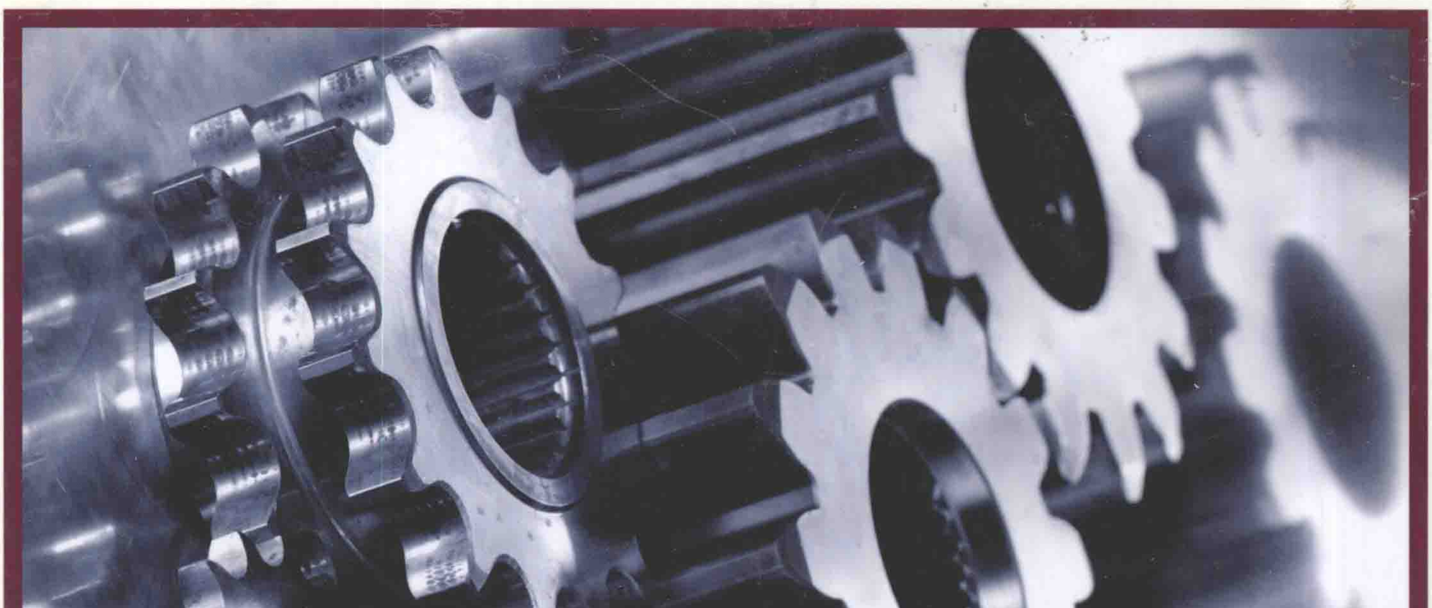
INTRODUCTORY

DIFFERENTIAL EQUATIONS

WITH BOUNDARY VALUE PROBLEMS

THIRD EDITION



ABELL & BRASELTON

Introductory Differential Equations with Boundary Value Problems

Third Edition

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Preface

Introductory Differential Equations with Boundary Value Problems began as the third edition of a text called *Modern Differential Equations* but with a different publisher. Perhaps, you are not surprised, but a change of publisher automatically makes a revision quite complicated, essentially involving a line-by-line revision of the text; the purpose of all artwork must be rethought and redone. All exposition, examples, and exercises are scrutinized for purpose.

When we were done with the "revision," we no longer saw *Modern Differential Equations* but rather a new text that we have titled *Introductory Differential Equations with Boundary Value Problems*.

Originally, *Modern Differential Equations* was "modern" because it was one of the first texts that nearly required access to a graphing calculator, computer algebra system, or numerical software package.

Computer algebra systems and sophisticated graphing calculators have changed the ways in which we learn and teach ordinary differential equations. Instead of focusing students' attention only on a sequence of solution methods, we want them to use their minds to understand what solutions mean and how differential equations can be used to answer pertinent questions. Now their use is expected in a standard course, so the term "modern" no longer applies to the text.

Interestingly, the metamorphosis in the teaching of differential equations described above occurred relatively "overnight" and coincided with our professional careers at Georgia Southern University. Our interest in the use of technology in the mathematics classroom began in 1990 when we started to use computer laboratories and demonstrations in our calculus, differential equations, and applied mathematics courses. Over the past years, we have learned some ways of how to and how not to use technology in the mathematics curriculum. In the early stages, we simply wanted to show students how they could solve more difficult problems by using a computer algebra system so that they could be exposed to the technology. However, we soon realized that we were missing the great opportunity of allowing students to discover aspects of the subject matter on their own. We revised our materials to include experimental problems and thought-provoking questions in which students are asked to make conjectures and investigate supporting evidence. We also developed application projects called *Differential Equations at Work*, not only to emphasize technology, but also to improve the problem-solving and communication skills of our students. To preserve the "wow" aspects of technology, we continue to use it to observe solutions in classroom demonstrations through such things as animating the motion of springs and pendulums. These demonstrations not only grab the attention of students, but also help them to make the connection between a formula and what it represents.

In presenting our findings to colleagues around the country, we quickly found out that others were interested in our work. As a result, we decided to develop a differential equations textbook to share this work with those who share our desire to improve mathematics education. This book is a culmination of years of "trials and tribulation" as the differential equations students at Georgia Southern can attest. Our hope is that its use will inspire students to open their eyes to the exciting discoveries that differential equations offer.

This book is designed to serve as a text for beginning courses in differential equations. Usually, introductory differential equations courses are taken by students who have successfully completed a first-year calculus course, and this text is written at a level readable for them.

TECHNOLOGY

The advantages of incorporating technology into mathematics courses are well known. Some of them include enhancing the ability to solve a variety of problems; helping students work examples; supporting varied, realistic, and illuminating applications; exploiting and improving geometric intuition; encouraging mathematical experiments; and teaching approximation. In addition, technology is implemented throughout this text to promote the following goals in the learning of differential equations:

1. *Solving problems*: Using different methods to solve problems and generalize solutions
2. *Reasoning*: Exploiting computer graphics to develop spatial reasoning through visualization
3. *Analyzing*: Finding the most reasonable solution to real problems or observing changes in the solution under changing conditions
4. *Communicating mathematics*: Developing written, verbal, and visual skills to communicate mathematical ideas
5. *Synthesizing*: Making inferences and generalizations, evaluating outcomes, classifying objects, and controlling variables

Students who develop these skills will succeed not only in differential equations, but also in subsequent courses and in the workforce.

The icon is used throughout the text to indicate those examples in which technology is used in a nontrivial way to develop or visualize the solution or to indicate the sections of the text, such as those discussing numerical methods, in which the use of appropriate technology is essential or interesting.

APPLICATIONS

Applications in this text are taken from a variety of fields, especially biology, physics, chemistry, engineering, and economics, and they are documented by references. These applications can be found in many of the examples and exercises, in separate sections and chapters of the text, and in the *Differential Equations at Work* subsections at the end of each chapter. Many of these applications are well suited to exploration with technology because they incorporate real data. In particular, obtaining closed form solutions is not necessarily "easy" (or always possible). These applications, even if not formally discussed in class, show students that differential equations is an exciting and interesting subject with extensive applications in many fields.

STYLE

To keep the text as flexible as possible, addressing the needs of both audiences with different mathematical backgrounds and instructors with varying preferences, *Introductory Differential Equations with Boundary Value Problems* is written in an easy-to-read, yet mathematically precise, style. It contains all topics usually included in standard differential equations texts. Definitions, theorems, and proofs are concise but worded precisely for mathematical accuracy. Generally, theorems are proved if the proof is instructive or has "teaching value." Of course, discussion of such proofs is optional in the typical classroom for which this text is written. In other cases, proofs of theorems are developed in the exercises or omitted. Theorems and definitions are boxed for easy reference; key terms are given in italic. Figures are used frequently to clarify material with a graphical interpretation.

FEATURES

Introductory Differential Equations with Boundary Value Problems is an extensive revision of the second edition of *Modern Differential Equations*. Particular features include the following:

- The text's companion Web site at http://elsevierdirect.com/9780123749352 contains PDFs of the text in addition to a variety of other resources. Typical resources include background material, proofs of some theorems, solutions to selected exercises, additional exercises, visualizations of certain topics (movies), and podcasts that students can download to their video-capable *iPod* or other compatible mv4 players.
- Because the mathematicians who developed the mathematics discussed in this text were (or *are*) still interesting in their own right, we have tried to include an image and interesting tidbit about their lives whenever possible hoping to help some students become more interested in the course. When credit is not given for a photo, it is because we have reasonable reason to believe that the image is in the public domain. If a copyright applies to an image and appropriate credit has not been given, please alert us so that we can correct the situation promptly.
- Chapter 2 has been reorganized so that the chapter begins with the basic theory of first-order equations. Some instructors will choose to omit Section 2.1 and proceed directly to 2.2 (Separable Equations). We have also stated the Existence and Uniqueness theorems for separable, linear, and exact equations.
- Chapter 3 includes several new applications, including an exploration of the logistic difference equation that is expanded on at the text's Web site as well as introducing the basic model of competition in a chemostat.
- Chapter 4 has been reorganized so that second-order linear homogeneous and nonhomogeneous equations are discussed first and then the second-order case is generalized to the situations for higher-order linear and nonhomogeneous equations. The second edition of *Modern Differential Equations* contained a single section on

series solutions. In this text, that section has been divided into two sections: power series solutions and series solutions about regular singular points. Additional material regarding series solutions is available at the text's Web site for those instructors who wish to emphasize this topic or for students who wish to explore the topic in greater detail.

- All graphics have been redone. In each case, the intent of the graphic has been questioned. In some cases, graphics have been eliminated; in other cases, they have been redone to emphasize their purpose.
- In Chapter 6, we have added a section on phase portraits.

PEDAGOGICAL FEATURES

Examples

Throughout the text, numerous examples are given, with thorough explanations and a substantial amount of detail. Solutions to more difficult examples are constructed with the help of graphing calculators or a computer algebra system and are indicated by an icon.

"Think About It!"

Many examples are followed by a question indicated by a icon. Generally, basic knowledge about the behavior of functions is sufficient to answer the question. Many of these questions encourage students to use technology. Others focus on the graph of a solution. Thus, "Think about it!" questions help students determine when to use technology and make this text more interactive.

Technology

Many students entering their first differential equations course have had substantial experience with various sophisticated calculators and computer algebra systems.

Our differential equations course attempts to encourage students to *use technology intelligently*. We have italicized the words "*use technology intelligently*" because they take on different meaning to different instructors because they depend on the instructor's philosophy, institution, and students. Students also interpret the phrase differently depending upon their instructor and exposure to technology.

In any case, many of us have limited resources and would prefer that our students have a good grasp of the fundamentals rather than be "wowed!" by nonsense. We have tried to use technology intelligently here. We believe that it should not be obtrusive, so you should not notice when we do. When required in an example or exercise, it should be obvious to an instructor and relatively easy to convince a student that there are two ways to solve a problem: the easy way and the hard way. We choose the hard way when there is instructional value to the approach. The icon is intended to alert students that technology is intelligently (and wisely) used to assist in solving the problem. Typically, the technology we have used is a computer algebra system, like *Mathematica* or *Maple*.

Technology is used throughout the text to explore many of the applications and more difficult examples, especially those marked with and the problems in the subsections *Differential Equations at Work*.

Answers to most odd exercises are included at the end of the text. More complete answers, solutions, partial solutions, or hints to selected exercises are available separately to students and instructors. *Differential Equations at Work* subsections describe detailed economics, biology, physics, chemistry, and engineering problems documented by references. These problems include real data when available and require students to provide answers based on different conditions. Students must analyze the problem and make decisions about the best way to solve it, including the appropriate use of technology. Each *Differential Equations at Work* project can be assigned as a project requiring a written report, for group work, or for discussion in class.

Differential Equations at Work also illustrate how differential equations are used in the real world. Students are often reluctant to believe that the subject matter in calculus, linear algebra, and differential equations classes relates to subsequent courses and to their careers. Each *Differential Equations at Work* subsection illustrates how the material discussed in the course is used in real life.

The problems are not connected to a specific section of the text; they require students to draw different mathematical skills and concepts together to solve a problem. Because each *Differential Equations at Work* is cumulative in nature, students must combine mathematical concepts, techniques, and experiences from previous chapters and math courses.

Exercises

Numerous exercises, ranging in level from easy to difficult, are included in each section of the text. In particular, the exercise sets for topics that students find most difficult are rich and varied. The abundant "routine" exercises encourage students to master basic techniques. Most sections also contain interesting mathematical and applied problems to show that mathematics and its applications are both interesting and relevant. Instructors will find that they can assign a large number of problems, if desired, yet still have plenty for review in addition to those found in the review section at the end of each chapter. Answers to most odd-numbered exercises are included at the end of the text; detailed solutions to selected exercises are included in the *Student Resource Manual*.

Chapter Summary and Review Exercises

Each chapter ends with a chapter summary highlighting important concepts, key terms and formulas, and theorems. The review exercises following the chapter summary of each chapter offer students extra practice on the topics in that chapter. These exercises are arranged by section so that students having difficulty can turn to the appropriate material for review.

Figures

This text provides an abundance of figures and graphs, especially for solutions to examples. In addition, students are encouraged to develop spatial visualization and reasoning skills, to interpret graphs, and to discover and explore concepts from a graphical point of view. To ensure accuracy, the figures and graphs have been completely computer-generated.

Historical Material

Nearly every topic is motivated by either an application or an appropriate historical note. We have also included images of paintings, drawings, or photographs of the many famous scientists and descriptions of the mathematics they discovered.

CONTENT

The highlights of each chapter are described briefly below.

Chapter 1 After introducing preliminary definitions, we discuss direction fields not only for first-order differential equations, but also for systems of equations. In this presentation, we establish a basic understanding of solutions and their graphs. We give an overview of some of the applications covered later in the text to point out the usefulness of the topic and some of the reasons we have for studying differential equations in the exercises.

Chapter 2 In addition to discussing the standard techniques for solving several types of first-order differential equations (separable equations, homogeneous equations, exact equations, and linear equations), we introduce several numerical methods (Euler's Method, Improved Euler's Method, Runge–Kutta Method) and discuss the existence and uniqueness of solutions to first-order initial value problems. Throughout the chapter, we encourage students to build an intuitive approach to the solution process by matching a graph to a solution without actually solving the equation.

Chapter 3 Not only do we cover most standard applications of first-order equations in Chapter 3 (orthogonal trajectories, population growth and decay, Newton's law of cooling, free-falling bodies), but we also present many that are not (due to their computational difficulty) in *Differential Equations at Work*.

Chapter 4 This chapter emphasizes the methods for solving homogeneous and nonhomogeneous higher-order differential equations. It also stresses the Principle of Superposition and the differences between the properties of solutions to linear and nonlinear equations. After discussing Cauchy–Euler equations, series methods are introduced, which includes a discussion of several special equations and the properties of their solutions/equations important in many areas of applied mathematics and physics.

Chapter 5 Several applications of higher-order equations are presented. The distinctive presentation illustrates the motion of spring-mass systems and pendulums graphically to help students understand what solutions represent and to make the applications more meaningful to them.

Chapter 6 The study of systems of differential equations is perhaps the most exciting of all the topics covered in the text. Although we direct most of our attention to solving systems of linear first-order equations with constant coefficients, technology allows us to investigate systems of nonlinear equations and observe phase planes. We also show how to use eigenvalues and eigenvectors to understand the general behavior of systems of linear and nonlinear equations. We have added a section on phase portraits in this edition.

Chapter 7 Several applications discussed earlier in the text are extended to more than one dimension and solved using systems of differential equations, in an effort to reinforce the understanding of these important problems. Numerous applications involving nonlinear systems are discussed as well.

Chapter 8 Laplace transforms are important in many areas of engineering and exhibit intriguing mathematical properties as well. Throughout the chapter, we point out the importance of initial conditions and forcing functions on initial-value problems.

Chapter 9 Chapter 9 focuses on boundary value, eigenvalue, and Sturm–Liouville problems and Fourier series, including generalized Fourier series. Technology is utilized to observe the convergence of these series and visualize the Gibbs phenomenon.

Chapter 10 In this chapter, we use the method of separation of variables to solve the one-dimensional heat equation, one-dimensional wave equation, Laplace's equation (in a rectangular and circular region), and the wave equation in a circular region. Exercises involving cylindrical and spherical coordinates are also included. Graphics are used so that students can see how the results obtained relate to the applications.

For a one-semester course introducing ordinary differential equations, many instructors will choose to cover topics from Chapters 1 to 7 or from Chapters 1 to 6 and Chapter 8. For a two-semester course, the instructor will easily be able to cover the remaining chapters of the text. In our introductory ordinary differential equations course, we typically cover most of Chapters 1, 2, 4, and 6, and instructors choose a variety of applications from Chapters 3, 5, and 7. In our applied mathematics course, we cover most of the material in Chapters 8–10.

- For a one-semester course targeted to the computational needs of most engineering majors, cover most topics from Chapters 1, 2, 4, 6, and 8.
- For a casual one-semester course directed toward math and math education majors, cover most mathematical topics in Chapters 1, 2, 4, and 6 and selected applications from Chapters 3, 5, and 7.
- For a second one-semester course directed toward math and math education majors, cover selected topics from Chapters 8–10.

SUPPLEMENTS

A Web site (http://www.elsevierdirect.com/companions/9780123749352) has specifically been created for *Introductory Differential Equations with Boundary Value Problems*. As described above, this Web site offers additional resources to instructors and students who have adopted the text.

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The development of this text has involved a thorough program of review, for both pedagogical and topical content and for accuracy. We gratefully acknowledge the contributions of our colleagues that were involved in the revision of this text.

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