Time-Varying Image Processing and Moving Object Recognition

V. Cappellini, editor

TIME-VARYING IMAGE PROCESSING AND MOVING OBJECT RECOGNITION

Proceedings of the International Workshop Florence, Italy, September 8–9, 1986

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PREFACE

The area of Digital Image Processing is of high actual importance in terms of research and applications. Through the interaction and cooperation with the near areas of Pattern Recognition and Artificial Intelligence, the new specific area of "Time-Varying Image Processing and Moving Object Recognition" is emerging of increasing interest. This new area is indeed contributing to impressive advances in several fields, such as communications, radar-sonar systems, remote sensing, biomedicine, moving vehicle recognition-tracking, traffic monitoring, automatic inspection and robotics.

This book represents the Proceedings of the Second International Workshop on Time-Varying Image Processing and Moving Object Recognition, held in Florence, September 8-9, 1986. Papers reported here provide an authorative and permanent record of the scientific and technical lectures, presented by selected speakers from several Nations. Some papers are more theoretical or of review nature, while others contain new implementations and applications. They are conveniently grouped into the following 7 fields:

- A. Digital Processing Methods and Techniques
- B. Digital Television
- C. Remote Sensing Image Processing
- D. Digital Processing of Biomedical Images
- E. Pattern Recognition Methods and Techniques
- F. 3-D Methods and Techniques in Computer Vision
- G. Tracking and Recognition of Moving Objects

New digital image processing and recognition methods, implementation techniques and advanced applications (television, remote sensing, biomedicine, traffic, robotics, etc.) are presented. In particular new approaches for solving 3-D problems (processing, description, recognition) are described. In overall the book presents - for the new outlined area - the state of the art (theory, implementation, applications) with the next-future trends.

This work will be of interest not only to researchers; professors and students in university departments of engineering, communications, computers and automatic control, but also to engineers and managers of industries concerned with computer vision, manufacturing, automation, robotics and quality control.

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A DIGITAL PROCESSING METHODS AND TECHNIQUES

REAL TIME IMAGE PROCESSING WITH DECOMPOSITION STRUCTURES

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A number of realization structures of two-dimensional digital filters suitable for real-time image processing are reviewed. These realizations are based on proper matrix decomposition methods which are characterized by inherent parallelism, modularity, flexibility and may be implemented via VLSI array processors or by using other efficient hardware techniques.

INTRODUCTION

Digital image processing was stimulated mainly by space research in the 1960's and includes sampling, quantization, compression, coding, transmission, enchancement, restoration and storage. Some of the most important present applications are described in the sequel [1]:

1) Telecommunications

Broadcast television (TV), image transmission through space or cables, fascimile, video conferencing and video phone are the main applications.

2) Biomedecine

The main applications here are radiography, echography, nuclear medicine, electron micrography, scintigraphy, theormography, computer tomography (CT), nuclear magnetic resonance (NMR). The images here may be dynamic or static; they may require either real-time or off-line techniques, image storage, retrieval and compression.

3) Remote Sensing

Weather prediction, remote sensing of earth resources, space imagery, satellite imagery, serial imagery, air reconnaissance, mapping, astronomy, are some of the applications here.

4) Non-natural images received by arrays

Seismic records, signals generated by radar, sonar and radioastronomical arrays, forward looking infrared (FLIR) and side looking radar (SLR), voice prints, sonograms, range-time and range-rate planes.

5) Other Applications

Other more recent applications include robotics, industrial radiographs, non des-

tructive testing, moving object recognition and tracking, traffic monitoring (such as planes, rockets, satellites, cars and fishes) traffic monitoring, vision in manufacturing processes, art processing and any multispectral or other form of mapping of scenes or objects, into a 2-D or m-D format.

The tremendous growth in both number of images and bit rates that have been recently experienced, coupled with the need for fast processing of these images, has led an intense interest for real-time image processing. Specifically the need for real-time image processing became evident with the expanding utilization of television imaging to the industrial, medical and military environments. The real time image processing may be defined as "the processing of images, that can be viewed on an appropriate display, with an apparent continuity". Namely for an image of dimension MxN and a TV scan rate of L images per second and if R operations per pixel are required, we reach the quantity $S = M \times N \times R \times L$, which represents the number of operations that must be performed per second, to achieve real-time image processing [2]. In the case where M = N = 256, L = 30 and R = 1,000 we obtain $S = 1.966 \cdot 10^9$ operations per second or 1.966 MOPs, while in the case of M = N = 1.024, L = 30 and R = 1.000 we obtain S = 31.457 MOPs. The requirements of real-time image processing can be addressed by developing algorithms and structures which are implemented by low-cost, very efficient, high-speed special purpose VLSI building blocks.

Two dimensional (2-D) and m-dimensional (m-D) filtering are concerned with the extension of 1-D filtering techniques to two and more dimensions. This subject has received considerable attention recently, due to its importance in enchancement and restoration [3,4]. The fundamental difficulty in the extention of 1-D techniques to 2-D and m-D is the lack of a Fundamental Theorem of Algebra in polynomials of two or more variables, The linear 2-D digital filters, due to their mathematical simplicity and easy realization have extensively studied and used for image processing applications [5,6]. However the linear filters tend to blur the object boundaries and edges which are important in edge detection and object recognition applications. Linear filters also are not appropriate when the Gaussian assumption is not valid or the noise is not signal dependent and/or additive. To face the problems which characterize the linear filters, numerous 2-D nonlinear filters have been recently studied and proposed. Specifically, median filters are among the better known [7]. Others are quadratic filters [8], homomorphic filters, α -trimmed filters, generalized and nonlinear mean filters and nonlinear order statistics filters.

Motivated by the desire to give a general realization method, for the sake of modularity a method was developed [9,10] for the exact expansion of a general 2-D or (m-D) real rational transfer function in terms of order one, each one of which is a function of one of the two (or m) variables only. This method was based on a decomposition of the matrix of coefficients of 2-D (or m-D) polynomials and it was also used by the authors for the construction of reconfigurable filters.

Special forms of the above general decomposition structure may be considered the following realizations which are based on special matrix decompositions forms:

- 1. Jordan decomposition (J) [11].
- Singular Value decomposition (SV) [12,13].
- 3. Lower-Upper triangular decomposition (LU) [14,15].
- 4. Walsh-Hadamard Transform decomposition (WHT) [9,16,17].

Detailed comparisons of the above matrix decomposition approaches w.r.t. various figures of merit have been presented in [12,17,18].

Moreover a modular VLSI implementation of 2-D digital filters, which is based on the expression of a 2-D high-order polynomial in terms of low-order polynomials, and uses the matrix decomposition approach, has been presented in [19]. Each one of these 2-D polynomials is implemented in a modular manner on a 2-D chip with the bit-sliced technique.

Other general realization structures for the block processing of 2-D signals, which are based on the matrix decomposition approaches, are presented in [20,21]. In [22] realization structures of FIR and IIR 2-D digital filters, which are based on the LU decomposition and are characterized by minimum cycle time idependetly of the filters order, are presented. Finally in [23] the first effort for the implementation of the decomposed FIR 2-D filters via VLSI systolic array structures has been presented.

2. THE GENERAL DECOMPOSITION STRUCTURE OF A 2-D TRANSFER FUNCTION

Consider at first the transfer function of a 2-D FIR digital filter of the form

$$H_{A}(z_{1},z_{2}) = \sum_{i=0}^{n_{1}} \sum_{j=0}^{m_{1}} n_{ij}z_{1}^{i}z_{2}^{j}$$
(1)

where $\{n_{ij}^{}\}$ are the filter's coefficients. Then $H(z_1,z_2)$ can be written in the form

$$H_{A}(z_{1},z_{2}) = Z_{1}^{T} N Z_{2}$$
 (2)

where

$$\mathbf{Z}_{1}^{\mathsf{T}} = \begin{bmatrix} 1 & \mathbf{z}_{1} & \dots & \mathbf{z}_{1}^{\mathsf{n}} \end{bmatrix}$$
 , $\mathbf{Z}_{2}^{\mathsf{T}} = \begin{bmatrix} 1 & \mathbf{z}_{2} & \dots & \mathbf{z}_{2}^{\mathsf{m}} \end{bmatrix}$

and $M = [n_{i-1}, j_{-1}]$ is a constant $(n_i+1)x(m_i+1)$ coefficient matrix. The matrix M may be written in general in a number of ways as a product of two constant matrices R and S, i.e. M = R S. If we substitute M = R S in (2) we obtain:

$$H_{A}(z_{1},z_{2}) = [Z_{1}^{T} R] [S Z_{2}]$$

$$= [r_{0}(z_{1})...r_{1}(z_{1})...r_{\xi}(z_{1})] [s_{0}(z_{2})...s_{1}(z_{2})...s_{\xi}(z_{2})]^{T}$$

$$= \int_{i=0}^{T} r_{i}(z_{1}) s_{i}(z_{2})$$
(3)

The block diagram of the realization of the FIR filter $H_A(z_1,z_2)$ via (3) is shown in Figure 1. The smallest necessary number of parallel branches for the realization of (2) equals to the rank of the decomposition i.e. $\tau \ge r = \operatorname{rank} N$. It has been shown in [10] that any 2-D IIR filter may be expressed in a decomposed form like that in (3). Specifically the following theorem has been established.

Theorem 1

The 2-D rational transfer function

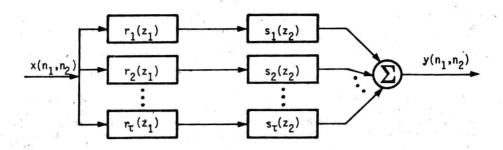


FIGURE 1 The general decomposition structure of a 2-D FIR digital filter in terms of separable 1-D digital filters.

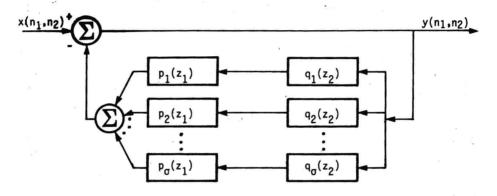
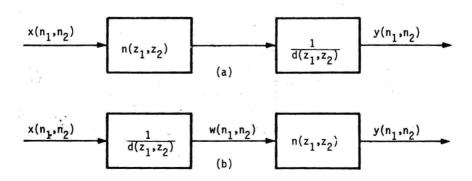


FIGURE 2 The general decomposition structure of a 2-D all pole IIR digital filter in terms of separable filters.



$$H(z_{1},z_{2}) = \frac{n(z_{1},z_{2})}{d(z_{1},z_{2})} = \frac{n(z_{1},z_{2})}{1+\bar{d}(z_{1},z_{2})} = \frac{\prod_{\substack{i=0 \ j=0 \ i=0 \ j=0 \ (i,j) \neq (0,0)}}^{n_{1}} \prod_{\substack{j=0 \ j=0 \ (i,j) \neq (0,0)}}^{m_{1}} \prod_{\substack{j=0 \ j=0 \ (i,j) \neq (0,0)}}^{m_{1}} q_{1}^{j} z_{2}^{j}$$

$$(4)$$

which describes an IIR 2-D digital filter, can be expressed by terms of the form $(z_1-z_{11}),(z_2-z_{2j})$ only, where z_{1j},z_{2j} are constants and $i=1,2,\ldots$, max (n_1,n_2) ; $j=1,2,\ldots$, max (m_1,m_2) . Feedback which contains both variables z_1 and z_2 is required to realize a nonseparable denominator.

To prove the above theorem, the polynomial $\bar{d}(z_1,z_2)$, which does not have a constant term, is expanded in a similar way as follows:

$$\bar{\mathbf{d}}(z_{1}, z_{2}) = Z_{1}^{T} D Z_{2} = [Z_{1}^{T} P] [Q Z_{2}]$$

$$= \sum_{i=0}^{\sigma} p_{i}(z_{1})q_{i}(z_{2})$$
(5)

where P,Q are $(n_2+1)\times\sigma$ and $\sigma x(m_2+1)$ matrices respectively, where $\sigma_{\geq p}=\text{rank}\textbf{D}.$ In order to avoid delay-free loops in the feedback branches, and therefore to obtain realizable structures, none of the products $p_1(z_1)q_1(z_2),\ i=0,\ldots,\sigma$ must contain a constant term. Decompositions of the form (5), which ensure the realizability, are obtained if we choose the matrix $\textbf{P}=\begin{bmatrix}p_{i-1},j_{-1}\end{bmatrix}$ to be a $(n_2x\,1)\,x\,(n_2x\,1)$ nonsingular square matrix with $p_{00}\ne 0$ and $p_{0i}=0,\ i=1,2,\ldots,n_2$ [10]. Then the matrix Q may be determined by $\textbf{Q}=\textbf{P}^{-1}\textbf{D}$, provided that P is nonsingular. Now taking into account the form of P and D, it results that the $(1,1)^{th}$ element q_{00} of the matrix Q is zero. The realization of the all-pole IIR filter $\textbf{H}_1=1/d(z_1,z_2)$ is shown in Figure 2.

In the sequel we will consider two forms of the general decomposition structure decribed above. These forms will be called forms I, II and are based on the direct form I, and II realizations respectively [3,19].

2.1. Decomposition Structure Form I

The direct form I realization results from the cascade configuration of the non-recursive FIR filter $H_A(z_1,z_2)=n(z_1,z_2)$ with the recursive all pole IIR filter $H_1(z_1,z_2)=1/d(z_1,z_2)$ (Figure 3a). The direct form II results from the cascade realization of the above two filters in reverse order (Figure 3b). Similarly the decomposition structure form I results from the cascade configuration of the nonrecursive array block, which realizes the FIR filter $H_A(z_1,z_2)=n(z_1,z_2)$ (Figure 1) and the recursive feedback block, which realizes the IIR filter $H_F(z_1,z_2)=1/d(z_1,z_2)$ (Figure 2). Due to the forms of P and Q in (5) there is no constant term in any of the products $p_i(z_1)q_i(z_2)$.

2.2. Decomposition Structure Form II

The decomposition structure form II result as the cascade configuration of the recursive all pole IIR filter $H_1(z_1,z_2)$ with the nonrecursive array FIR filter $H_A(z_1,z_2)$. Although the order of the subfilters is reversed, the overall transfer function (1) is not affected since we deal with linear, space-invariant (LSI) systems [24]. It is seen from Figure 3b that the intermediate variable $W(z_1,z_2)$, which represents the output of the filter $H_1(z_1,z_2)$, is related to the input and output by the relations

$$W(z_1,z_2) = X(z_1,z_2) - \bar{d}(z_1,z_2) W(z_1,z_2)$$
 (6a)

$$Y(z_1,z_2) = n(z_1,z_2) W(z_1,z_2)$$
 (6b)

where $X(z_1,z_2)$, $Y(z_1,z_2)$ are the z transforms of the input $x(n_1,n_2)$ and the output $y(n_1,n_2)$ respectively. The form II is generally characterized by reduced storage requirements,less hardware cost and increased throughput rate. Moreover there is possibility some of the delay elements z_1,z_2 to be shared between the feedback and forward branches. Specifically if $n_1=n_2$, we could choose the auxiliary matrix $\mathbf R$ to be equal to $\mathbf P$. Then the polynomials $\mathbf r_1(z_1)$ would appear as common factors in the feedback and array blocks, thus resulting in a reduction of the storage requirements and the number of delays and registers. Similarly, if $m_1=m_2$ we could choose $\mathbf S=\mathbf Q$. In the general case where $n_2>n_1$ and $m_2>m_1$, we consider an augmented coefficient matrix $\hat{\mathbf N}$ of dimensions $(n_2+1)\times(m_1+1)$. Then the coefficient matrices $\hat{\mathbf N}$, $\mathbf D$, may be decomposed as $\hat{\mathbf N}=\mathbf R\mathbf S$, $\mathbf D=\mathbf R\mathbf Q$ where $\mathbf R$ is a $(n_2+1)\times(n_2+1)$ square auxiliary nonsingular matrix. Then the matrices $\mathbf S$, $\mathbf Q$ are determined by $\mathbf S=\mathbf R^{-1}\hat{\mathbf N}$, $\mathbf Q=\mathbf R^{-1}\mathbf D$. Using the decomposed forms of $n(z_1,z_2)$, $d(z_1,z_2)$, we arrive at the following equations of the output $\mathbf Y(z_1,z_2)$ and the output of the feedback branch $\mathbf V(z_1,z_2)=d(z_1,z_2)\mathbf W(z_1,z_2)$, as follows:

$$Y(z_1,z_2) = \sum_{i=0}^{\xi} r_i(z_1) s_i(z_2) W(z_1,z_2)$$
 (8a)

$$V(z_1,z_2) = \sum_{i=0}^{\xi} r_i(z_1)q_i(z_2)W(z_1,z_2)$$
 (8b)

where $\xi = \max(\tau, \sigma)$. Thus the factors $r_i(z_1)$, $i = 0, 1, \ldots, \xi$ may be generally shared by the forward and feedback branches.

3. SPECIFIC MATRIX DECOMPOSITION APPROACHES

The general decomposition structure which has been reviewed in section 2 consists the basis for a number of known useful decompositions. These realizations result easily from the general one by specifying properly the auxiliary matrices R and S. In the following we refer very concisely to the most dominant decomposition structures.

3.1. The Jordan form decomposition (J) [11]

This approach is based on matrix diagonalization. We consider the square augmented matrix N (in the general case where $n_1\neq m_1$), where

$$\hat{\mathbf{N}} = \begin{bmatrix} \mathbf{N} & \mathbf{0} \end{bmatrix}$$
, if $\mathbf{n}_1 > \mathbf{m}_1$ or $\hat{\mathbf{N}} = \begin{bmatrix} -\mathbf{N} \\ \mathbf{0} \end{bmatrix}$, if $\mathbf{n}_1 < \mathbf{m}_1$ (10)

Then we may write $\hat{N}_i = \hat{H} J H^{-1}$ where H is the transforming matrix consisting of the eigenvectors of \hat{N} , and J is the Jordan matrix of \hat{N} . In the present case R and S may be identified by H J, H^{-1} or H, $J H^{-1}$ respectively.

3.2. The Singular Value decomposition (SV) [12,13]

The coefficient matrix N is written in the form