

MECHANICAL ENGINEERING

BTEC NATIONAL OPTION UNITS

ALAN DARBYSHIRE



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Mechanical Engineering

Introduction

Welcome to the challenges of mechanical engineering! This book has been written to help you get through the core unit Mechanical Principles and a further five optional units which may form part of your BTEC National Certificate or Diploma award in mechanical engineering. It accompanies the book BTEC National Engineering by Mike Tooley and Lloyd Dingle and provides the essential underpinning knowledge required by a student who wishes to pursue a career in mechanical engineering.

The book has been written by a highly experienced lecturer with over thirty years experience of industry and higher education. It has been designed to cover the requirements of the revised and updated BTEC Engineering programme. The author has adopted a common format and approach throughout the book with numerous student activities, examples, problems and key points.

About the BTEC National Certificate and Diploma

The BTEC National Certificate and National Diploma qualifications have long been accepted by industry as appropriate qualifications for those who are about to enter industry or who are receiving training at the early stages of employment in industry. At the same time, these qualifications have become increasingly acceptable as a means of gaining entry into higher education.

BTEC National programmes in Engineering attract a very large number of registrations per annum such that there is in excess of 35 000 students currently studying for these qualifications in the UK by both part-time and full-time modes of study.

The BTEC National syllabus was recently reviewed and extensively updated and new programmes have been launched with effect from September 2002. The new scheme is likely to be adopted by *all* institutions that currently offer the programme as well as a number of others who will now be able to mix and match parts of the BTEC qualification with vocational GCSE and AVCE awards.

Many organizations have contributed to the design of the new BTEC National Engineering programme including the Qualifications and Cirriculum Authority (QCA), the Engineering Council, and several National Training Organizations (NTO).

The Engineering Council continues to view the BTEC National Certificate/Diploma as a key qualification for the sector. They also recognise that BTEC National qualifications are frequently used as a means of entry to higher education courses, such as HNC/HND programmes and Foundation Degree courses.

In revising and updating the Engineering BTEC programme, Edexcel has taken into consideration a number of issues, including:

- Occupational standards and NTO requirements.
- Professional requirements (particularly with regard to the engineering technician).
- Progression into employment.
- Progression to Higher National qualifications (with particularly close match in the core units of Engineering Science and Mathematics).
- A flexible course structure that is commensurate with the broad aims of curriculum 2000.
- Relevant QCA criteria.
- External assessments as required.
- Key skills signposting.
- Several other issues, including the way in which the study of the area can contribute to an understanding of spiritual, moral, ethical, social and cultural issues.

How to use this book

Chapter 1 of the book covers the core unit Mechanical Principles, which is essential for a BTEC National Certificate/Diploma award in Mechanical Engineering. Succeeding chapters cover the optional units Further Mechanical Principles, Mechanical Technology, Engineering Materials, Fluid Mechanics and Thermodynamics. Each chapter contains Text, 'Key points', 'Test your knowledge' questions, Examples, Activities and Problems.

The 'Test your knowledge' questions are interspersed with the text throughout the book. These questions allow you to check your understanding of the preceding text. They also provide you with an opportunity to reflect on what you have learned and consolidate this in manageable chunks.

Most 'Test your knowledge' questions can be answered in only a few minutes and the necessary information, formulae, etc., can be gleaned from the preceding text. Activities, on the other hand, make excellent vehicles for gathering the necessary evidence to demonstrate that you are competent in Key Skills. Consequently, they normally require a significantly greater amount of time to complete. They may also require additional library or resource area research time coupled with access to computing and other information technology resources.

Many tutors will use 'Test your knowledge' questions as a means of reinforcing work done in class whilst Activities are more likely to be 'set work' for students to do outside the classroom. Whether or not this approach is taken, it is important to be aware that this student-centred work is designed to complement a programme of lectures and tutorials based on the BTEC syllabus. Independent learners (i.e. those not taking a formal course) will find complete syllabus coverage in the text.

The units Mechanical Principles, Further Mechanical Principles, Fluid Mechanics and Thermodynamics involve a considerable amount of mathematical calculation. The worked examples will show you this should be done and in order to successfully

tackle the work you will need to have a good scientific calculator (and know how to use it). The units Mechanical Technology and Engineering Materials are essentially descriptive and investigative. Access to a computer with word processing and drawing software will be of advantage when carrying out activities and other setwork in these units.

Finally, here are some general points to help you with your studies:

- Allow regular time for reading get into the habit of setting aside an hour, or two, at the weekend. Use this time to take a second look at the topics that you have covered during the week or that you may not have completely understood.
- Make notes and file these away for future reference lists of facts, definitions and formulae are particularly useful for revision.
- Look out for inter-relationships between subjects and units you will find many ideas and a number of themes that crop up in different places and in different units. These can often help to reinforce your understanding.
- Don't expect to find all the subjects and topics within a course equally interesting. There may be parts that, for a whole variety of reasons, don't immediately fire your enthusiasm. There is nothing unusual about this; however do remember that something that may not appear particularly useful now may become crucial at some point in the future.
- However difficult things may get, don't be tempted to give up and don't be afraid to ask for assistance! Engineering is not, in itself, a difficult subject, rather it is a subject that demands logical thinking and an approach in which each new concept builds upon those that have gone before.
- Finally, don't be afraid to put your ideas into practice. Engineering is about doing - get out there and do it!

Good luck with your BTEC Engineering studies!

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Chapter 1 Mo

Summary

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Key point

A system of concurrent coplanar forces can be reduced to a single resultant force.

Key point

A system of non-concurrent coplanar forces can be reduced to a single resultant force and a resultant couple or turning moment. balancing force, which will hold the body in a state of static equilibrium, is called the *equilibrant* of the system.

When a body is subjected to a system of non-concurrent coplanar forces, there is a tendency for the forces not only to make it move in a particular direction, but also to make it rotate. Such a non-concurrent system can be reduced to a single resultant force and a resultant couple. If the body is to be held at rest, an equilibrant must again be applied which is equal and opposite to the resultant force. This alone however will not be sufficient. A balancing couple or turning moment, must also be applied which is equal and opposite to the resultant couple.

If you have completed the core unit Science for Technicians, you will know how to find the resultant and equilibrant of a coplanar force system graphically by means of a force vector diagram. We will use this method again shortly but you now need to know how to do the same using mathematics.

Sign convention

When you are using mathematics to solve coplanar force system problems, you need to adopt a method of describing the action of the forces and couples. The following sign convention is that which is most often used (Figure 1.2):

- (i) Upward forces are positive and downward forces are negative.
- (ii) Horizontal forces acting to the right are positive and those acting to the left are negative.
- (iii) Clockwise acting moments and couples are positive and anticlockwise acting ones are negative.

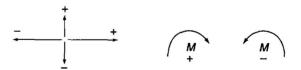


Figure 1.2 Sign convention

Resolution of forces

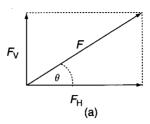
Forces which act at an angle exert a pull which is part horizontal and part vertical. They can be split into their horizontal and vertical parts or *components*, by the use of trigonometry. When you are doing this, it is a useful rule to always measure angles to the horizontal.

In Figure 1.3(a), the horizontal and vertical components are both acting in the positive directions and will be

$$F_{\rm H} = +F\cos\theta$$
 and $F_{\rm V} = +F\sin\theta$

In Figure 1.3(b), the horizontal and vertical components are both acting in the negative directions and will be

$$F_{\rm H} = -F\cos\theta$$
 and $F_{\rm V} = -F\sin\theta$



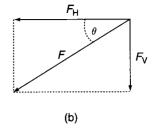


Figure 1.3 Resolution of forces

Forces which act upward to the left or downward to the right will have one component which is positive and one which is negative. Having resolved all of the forces in a coplanar system into their horizontal and vertical components, each set can then be added algebraically to determine the resultant horizontal pull, $\Sigma F_{\rm H}$, and the resultant vertical pull, $\Sigma F_{\rm V}$. The Greek letter Σ (sigma) means 'the sum or total' of the components. Pythagoras' theorem can then be used to find the single resultant force R, of the system.

That is.

$$R^{2} = (\Sigma F_{H})^{2} + (\Sigma F_{V})^{2}$$

$$R = \sqrt{(\Sigma F_{H})^{2} + (\Sigma F_{V})^{2}}$$
(1.1)

The angle θ , which the resultant makes with the horizontal can also be found using

$$\tan \theta = \frac{\Sigma F_{\rm V}}{\Sigma F_{\rm H}} \tag{1.2}$$

With non-concurrent force systems, the algebraic sum of the moments of the vertical and horizontal components of the forces, taken about some convenient point, gives the resultant couple or turning moment. Its sign, positive or negative, indicates whether its direction is clockwise or anticlockwise. This in turn can be used to find the perpendicular distance of the line of action of the resultant from the chosen point.

Example 1.1

Find the magnitude and direction of the resultant and equilibrant of the concurrent coplanar force system shown in Figure 1.4.

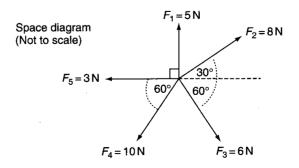


Figure 1.4

When you resolve the forces into their horizontal and vertical components it is essential to use the sign convention. A logical way is to draw up a table as follows with the forces, and their horizontal and vertical components, set out in rows and columns.

Force	Horizontal component	Vertical component
$F_1 = 5 \text{ N}$ $F_2 = 8 \text{ N}$ $F_3 = 6 \text{ N}$ $F_4 = 10 \text{ N}$ $F_5 = 3 \text{ N}$	$\begin{array}{c} 0 \\ +8\cos 30 = +6.93 \text{N} \\ +6\cos 60 = +3.0 \text{N} \\ -10\cos 60 = -5.0 \text{N} \\ -3.0 \text{N} \end{array}$	+5.0 N $+8 \sin 30 = +4.0 \text{ N}$ $-6 \sin 60 = -5.2 \text{ N}$ $-10 \sin 60 = -8.66 \text{ N}$ 0
Totals	$\Sigma F_{H} = +1.93 \mathrm{N}$	$\Sigma F_{V} = -4.86 \mathrm{N}$

The five forces have now been reduced to two forces, ΣF_H and ΣF_V . They can now be drawn as vectors (Figure 1.5).

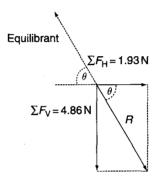


Figure 1.5

The resultant R, is found by Pythagoras as follows:

$$R = \sqrt{1.93^2 + 4.86^2}$$

$$R = 5.23 \text{ N}$$

The angle θ is found from

$$\tan \theta = \frac{\Sigma F_V}{\Sigma F_H} = \frac{4.86}{1.93} = 2.52$$

 $\theta = \tan^{-1} 2.52$
 $\theta = 68.3^{\circ}$

The equilibrant, which is required to hold the system in a state of static equilibrium, is equal to the resultant but opposite in sense.

Bow's notation

Example 1.1 can be solved graphically by means of a force vector diagram drawn to a suitable scale. The process is known as *vector addition*. The force vectors are taken in order, preferably working clockwise around the system, and added nose to tail to produce a polygon of forces.

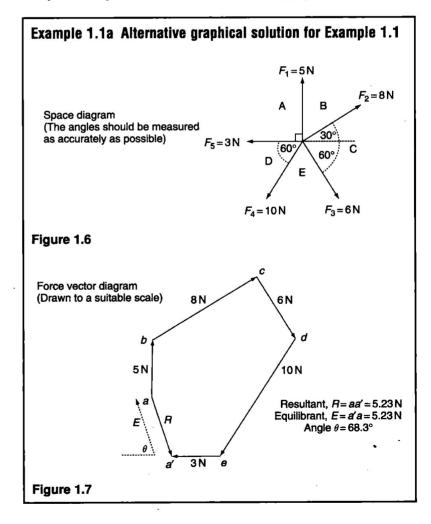
Should the final vector be found to end at the start of the first, there will be no resultant and the system will be in equilibrium. If however there is a gap between the two, this when measured from

Key point

Use capital letters and work in a clockwise direction when lettering space diagram using Bow's notation. the start of the first vector to the end of the last, represents the magnitude, direction and sense of the resultant. The equilibrant will, of course, be equal and opposite. It must be remembered that when you solve problems graphically, the accuracy of the answers will depend on the accuracy of your measurement and drawing.

Bow's notation is a useful method of identifying the forces on a vector diagram and also the sense in which they act. In the space diagram, which shows the forces acting at the point of concurrence, the spaces between the forces are each given a capital letter. Wherever possible, the letters should follow a clockwise sequence around the diagram. In the solution shown in Figures 1.6 and 1.7, the force F_1 is between the spaces A and B, and when drawn on the vector diagram it is identified by the lower case letters as force ab.

The clockwise sequence of letters on the space diagram, i.e. A to B, gives the direction of the force on the vector diagram. The letter a is at the start of the vector and the letter b is at its end. Although arrows have been drawn to show the directions of the vectors, they are not really necessary and will be omitted in future graphical solutions.



You can use the same graphical method to find the magnitude and direction of the resultant of a non-concurrent system of coplanar forces. Generally, however, it is best to use the analytical method as in Example 1.2. In problems, you are also usually asked to find the resultant couple or turning moment and this has to be done by calculation.

Key point

When using the analytical method of solution it is a good idea to measure angles from the horizontal. All of the horizontal components of the forces will then be given by $F_{\rm H} = F\cos\theta$ and all of the vertical components will be given by $F_{\rm V} = F\sin\theta$.

When drawing up a table for each force and its components, two extra columns are required. These are for the moments of the components, taken about some convenient point, and their total gives the resultant turning moment. With a little intuition, you can then determine the position of the line of action of the resultant.

Example 1.2

Determine the magnitude and direction of the resultant of the coplanar forces acting on the component shown in Figure 1.8. Determine also the perpendicular distance of its line of action from the corner A.

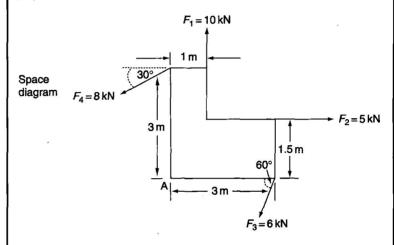


Figure 1.8

Force F (kN)	Horizontal component F _H (kN)	Vertical component F _V (kN)	Moment of F _H about A (kN m)	Moment of F _V about A (kN m)
10 5 6 8	$0+5-6\cos 60 = -3-8\cos 30 = -6.93$	$+10$ 0 $-6 \sin 60 = -5.2$ $-8 \sin 30 = -4.0$		$-(1 \times 10) = -10$ 0 +(5.2 \times 3) = +15.6
Totals	$\Sigma F_{H} = -4.93 \text{ kN}$	$\Sigma F_V = + 0.8 \mathrm{kN}$	ΣM =-7.7 kN m	

Note: When taking moments of the components about A, disregard the plus or minus sign in the components' columns. The sign of the moments is determined only by whether they are clockwise or anticlockwise about A.

The resultant of the force system is again found using Pythagoras (Figure 1.9).

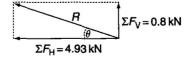


Figure 1.9

$$R = \sqrt{\Sigma F_{\rm H}^2 + \Sigma F_{\rm V}^2}$$

$$R = \sqrt{4.93^2 + 0.8}$$

$$R = \sqrt{4 \text{ QQ kN}}$$

The angle θ is a again given by

$$\tan \theta = \frac{\sum F_V}{\sum F_H} = \frac{0.8}{4.93} = 0.162$$

 $\theta = \tan^{-1} 0.162$
 $\theta = 9.22^{\circ}$

Let the perpendicular distance of the line of action of the resultant from the corner A be a. Because the resultant turning moment is negative, i.e. anticlockwise about A, the line of action of the resultant must be above A (Figure 1.10).

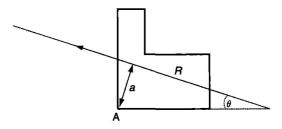


Figure 1.10

Now it must be that,

Resultant moment about A = moment of resultant about A

$$\Sigma M = R \times a$$

$$a = \frac{\Sigma M}{R} = \frac{7.7}{4.99}$$

$$a = 1.54 \text{ m}$$

Test your knowledge 1.1

- 1. What are coplanar forces?
- 2. What are concurrent forces?
- 3. What are the conditions necessary for a body to be in static equilibrium under the action of a coplanar force system?
- 4. What are the resultant and equilibrant of a coplanar force system?
- 5. What can a nonconcurrent coplanar force system be reduced to?

Activity 1.1

A connecting plate, which links the members in an engineering structure, is acted upon by four forces as shown in Figure 1.11. Calculate the magnitude, direction and sense of the resultant force, and the perpendicular distance of its line of action from the point A at which the 1 kN force acts.

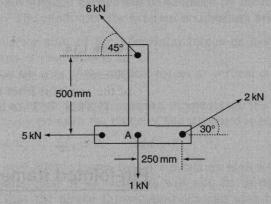


Figure 1.11

Problems 1.1

1. Determine the magnitude and direction of the resultant force for the coplanar force system shown in Figure 1.12.

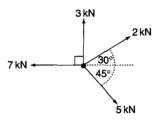


Figure 1.12

[1.79 kN, 15° to horizontal, upward to the left.]

2. Determine the magnitude and direction of the equilibrant required for the coplanar force system shown in Figure 1.13.

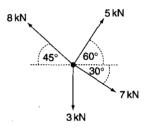


Figure 1.13

[4.54 kN, 50.2° to horizontal, downward to the left.]

3. Determine the magnitude and direction of the resultant force and turning moment for the force system shown in Figure 1.14.

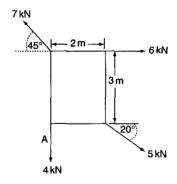


Figure 1.14

Determine also the perpendicular distance of the line of action of the resultant from the corner, A.

[5.8 kN, 7.53° to horizontal, downward to right, 6.57 kN m clockwise about A, 1.13 m.]

Pin-jointed framed structures

Examples of framed structures which you see in everyday life are bicycles, roof trusses, electricity pylons and tower cranes. They are

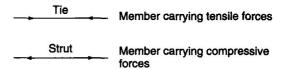


Figure 1.15 Representation of structural members

made up of members which are joined at their ends. Some of these are three-dimensional structures whose analysis is complex and it is only the two-dimensional, or coplanar structures, which we shall consider. There are three kinds of member in these structures (Figure 1.15).

- 1. Ties, which are in tension are shown diagrammatically with arrows pointing inwards. You have to imagine yourself in the place of a tie. You would be pulling inwards to stop yourself from being stretched. The arrows describe the force which the tie exerts on its neighbours to keep the structure in position.
- 2. Struts, which are in compression are shown diagrammatically with arrows pointing outwards. Once again, you have to imagine the way you would be pushing if you were in the place of a strut. You would be pushing outwards to keep the structure in position and to stop yourself from being squashed.
- 3. The third type is redundant members. A perfect framed structure is one which has just sufficient members to prevent it from becoming unstable. Any additional members, which may have been added to create a stiffer or stronger frame, are known as redundant members. Redundant members may be struts or ties or they may carry no load in normal circumstances. We shall avoid framed structures with redundant members as very often they cannot be solved by the ordinary methods of statics.

In reality the members are bolted, riveted or welded together at their ends but in our analysis we assume that they are pin-jointed or hinged at their ends, with frictionless pins. We further assume that because of this, the only forces present in the members are tensile and compressive forces. These are called *primary forces*. In practice there might also be bending and twisting forces present but we will leave these for study at a higher level.

When a structure is in a state of static equilibrium, the external active loads which it carries will be balanced by the reactions of its supports. The conditions for external equilibrium are:

- 1. The vector sum of the horizontal forces or horizontal components of the forces is zero.
- 2. The vector sum of the vertical forces or vertical components of the forces is zero.
- 3. The vector sum of the turning moments of the forces taken about any point in the plane of the structure is zero. That is,

$$\Sigma F_{\rm H} = 0, \qquad \Sigma F_{\rm v} = 0, \qquad \Sigma M = 0$$

We can also safely assume that if a structure is in a state of static equilibrium, each of its members will also be in equilibrium. It follows that the above three conditions can also be applied to individual members, groups of members and indeed to any internal part or section of a structure.

Key point

When a body or structure is in static equilibrium under the action of three concurrent external forces, the forces must be concurrent.