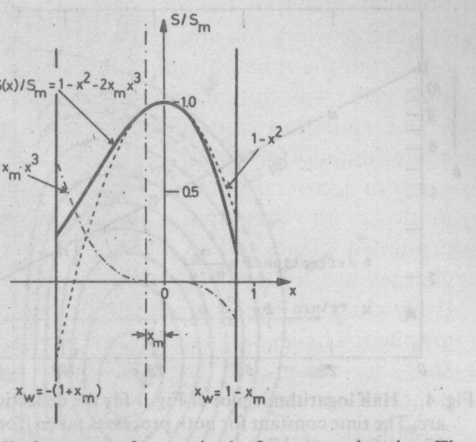


Fig. 3. Scheme of displacement of an arc in the first approximation. The parabolic shape of the temperature distribution is displaced by superposition of a third-order curve. The fourth-order parabola would improve the satisfaction of the boundary condition $S_w = 0$.

$(\nabla^2 S)$ in opposite directions

$$\nabla_m a + \nabla_m(\nabla^2 S) = 0. \qquad (25)$$

So, if this steady state is disturbed again by switching off the radiation the repelling "force" is the same as has been during the deflection. Therefore the time constant for both processes must be the same. Its magnitude τ can be calculated by means of the initial velocity given by (16′) and by the relative displacement x_m either measured or calculated. For the deflection the equation

$$X_A(t) = R x_m(1 - \exp(-t/\tau)) \qquad (26a)$$

holds, where X_A means the distance of the arc from the tube axis and for the return

$$X_A(t) = R x_m \exp(-t/\tau). \qquad (26b)$$

The time derivation of both these equations yields for the absolute value of the time constant

$$\tau = R x_m / V_{AO}. \qquad (27)$$

As the relative displacement x_m is always proportional to $(\partial a/\partial x)_m$, for small deflections

$$x_m = D\left(\frac{\partial a}{dx}\right)_m$$

and as the same holds for V_{AO} according to (16′) the inhomogeneous heating cancels for τ, whereby it becomes

$$\tau = D\left(\frac{\partial^2 S}{\partial x^2}\right)_m \left(\frac{\rho c_p}{\kappa}\right)_m. \qquad (28)$$

In the simple case of the parabola arc (23) yields

$$\tau = \frac{R^2}{8}\left(\frac{\rho c_p}{\kappa}\right)_m. \qquad (29)$$

Thus τ is inversely proportional to the thermal diffusivity.

As may be seen from Fig. 4 Schwertl [1] found in a half logarithmic plot the same straight line for both the displacement and the return the time constant of which amounts to almost 0.1 s. Actually the motion of the arc

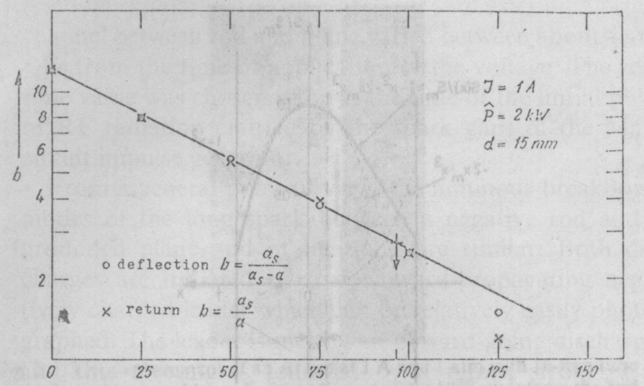


Fig. 4. Half logarithmic plot of Fig. 1 for the deflection and return of the arc. The time constant for both processes taken from the straight line is 97 ms (Schwertl [1]).

can be observed visually at least in the last stages during the switching on and off of the irradiation.

VIII. Motion and Displacement by Bending

A very important cause becomes active if an originally wall-stabilized arc in a straight cylinder suddenly is bent to a closed circle, where only the electrode regions cause a deviation from toroidal symmetry. The energy equation receives two terms more than in the cylindrical case, both of them depending strongly on the radial torus direction x beginning in the center of the tube (no longer reduced by R). The first of these additional terms is due to the fact that the electric field strength E can be described as the gradient of a potential the planes of which are directed radially in the case of the torus. Therefore, the electric field strength decreases in proportion to the main radius of the torus. Consequently the formula for $E(x)$ is

$$E(x) = E_0\left(1 - \frac{kx}{1 + kx}\right) \tag{30}$$

where E_0 is the field strength in the middle of the tube, and $k = 1/R_t$ and R_t is the principal curvature of the torus. The second additional term is the third part of the heat conduction loss in Cartesian coordinates

$$\nabla^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \tag{31}$$

which vanishes in the straight arc but appears in the curved arc due to the second-order character. This additional term is connected with the temperature gradient as follows:

$$\frac{\partial^2 S}{\partial z^2} = \frac{k}{1 + kx} \cdot \frac{\partial S}{\partial x}. \tag{32}$$

This term is dependent on the x direction even at the maximum. The energy equation for the bent arc reads in non-reduced Cartesian coordinates

$$\sigma E_0^2 + \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} - \sigma E_0^2\left(\frac{2kx}{1 + kx} - \left(\frac{kx}{1 + kx}\right)^2\right) + \frac{k}{1 + kx} \cdot \frac{\partial S}{\partial x} = 0 \tag{33}$$

Fig. 5. Cyclic arc in a torus chamber with square cross section 1×1 cm^2 in Ar with 1 atm and 1 A. The arc touches the inner wall while the outer wall may be recognized as the outer white circle (Nathrath [3]).

in which the two new terms cause by their inhomogeneity a motion of the arc towards the center of the torus. For the calculation of the arc velocity according to (14) we differentiate the energy equation with respect to the radial x direction and apply it to the perhaps excentric maximum

$$\left(\frac{\partial^3 S}{\partial x^3}\right)_m + \left(\frac{\partial^3 S}{\partial x \partial y^2}\right)_m - 2k\sigma_m E_0^2 + k\left(\frac{\partial^2 S}{\partial x^2}\right)_m = 0 \tag{34}$$

where a Taylor expansion has been used for the S distribution around the maximum and quantities of second order in the curvature k have been omitted. Immediately after having bent the tube the third-order terms disappear and between the two remaining parts exists according to the energy equation (33) with cylindric symmetry $k=0$ and with $(\partial^2 S/\partial x^2)_m = (\partial^2 S/\partial y^2)_m = (\partial^2 S/\partial r^2)_m$ the simple relation

$$\sigma_m E_0^2 = -2(\partial^2 S/\partial x^2)_m = -2\kappa_m(\partial^2 T/\partial x^2)_m. \tag{35}$$

Thereby, the initial velocity becomes according to (14)

$$V_{AM,x} = -5k\left(\frac{\kappa}{\rho c_p}\right)_m. \tag{36}$$

Since in the argon arc at low currents the electron temperature exceeds the gas temperature the electric conductivity σ becomes dependent on the field strength as well

$$\sigma = \sigma(S, E). \tag{37}$$

In this case, the differentiation has also to be applied to the conductivity with respect to E whereby on the right of (36) the factor $(1 - (\partial \ln \sigma/\partial \ln E^2)_m)$ has to be included. The negative sign in (36) indicates that the direction of motion is onto the main torus center. Because this process is based again on the inhomogeneity of heating and cooling, no mass flow occurs except the equalization flow reducing slightly the computed arc velocity. This experiment, indeed is a theoretical one only, but the consequence of the tube curvature has been investigated as the displacement of the arc in direction of the torus center by Nathrath [3]. Consistence between theory and experiment could not be achieved unless the deviation from thermal equilibrium in σ and the effect of the self-magnetic field were taken into account. Fig. 5 shows a photograph of an argon arc with one ampere at atmospheric pressure in a toroidal container with a

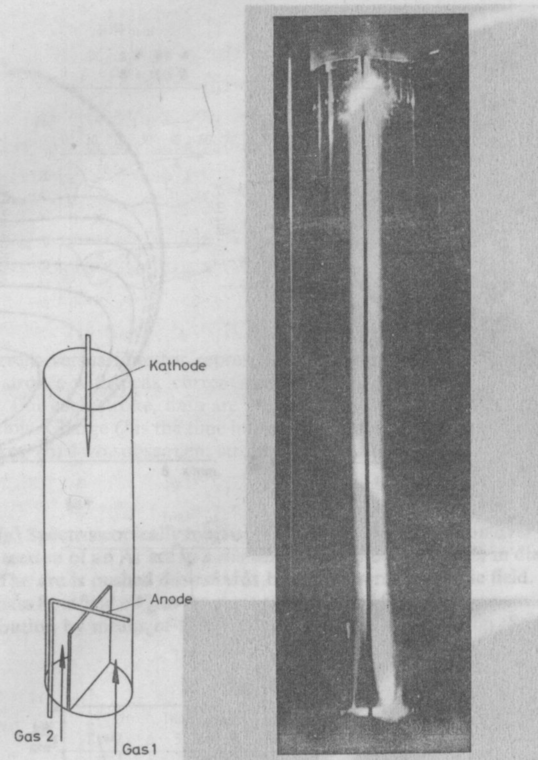


Fig. 6. (a) Scheme of a device for arc deflection in a gradient of gas properties produced by diffusion (Raeder [4]). (b) Arc displaced at the lower electrode due to the better conductivity of Ar at the right in comparison with N_2 at the left. The black line along the axis is a wire mark. At the left the reflected image again (Raeder [4]).

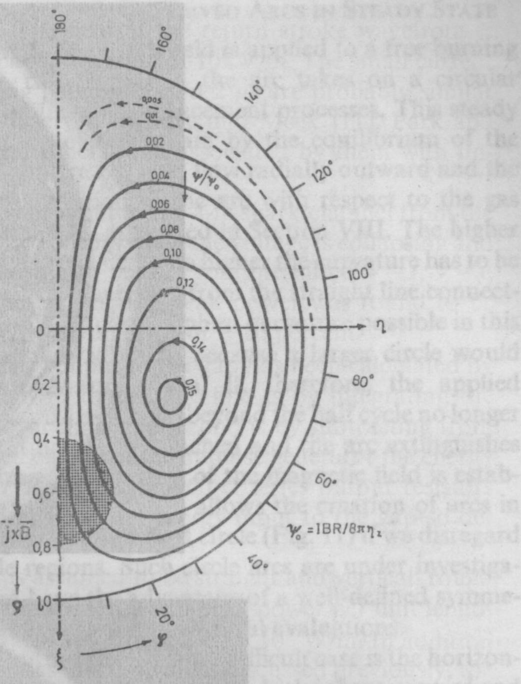


Fig. 7. Theoretically calculated flow field perpendicular to an arc displaced by a transverse magnetic field (Seeger [2]).

1-cm^2-square cross section and a main torus radius of 3.3 cm. Apparently the arc is pushed close to the inner wall of the chamber. The outer wall is the outer white circle.

For the steady state of displacement the inhomogeneous heating and cooling by the ohmic term and the dissipative term $(\partial^2 S/\partial z^2)_m$ in the maximum have to be compensated by an opposite inhomogeneity of the two other dissipative terms $(\partial^2 S/\partial x^2)_m$ and $(\partial^2 S/\partial y^2)_m$. Taking just linear parts in the curvature and using reduced coordinates x and y by R there follows

$$S_{3x,m} + S_{x2y,m} + kR(S_{2x,m} - 2\sigma_m E_0^2 R^2) = 0. \quad (38)$$

This equation shows that the third term in the case of arc bending is analogous to the expression $a_{x,m}R^2$ in the case of directed irradiation. By a procedure similar to that explained in the first case the approximate relations for a curved arc like those in (23) for an irradiated arc can be derived:

$$S(x, y) = \sigma_m E_0^2 R^2/4\{1 - (1 + 2x_m x)(x^2 + y^2)\}$$

$$S_{3,m} + S_{x2y,m} = -16 S_m x_m; \quad x_m = -5kR/8. \quad (23')$$

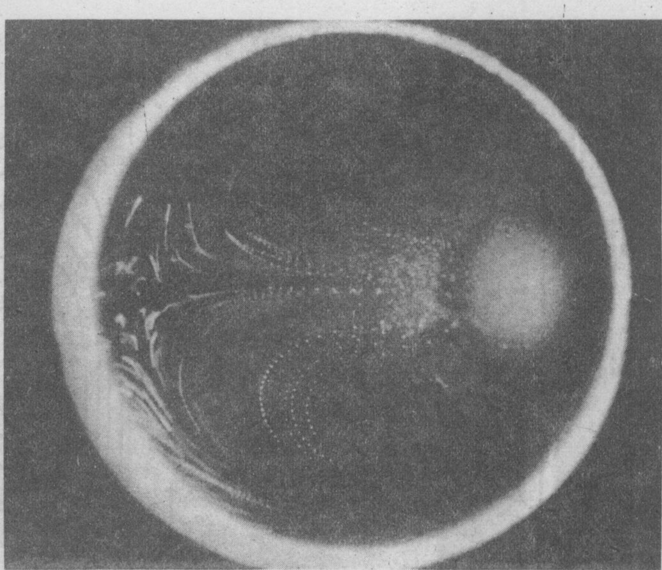
IX. Displacement by a Gradient of Gas Properties

In the argument of the gradient in (14) for the relative arc velocity V_{AM} a few coefficients characteristic for the gas employed are involved, e.g., mass density ρ, specific heat c_p, heat conductivity κ, and electric conductivity σ. If one succeeds in diffusing two different gases one into each another a gradient of the internal energy supply even in the temperature maximum is established whereby the conditions for the arc motion are fulfilled again. Just regarding a gradient of electric conductivity σ in the temperature maximum the ohmic heating σE^2 is higher on the side of better conductivity and we may predict, that an arc burning in this region will move into this direction.

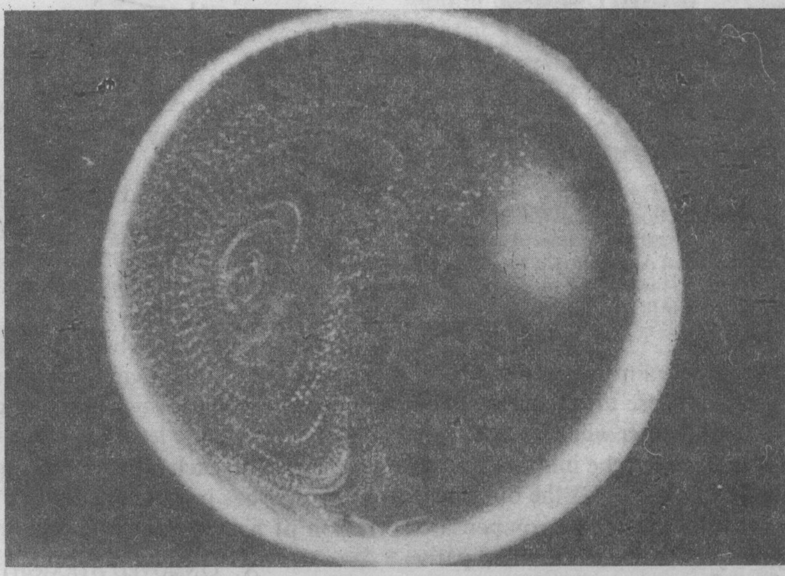
This experiment has been carried out by Raeder [4] using a metal tube with a diametral separation wall forming two channels through which the two different gases were running. After having left the end of this tube the gases begin to diffuse into each other and produce thereby a gradient of gas properties in which actually a burning arc tends to the region of higher electric conductivity (Fig. 6).

X. Unmoved Straight Arcs Due to Dynamic Equilibrium

The second type of arcs to be discussed should be those where the arc velocity $V_A = V_{AM} + V_M$ disappears due to the equilibrium between the relative motion of the arc with respect to the matter and the mass flow velocity. Experiments in which those properties are realized are wall-stabilized arcs exposed to a transverse magnetic field B. Just after the application of the magnetic field, the Lorentz forces push the hot arc gas into their direction and the arc follows this motion. But during this deflection a gradient of the heat conduction term in the energy equation is produced whereby the motion is retarded and ends up with a discrete displacement. In this new steady state the Lorentz forces are still in action creating besides a pressure gradient a double whirl closed along the wall and the diameter through the displaced arc. The steady motion is of course a pure magneto-hydrodynamic problem and has been treated by Seeger [5] using Greens function (Fig. 7). A corresponding experiment has been performed by Rosenbauer [6] who operated an Ar arc in a quartz tube of 10 mm in diameter at atmospheric pressure with a few amperes and a few gauss



(a)



(b)

Fig. 8. (a) End-on photo of a wall stabilized Ar arc at 1 atm and 1 A in a tube of 10 mm in diameter displaced by a transverse magnetic field of 4 G. The double whirl mass flow is marked by small Al_2O_3 particles illuminated intermittently. The total apparatus is free falling during the exposure. (b) Disturbance of the mass flow patterns by gravitational forces in the apparatus fixed in the laboratory system (Rosenbauer [6]).

transverse magnetic field. In order to observe end-on the double whirl motion driven by the arc as a pump he introduced small alumina particles illuminated intermittedly. The dotted traces of those particles photographed from the end of the discharge tube show indeed the vector field of mass flow in the shape of a double whirl where the direction of the traces coincides with the direction of the flow and where the distance between two neighboring dots is a measure for the amount of the velocity [Fig. 8(a)]. Those exposures can be successful only if the normal disturbance by the gravity field is eliminated, in other words all the apparatus have to be dropped during the introduction of the particles, the equalization of mass flow, and the moment of exposure. Otherwise the expected double whirl is superposed by a single whirl turning in that direction which the deflected arc prescribes [Fig. 8(b)]. The velocity of the gas amounts to the order of less than 1 m/s.

These difficulties due to the gravitational forces can be avoided when higher arc currents and higher transverse fields, correspondingly, are applied. Under these circumstances the air-cooled quartz tube used by Rosenbauer [6] has to be replaced by a stack of water-cooled and insulated copper plates, i.e., by the cascade arc chamber. Such investigations have been carried out by Sauter [7] who again used an atmospheric Ar arc with a tube of 10 mm in diameter but in a current range from 40 to 120 A and a magnetic field

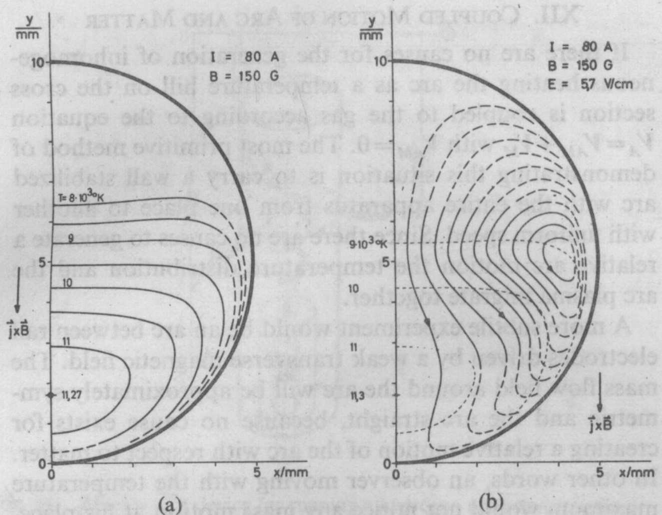


Fig. 9. (a) Spectroscopically measured temperature distribution over the cross section of an Ar arc in a cascade arc chamber of 10 mm in diameter. The arc is pushed downwards by a transverse magnetic field. (b) The mass flow field ρV_M in the same arc evaluated from the temperature distribution by means of the energy and continuity equation (Sauter [7]).

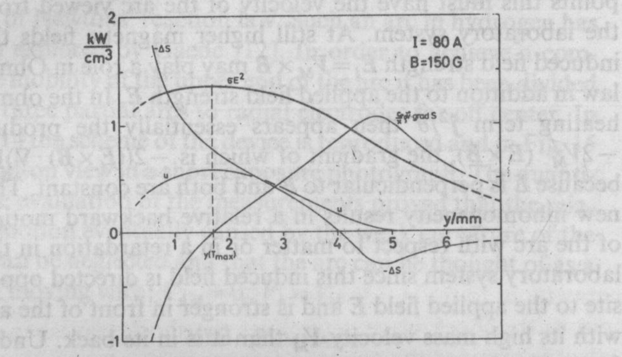


Fig. 10. The various terms in the equation of internal energy plotted along the tube diameter through the arc. Notice the opposite equality of the gradient of the convection term $\rho c_p V_m \cdot \nabla T$ and the dissipative term $V^2 S$ in the vicinity of the temperature maximum where neither the ohmic nor the radiation term have a gradient. (Sauter [7]).

range from 75 to 225 G. Under these intense conditions the gravitational forces cause no trouble at all. First of all, the temperature field has been measured spectroscopically so that the velocity could be evaluated by means of the convective term $\rho \cdot c_p \cdot V_M \cdot \nabla T$ in the energy equation including the mass continuity equation. Fig. 9 shows an example for the T and $\rho \cdot V_M$ field. Again one recognizes the double whirl in the streaming and furthermore the flow around the hot arc like around an obstacle. For our problem the plot of the different terms of the energy equation along the preferred diameter through the arc (Fig. 10) is of interest because around the temperature maximum the gradient of the heat conduction term compensates exactly the convection term while neither the ohmic heating nor the radiation loss term have a gradient in the maximum. So the lack of motion in this case is actually caused by the counteraction of the mass motion on the one hand and the relative arc motion caused by the inhomogeneity of the dissipating term on the other hand.

XI. Various Types of Curved Arcs in Steady State

If a transverse magnetic field is applied to a free burning arc between two electrodes the arc takes on a circular curved shape after the displacement processes. This steady state is to be explained again by the equilibrium of the magnetically generated mass flow radially outward and the relative inward-motion of the arc with respect to the gas due to the curvature as treated in Section VIII. The higher the applied magnetic field, the higher the curvature has to be and with it the displacement from the straight line connecting the electrodes. But the highest curvature possible in this geometry has the half cycle because a larger circle would decrease in curvature again. If, therefore, the applied magnetic field pushes the arc beyond the half cycle no longer a steady position can be reached and the arc extinguishes unless a decreasing gradient of the magnetic field is established. Such a configuration allows the creation of arcs in the shape of an almost closed circle (Fig. 11) if we disregard the electrode regions. Such circle arcs are under investigation now and have the advantage of a well-defined symmetry which is valuable for theoretical evaluations.

A similar but somewhat more difficult case is the horizontal arc in free convection (Fig. 12) which is bent upward and permeated by a mass flow V_M vertically. The direction of the relative velocity of the arc, however, shows to the center of the arc curvature and includes thereby an obtuse angle with the mass flow direction. Because of this geometry the arc forms a catenary. The sum of these both inclined vectors yields a direction of the arc velocity tangential to its axis. This result has to be interpreted as a steady position of the arc because components of motion along the direction of the arc axis cannot be observed.

In connection with these phenomena Mentel [9] made an interesting observation: A hydrogen arc at atmospheric pressure burns in a quartz tube of about 20 mm in diameter as a completely straight red filament in the axis up to about 10 A. But if the current is increased to 10.5 A the arc deforms itself to a standing screw without motion and rotation (Fig. 13). The experimental and theoretical investigations on this anomalous behavior of the hydrogen arc revealed that there compete four causes altogether leading at this elevated current to the deformed steady arc. These are on one hand the Lorentz force generated by the self-magnetic field of the arc shaped similar to a coil and driving the arc plasma radially outwards, and on the other hand the gradient of three terms in the equation of the internal energy due to the curvature, the first being the ohmic heating, the second being the heat conduction term taken tangentially to the curved arc, and finally that taken in outward direction accounting for the approach to the wall. Theory and experiment agree very well, the former predicting 10.3 A for the onset of this screw deformation and its pitch amounting to twice the tube diameter. Apparently the destabilizing Lorentz forces overcome the stabilizing thermal "forces" after transgression of the critical current. So again this screwed state has to be explained by the dynamic equilibrium between mass motion outwards and opposite relative motion of the arc inwards.

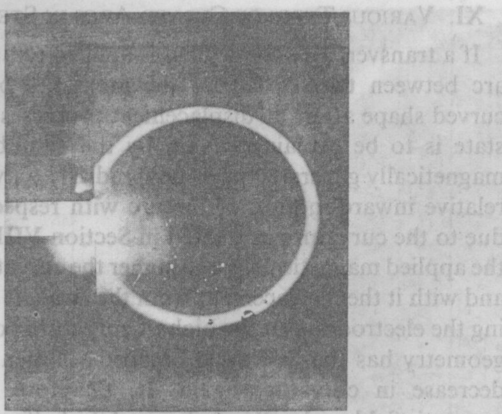


Fig. 11. Ar arc between a quartz plate and an enamel plate a few milimeters distant at 1 A and 1 atm. The arc is bent to an almost closed circle by a transverse magnetic field decreasing outwards. The equilibrium state is due to the balance between the magnetically created mass flow outwards and the relative back motion of the arc caused by the curvature (Simon [8]).

Fig. 12. Horizontal mercury arc at elevated pressure bent upwards by the vertically directed thermal lift. The equilibrium between the mass flow upwards and the relative back motion due to curvature forms a catenary. This phenomenon has given the arc its name.

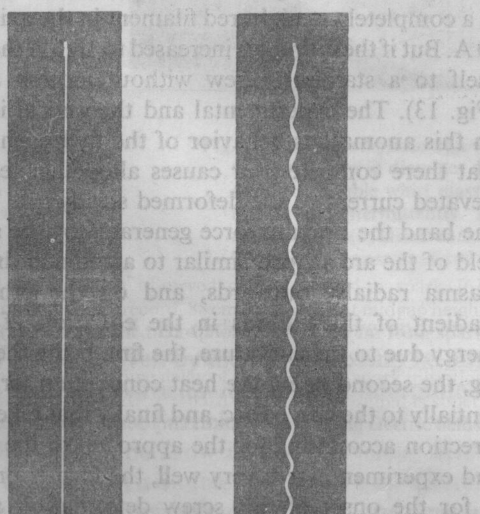


Fig. 13. A H_2 arc at atmospheric pressure in a water cooled quartz tube of 20 mm in diameter at 10 and 10.5 A, respectively. In this current interval the arc passes over from a straight filament in the tube axis to a standing screwed shape (Mentel [9]).

XII. Coupled Motion of Arc and Matter

If there are no causes for the generation of inhomogeneous heating the arc as a temperature hill on the cross section is coupled to the gas according to the equation $V_A = V_{AM} + V_M$ with $V_{AM} = 0$. The most primitive method of demonstrating this situation is to carry a wall stabilized arc with the entire apparatus from one place to another with uniform speed. Since there are no causes to generate a relative arc motion the temperature distribution and the arc plasma migrate together.

A more subtile experiment would be an arc between rail electrodes driven by a weak transverse magnetic field. The mass flow field around the arc will be approximately symmetric and the arc straight, because no cause exists for creating a relative motion of the arc with respect to matter. In other words, an observer moving with the temperature maximum would not notice any mass motion at his place. At higher magnetic forces and mass velocities the viscosity effects come into play and probably give rise to the ocurrence of two stagnation points before and behind the arc in between which a double whirl guarantees the steadiness of the mass flow field. Relating to the mass in the stagnation points this must have the velocity of the arc viewed from the laboratory system. At still higher magnetic fields the induced field strength $\boldsymbol{E}_i = \boldsymbol{V}_M \times \boldsymbol{B}$ may play a role in Ohm's law in addition to the applied field strength $\boldsymbol{E}$. In the ohmic heating term j^2/σ then appears essentially the product $-2\boldsymbol{V}_M \cdot (\boldsymbol{E} \times \boldsymbol{B})$, the gradient of which is $-2((\boldsymbol{E} \times \boldsymbol{B}) \cdot \nabla)\boldsymbol{V}_M$ because $\boldsymbol{E}$ is perpendicular to $\boldsymbol{B}$ and both are constant. This new inhomogeneity results in a relative backward motion of the arc with respect to matter or in a retardation in the laboratory system since this induced field is directed opposite to the applied field $\boldsymbol{E}$ and is stronger in front of the arc with its high mass velocity $\boldsymbol{V}_M$ than it is in its back. Under favorable circumstances it might be possible that the relative motion due to the inhomogeneity of the counterinduction exceeds the opposite mass motion produced by the Lorentz forces. This could be a possible hypothesis for the explanation of the retrograde motion. Anyhow, the retrograde motion can be explained only by an overwhelming of the mass flow in amperian direction by an opposite motion due to inhomogeneous heating.

In practice, experiments with rail electrodes have their difficulties because of which it is preferred often to work with a standing arc exposed to a homogeneous mass flow from the direction opposing the Lorentz force (cf. e.g., Murphree and Carter [10]). Another example for a steady motion of an arc virtually combined with a flow of matter is the so-called spinning arc first observed by Guillery [11]. This phenomenon exists in a definite deformation of an arc into an expanding helix between a point electrode and a flat one on which the end of the arc is movable. This arc helix rotates as a rigid body about its axis but opposite to the direction of the winding. The reasons for the formation of such an arc are similar to those treated in Section XI. The rotation can be explained roughly by the action of Lorentz forces produced by the arc configuration itself so that the axial component of the magnetic field interacts