Superconductivity

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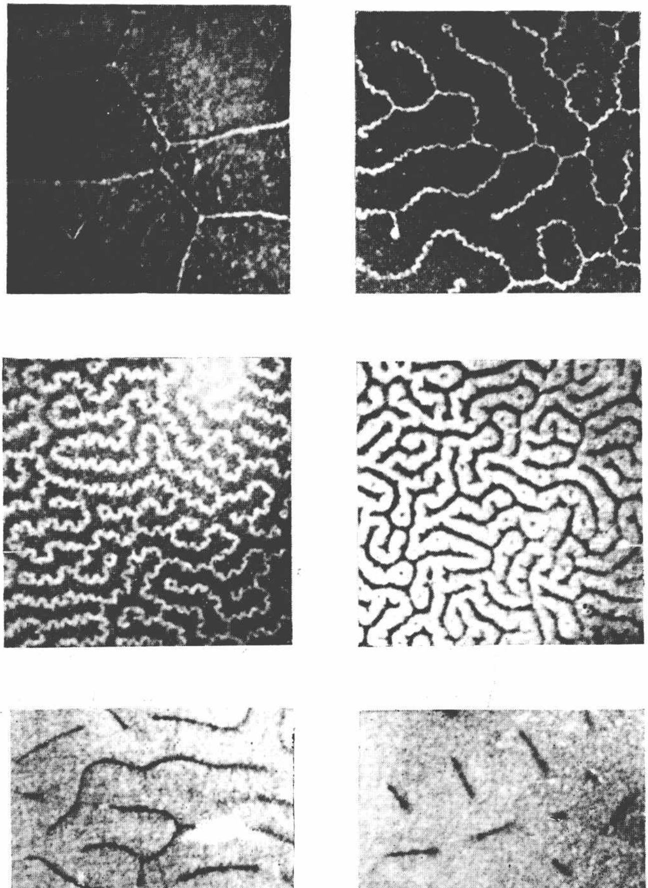

Powder patterns of the intermediate state, showing the shrinking of the superconducting (dark) regions as h takes on the values (left to right, top to bottom) 0, 0·08, 0·27, 0·53, 0·79, and 0·90.

(After Faber, 1958. Reproduced by kind permission of the Royal Society and the author.) *Proc. Roy. Soc. A248, 464, plate 25.*

For Carla

Acknowledgements

This book has grown, beyond recognition, from a set of lecture notes written and used during my stay at the Institut Fourier of the University of Grenoble in 1959–60. I should like once again to thank my hosts, Professors Néel and Weil and Dr Goodman, for a stimulating and pleasant year. I am very grateful to a large number of people who have helped me with written or oral comments, with news of their unpublished work, with preprints, and with copies of graphs. In particular I thank Drs Coles, Collins, Cooper, Douglass, Faber, Garfunkel, Goodman, Masuda, Olsen, Pippard, Schrieffer, Shapiro, Swihart, Tinkham, Toxen, and Waldram. My colleagues Lindenfeld, McLean, and Weiss provided much helpful discussion. Above all my gratitude is due to Bernard Serin, from whose guidance and friendship I have profited for many years. He found the time to read the entire first draft of the manuscript and suggested many improvements, not all of which I have been wise enough to incorporate.

September 1961 E. A. LYNTON

Preface to the Third Edition

This monograph has been brought up to date by changes and additions in all chapters. In addition a complete revision of Chapters V, VI, and VII reflects both the increasingly important role of the Ginzburg-Landau theory and the extensive theoretical and experimental concentration on type II superconductors during recent years. Chapter V has been considerably expanded and now includes – in more logical sequence – the discussion of size effects. Chapter VII is entirely devoted to type II superconductivity, and treats this topic in much greater detail than the previous edition. Further additions to the present edition are the sections on the Josephson effect and on macroscopic coherence in Chapter XI.

In preparing this third edition I have continued to profit from helpful comments of my colleagues at Rutgers University. I am also most grateful to the many preprints sent to me prior to their publication, which made my work much easier.

September 1967 E. A. LYNTON

Contents

Introduction

Although the fascinating phenomenon of superconductivity has been known for fifty years, it is largely through the concentrated experimental and theoretical work of the past decade that a basic (though as yet very incomplete) understanding of the effect has been reached. Far from being an oddity of little physical interest it has been shown to be a co-operative phenomenon of basic importance and with close analogies in a number of fields. At the present time one important period in the development of the subject has been completed, and the next is already well under way, with much effort in theory and experiment to carry our understanding from the general to the particular, from the idealized superconductor to the specific metal. Somewhat coincidentally, there now also is great interest in possible practical applications of superconductivity.

This monograph is a largely descriptive introduction to superconductivity, requiring no more than an undergraduate physics background, and written to serve two functions. It can be a first survey and a stepping stone toward more intensive study for those who intend to become actively engaged in the further development of superconductivity, be it in basic research or in technical applications. Such readers will benefit from the extensive bibliography, listing more than 450 books and articles. At the same time the book is sufficiently complete in its description both of experimental details and of theoretical approaches to be a basic reference for those who wish to be acquainted with the present state of superconductivity. It will enable them to follow further developments as they appear in the scientific and technical literature.

The contents of the book can be grouped into a number of sections which treat the subject of superconductivity in successive layers with increasing resolution of detail. The first three chapters introduce the reader to the principal characteristics of bulk superconductors, and treat these in terms of the basic phenomenological models of London and of Gorter-Casimir. With this section the reader thus acquires a broad outline and a general understanding of the thermodynamic and the static electromagnetic behaviour of idealized, bulk superconductors. The treatment of the subject is then pursued in greater detail along two essentially parallel directions. In the section comprising Chapters IV–VII are discussed those aspects of the behaviour of superconductors which lead to the non-local treatments of Pippard and of Ginzburg and Landau. These more sophisticated phenomenological models account for an interphase surface energy, in terms of which the later chapters of this section describe the intermediate state, phase nucleation, propagation, and supercooling, superconductors of the second kind, and the magnetic behaviour of specimens of small dimensions. Chapters VIII–X can be read without a study of the preceding section (IV–VII) and describe in much detail those characteristics of a superconductor which during the past decade have indicated the microscopic nature of superconductivity, and have led to the theory of Bardeen, Cooper, and Schrieffer. The fundamental aspects of this theory are presented with a minimum of mathematics.

The book closes with a chapter on the behaviour of alloys and compounds, and with one on superconducting devices.

In describing the principal empirical characteristics of superconductors I have tried to include only the key experiments through which the phenomenon in question was established, as well as more recent work which gives the most detailed or the most precise information. It is both unnecessary and impossible in a monograph of this small size to be encyclopaedic either in the enumeration of all pertinent experiments, or in the description of superconducting behaviour in minute detail. My selection of what aspects of the latter to emphasize may appear arbitrary, especially to those whose work has been slighted. The choice was not a judgement of the scientific value of such work, but rather of its didactic usefulness in illuminating the elementary characteristics of superconductors.

CHAPTER I

Basic Characteristics

1.1. Perfect conductivity and critical magnetic field

The behaviour of electrical resistivity was among the first problems investigated by Kamerlingh Onnes after he had achieved the liquefaction of helium. In 1911, measuring the resistance of a mercury sample as a function of temperature, he found that at about 4°K the resistance falls abruptly to a value which Onnes' best efforts could not distinguish from zero. This extraordinary phenomemon he called *superconductivity*, and the temperature at which it appears the critical temperature, T_c (Kamerlingh Onnes, 1913).

When a metallic ring is exposed to a changing magnetic field, a current will be induced which attempts to maintain the magnetic flux through the ring at a constant value. For a body of resistance R and self-inductance L, this induced current will decay as

$$I(t) = I(0)\exp(-Rt/L). \tag{I.1}$$

$I(t)$ can be measured with great precision, for example, by observing the torque exerted by the ring upon another, concentric one which carries a known current. This allows the detection of much smaller resistance than any potentiometric method. A long series of such measurements on superconducting rings and coils by Kamerlingh Onnes and Tuyn (1924), Grassman (1936), and others recently culminated in an experiment by Collins (1956), in which a superconducting ring carrying an induced current was kept below T_c for about two and a half years. The absence of any detectable decay of the current during this period allowed Collins to place an upper limit of 10^{-21} ohm-cm on the resistivity of the superconductor.† This can be compared to the value of 10^{-9} ohm-cm for the low temperature resistivity of the purest copper.

There is, therefore, little doubt that a superconductor is indeed a

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† Quinn and Ittner (1962) have lowered this upper limit to 10^{-23} ohm-cm by looking for the time decay of a current circulating in a thin film tube.

perfect conductor, in the interior of which any slowly varying electric field vanishes. A current induced in a superconducting ring of thickness larger than a few hundred Å will persist an immeasurably long time without dissipation. For wire of smaller diameter Little (1967) has shown that thermodynamic fluctuations will cause a finite lifetime for the decay of the persistent current. The existence of such fluctuations has been demonstrated by Parks and Goff (1967).

Below T_c, the superconducting behaviour can be quenched and normal conductivity restored by the application of an external magnetic field. This field, H_c, is called the *critical* or *threshold magnetic field*, and, as shown in Figure 1, it varies approximately as

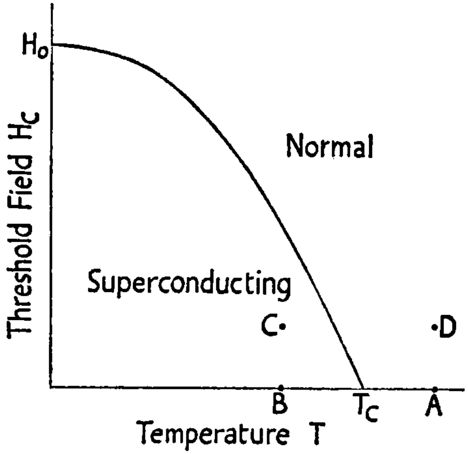


FIG. 1

$$H_c \approx H_0\left[1-\left(\frac{T}{T_c}\right)^2\right], \tag{I.2}$$

where $H_0 = H_c$ at $T = 0°$K. It is convenient to introduce reduced coordinates $t \equiv T/T_c$, and $h(t) \equiv H_c(T)/H_0$, in terms of which

$$h \approx 1-t^2. \tag{I.2a}$$

The actual temperature variation of h is more accurately represented by a polynomial in which the coefficient of the t^2 term differs from unity by a few per cent.

The superconductivity of a wire or film carrying a current can be quenched when this reaches a critical value. For specimens sufficiently thick so that surface effects can be ignored, the critical current is that which creates at the surface of the specimen a field equal to H_c. Smaller samples remain superconducting with much higher currents than those calculated from this criterion, which is called Silsbee's rule (Silsbee, 1916).

1.2. Superconducting elements and compounds

Table I lists all presently known superconducting elements and their

TABLE I

Element	T_c (°K)	H_0 (gauss)
Aluminium	1·19	99
Cadmium	0·56	30
Gallium	1·09	51
Indium	3·404	293
Iridium	0·14	~20
Lanthanum-α	~5	—
Lanthanum-β	6·0	1600
Lead	7·18	803
Mercury-α	4·153	411
Mercury-β	3·95	340
Niobium	9·46	1944
Osmium	0·66	65
Protactinium	1·4	—
Rhenium	1·698	198
Ruthenium	0·49	66
Tantalum	4·482	830
Technetium	7·75	1410
Thallium	2·39	171
Thorium	1·37	162
Tin	3·722	309
Titanium	0·39	100
Tungsten	0·012	1070
Uranium-α	0·68	~2000
Uranium-β	1·80	—
Vanadium	5·414	1370
Zinc	0·875	53
Zirconium	0·546	47

(see Roberts (1966) for most references) characteristic H_0 and T_c. In addition there have been found by many investigators, in particular by Matthias and co-workers, by Alekseevskii and co-workers, and by Zhdanov and Zhuravlev (see Matthias, 1957; Roberts, 1966), a very large number of alloys and compounds which also become superconducting. Some of these compounds consist of metals, only one of which by itself becomes superconducting, some have constituents of which neither by itself is superconducting, and some even are semiconductors. The possibility of superconductivity in semiconductors and semimetals has been discussed by M. L. Cohen (1964), and both GeTe (Hein *et al.*, 1964) and $SrTiO_3$ (Schooley *et al.*, 1964, Ambler *et al.*, 1966) have been found to be superconducting at very low temperatures.

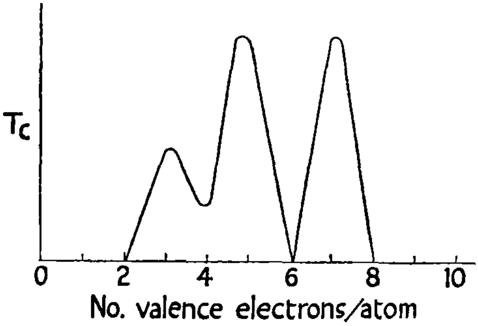


FIG. 2

The critical temperature of known superconductors range from very low values up to 20·05°K for a solid solution between Nb_3Al and Nb_3Ge (Matthias *et al.*, 1967). Matthias (1957) has pointed out a number of regularities in the appearance of superconductivity and in the values of T_c, the principal of which are the following:

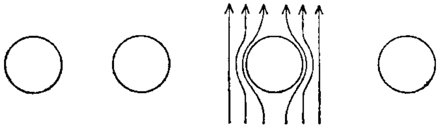
(1) Superconductivity has been observed only for metallic substances for which the number of valence electrons Z lies between about 2 and 8.

(2) In all cases involving transition metals, the variation of T_c with the number of valence electrons shows sharp maxima for $Z = 3$, 5, and 7, as shown in Figure 2.

(3) For a given value of Z, certain crystal structures seem more favourable than others, and in addition T_c increases with a high power of the atomic volume and inversely as the atomic mass.

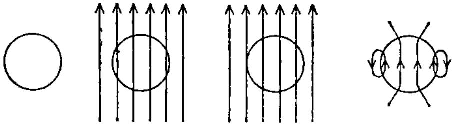
1.3. The Meissner effect, and the reversibility of the S.C. transition

If a perfect conductor were placed in an external magnetic field, no magnetic flux could penetrate the specimen. Induced surface currents would maintain the internal flux, and would persist indefinitely. By the same token, if a *normal* conductor were in an external field before it became perfectly conducting, the internal flux would be locked in by induced persistent currents even if the external field were removed. Because of this, the transition of a merely perfectly conducting specimen from the normal to the superconducting state would not be reversible, and the final state of the specimen would depend on the path of the transition.



$A: H_e = 0$, $T > T_c$. (*a*)
$B: H_e = 0$, $T < T_c$. (*b*)
$C: 0 < H_e < H_c$, $T < T_c$. (*c*)
$B: H_e = 0$, $T < T_c$. (*d*)

FIG. 3



$A: H_e = 0$, $T > T_c$. (*a*)
$D: H_e > 0$, $T > T_c$. (*b*)
$C: 0 < H_e < H_c$, $T < T_c$ (*c*)
$B: H_e = 0$, $T < T_c$. (*d*)

FIG. 4

As an example, Figures 3 and 4 show the flux configuration for a perfectly conducting sphere taken from point A in Figure 1 to point C