PHYSICS TEXTBOOK

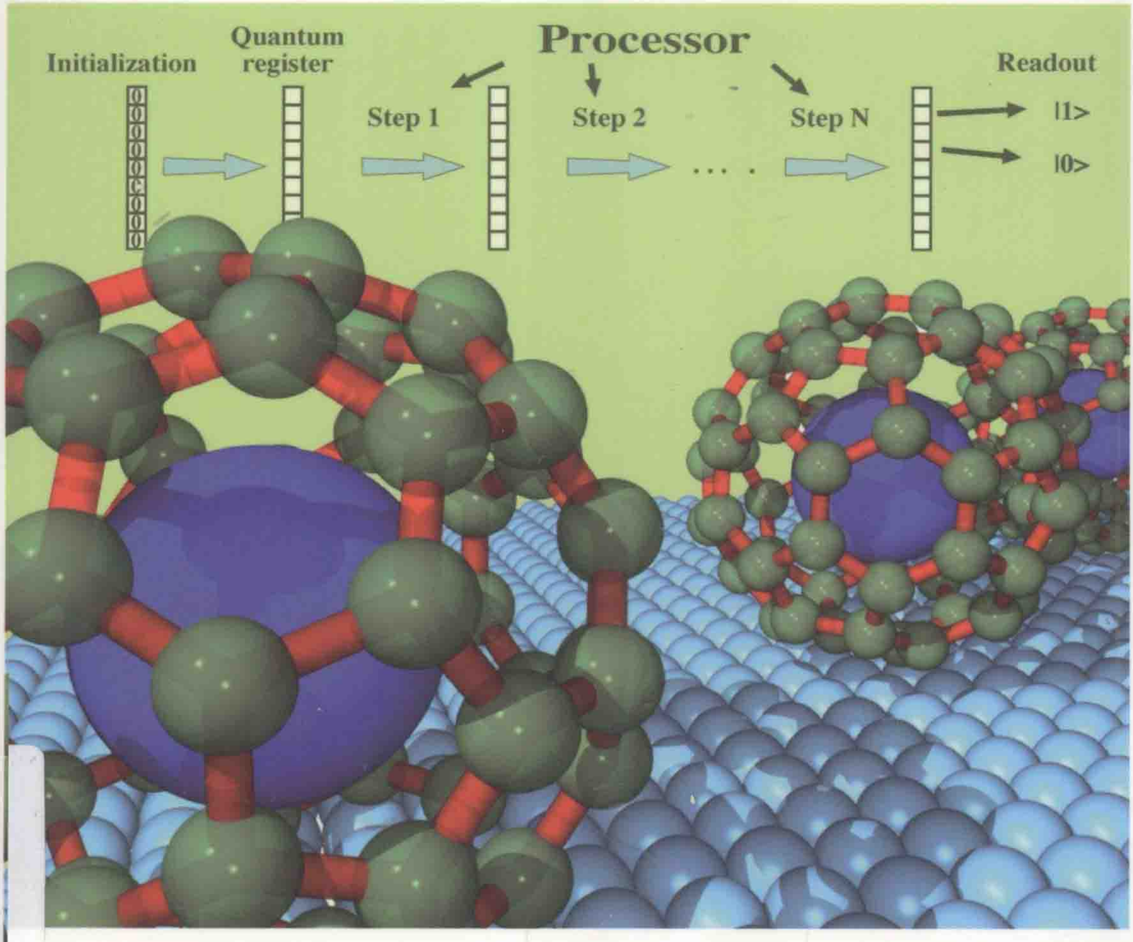
Joachim Stolze, Dieter Suter

WILEY-VCH

Quantum Computing

A Short Course from Theory to Experiment

Second, Updated and Enlarged Edition



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Cover
Quantum computation requires a physical basis to store the information. This is represented by the row of endohedral fullerenes (N@C60 or P@C60) that could serve as "qubits" or quantum bits. The truth table of the reversible logical operation CNOT symbolizes the quantum algorithms from which quantum computers derive their power, while the trajectory on the sphere represent how such a logic operation (in this case the Hadamard gate H) is implemented as a rotation of a spin 1/2. The background contains representations of an ancient mechanical computer and a current palmtop computer.

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Preface to the Second Edition

Quantum information processing has continued to be a fascinating field of research ever since we first taught the course on which this book is based. New research results are being published on a daily basis, as seen on xxx.arxiv.org or in the Virtual Journal of Quantum Information, www.vjquantuminfo.org, and large review articles are trying to keep track of the progress. That situation would have suggested a substantial increase in the extent of this book between the first and second editions. However, it has always been our concept to bring advanced students of physics into contact with the principal ideas in a short course after which they should be able to further explore the field by means of the many excellent monographs and reviews available. Consequently we refrained from a substantial extension, but only added a little material in areas where particularly important progress has been achieved. Of course the choice of these areas is partly a matter of taste, and we felt that although theorists have been very busy in quantum information processing, we should restrict ourselves to important progress in experimental techniques. In particular, we have added a little material on techniques employing single photons, and some more on neutral atoms in optical traps. Neutral atoms trapped in standing-wave fields have provided scientists with the fascinating possibility to study clean artificial crystals with tunable interactions between particles. This may be viewed as a realization of the long-standing idea of a quantum simulator.

To improve the usability of the book we have added a few end-of-chapter problems, many of them related to technical details from the body of the chapter. Slight changes and additions can be found throughout the book and the references. Of course we have detected and corrected numerous small errors and we expect to hear from our readers about those which we did not detect as well as the newly introduced ones.

Joachim Stolze and Dieter Suter

Dortmund, December, 2007

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Preface to the First Edition

During the past decade the field of quantum information processing has experienced extremely rapid progress. Many physicists and computer scientists have become interested in this exciting new field, and research activities were started in many places, including the University of Dortmund, where several groups from experimental and theoretical condensed-matter physics and from computer science, joined forces in a program called "Materials and methods for quantum information processing". Since that program involved graduate students from several countries, and with different scientific backgrounds, we decided to teach an introductory course on the fundamentals of quantum information processing. The idea was to provide the graduate students working on highly specialized research projects in, for example, magnetic resonance, semiconductor spectroscopy, or genetic algorithms, with a common language and background connecting their areas of research. In that course we tried to discuss on equal footing both theoretical foundations and experimental opportunities and limitations. The present book contains the material presented in our course, in an edited and slightly updated form.

We are well aware of the existence of a number of excellent books and courses relevant to our subject. Nevertheless, we feel that a compact introduction to both theory and experiment aimed at advanced students of physics is still lacking. We assume that our readers have a reasonably good background in physics, notably in quantum mechanics, plus some knowledge in introductory statistical mechanics and solid-state physics. We did not attempt to make our book self-contained by explaining every concept which is needed only occasionally. We do hope, however, that we have succeeded in explaining the basic concepts from quantum mechanics and computer science which are used throughout the book and the whole field of quantum computing and quantum communication.

We are grateful to the students who attended our course or participated in a seminar based partly on the course material. Their questions and comments were helpful in shaping the material. Of course all errors and inaccuracies (which are present, no doubt) are entirely our own responsibility. We would like to express our thanks to many colleagues for many things: to Bernd Burghardt for LaTeX help, to Hajo Leschke for clarifying remarks, to Heinz Schuster and Claudius Gros for encouragement, to Michael Bortz, Hellmut Keiter (who fought his way through the entire manuscript when it was still in an intermediate state), and André Leier for reading parts of the manuscript, and to André Leier for also supplying material on quantum error correction.

Joachim Stolze and Dieter Suter

Dortmund, March, 2004

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Symbols and Abbreviations

Symbol	**Explanation** (Chapter)
$\oplus$	XOR logical operation, addition modulo 2 (3)
$\otimes$	direct product (4)
$\lvert\uparrow\rangle, \lvert\downarrow\rangle$	basis states of spin 1/2 (2)
$\lvert 0\rangle, \lvert 1\rangle$	basis states of qubit (1)
$\mathbf{1}$	unit operator (4)
$\mathbf{A}$	operators: boldface (4)
$\mathbf{A}_i$	gate operators (6)
$[A, B] = AB - BA$	commutator (2)
$\mathbf{a}^\dagger$, $\mathbf{a}$	creation and annihilation operators (6)
$\vec{B} = (B_x, B_y, B_z)$	magnetic flux density in frequency units (4)
$\vec{B}$	magnetic flux density, magnetic field (4,9)
B_1	radio-frequency amplitude (10)
$\vec{B}_{\mathrm{rf}}$	radio-frequency magnetic field (10)
$\mathbf{b}^\dagger$, $\mathbf{b}$	creation and annihilation operators (6)
C	concurrence (entanglement measure) (4)
c	vacuum speed of light (2)
$\mathbf{c}^\dagger$, $\mathbf{c}$	creation and annihilation operators (6)
CHSH	Clauser, Horne, Shimony, and Holt (inequality) (4)
$\{c_i\}$	code (13)
CMOS	complementary metal-oxide semiconductor (2)
CNOT	controlled NOT operation (1, 3)
CSS	Calderbank, Shor, and Steane (codes) (7)
D	diffusion constant (6)
d	dimension of Hilbert space (4)
$d[T]$	description of Turing machine T (3)
δ_{ij}	Kronecker symbol (4)
det	determinant (4)
DFS	decoherence-free subspace (7)
E	energy (2)
$\mathbf{E}_i$	POVM element (13)
EPR	Einstein–Podolsky–Rosen (4)
EPR	electron paramagnetic resonance (12)
ε_i	energy eigenvalue (4)

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F	total (electronic and nuclear) angular momentum (11)
$\mathbf{F}_\alpha$	generator of decoherence (7)
FET	field effect transistor (1)
FFT	fast Fourier transformation (8)
FID	free induction decay (10)
γ	gyromagnetic ratio (9)
gcd	greatest common divisor (8)
$\mathcal{H}$	Hamiltonian operator (4)
$\mathbf{H}$	Hadamard gate (4)
$\mathfrak{H}$	Hilbert space (4)
$H(p)$	binary entropy (13)
$I(X:Y)$	mutual information content (13)
$i = \sqrt{-1}$	imaginary unit (2)
int	integer part (of a real number) (8)
k_B	Boltzmann constant (2)
$\{k_i\}$	key (13)
λ_i	eigenvalue of observable (9)
ln	natural logarithm (2)
M_1, M_2	classical bits (13)
m_F	total magnetic quantum number (11)
$\mathbf{M}_i$	measurement operator (13)
$\{m_i\}$	message (13)
$\vec{n}$	unit vector (5)
NMR	nuclear magnetic resonance (10)
NP	nondeterministic polynomial (complexity class) (3)
ω	angular frequency (7)
ω_L	Larmor frequency (7)
P	polynomial (complexity class) (3)
$\vec{P}$	polarization vector (4)
$p(\vec{r},t)$	probability density (6)
$p(x), p(y)$	probabilities (13)
$p(x\|y)$	conditional probability (13)
$p(x,y)$	simultaneous probability (13)
$\mathbf{P}_i = \|a_i\rangle\langle a_i\|$	projection operator (4)
p_i	probability (4)
p_{ik}	probability (6)
Φ	magnetic flux (12)
Φ_0	magnetic flux quantum (12)
POVM	positive operator-valued measure (4)
$\|\psi\rangle$	quantum state (1)
$\mathbf{q}_i^\dagger$, $\mathbf{q}_i$	creation and annihilation operators (6)
QFT	quantum Fourier transformation (8)
QND	quantum nondemolition detection (1)

$\mathbf{R}_{\vec{n}}(\theta)$	rotation operator (5)
$\mathbf{R}_k$	phase gate (8)
RF	radio-frequency (10)
$\boldsymbol{\rho}$	density operator (2)
S	entropy (2)
$S(X)$	information content, Shannon entropy (13)
$S(X\|Y)$	conditional entropy (13)
$S(\boldsymbol{\rho})$	von Neumann entropy (13)
$\mathbf{S}^+, \mathbf{S}^-$	spin raising and lowering operators (4)
$\vec{\mathbf{S}} = (\mathbf{S}_x, \mathbf{S}_y, \mathbf{S}_z)$	spin - 1/2 operators (4)
STM	scanning tunneling microscope (12)
T	absolute temperature (2)
T	Turing machine (3)
T_2	dephasing time (7)
$\theta^{(3)}$ =CCNOT=C^{2}NOT	Toffoli gate (3)
Tr	trace operation (2)
TTL	transistor to transistor logic (1)
U	universal Turing machine (3)
$\mathbf{U}$	unitary transformation (8)
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$	Pauli operators (4)
$\hat{x}, \hat{y}, \hat{z}$	unit vectors along coordinate axes (4, 5, 13)
X, Y	random variables (13)
x, y	possible values of random variables (13)
xy	x AND y (3)

Contents

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