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Jacob Bear is one of the world's foremost hydrologists. Presently, he has 67 publications to his credit and is Professor of Hydrology and Deputy Vice President at the Technion-Israel Institute of Technology in Haifa, where he has also been head of both the Hydraulics Laboratory and the Water Resources Research Center. In addition to his years at the Technion, Dr. Bear has taught and developed courses in hydrology at the Massachusetts Institute of Technology, the University of California at Berkeley, the University of Wisconsin, Princeton University, and New Mexico Institute of Mines and Technology. He has also been a planning engineer, a hydrologist, and a consultant on hydrology to Water Planning for Israel Ltd., the Ministry of Agriculture of Israel, and, most recently, Yalon-Balasha, consultants and engineers on large scale development projects in Iran and the Benue Plateau in Nigeria.

A native of Israel, Dr. Bear graduated with honors from the Technion in 1953, where he also received his Dipl. Eng. and M.Sc. degrees in 1954 and 1957 respectively. In 1960 he obtained his Ph.D at the University of California at Berkeley. His studies abroad include scholarships at the Hague and the University of California at Berkeley, and a three-month U.N. scholarship for a study tour in the Netherlands, France and the U.S.A.

Active in many related fields, Dr. Bear's memberships include the Israel Association of Geodesy and Geophysics, the International Association for Hydraulic Research, the National Committee of the Hydrological Decade, the editorial board of the *Israel Journal of Earth Sciences,* and Israel's Water Planning Committee. He has been chairman of the Hydrology Section of Israel Association of Geodesy and Geophysics, the National Committee of the Hydrological Decade, and the Section of Flow through Porous Media of the International Association for Hydraulic Research. A speaker at numerous international conferences, Dr. Bear's lectures include discussions on intercepting fresh water above the interface in a coastal aquifer, the optimal yield of an aquifer, hydrodynamic dispersion in porous media, immiscible displacement in fractured porous media, and hydrologic education in Israel.

Preface

This book is an attempt to present, in an ordered manner, the theory of dynamics (actually, also of statics) of fluids in porous media, as applicable to many disciplines of science and engineering. For some years I have taught courses on flow through porous media, and have treated this subject as a part of other courses, such as ground water hydrology, while at the Technion—Israel Institute of Technology, at M.I.T. where I spent my sabbatical leave (1966–7), and at several other institutions. I have felt the lack of a suitable textbook on this subject. Ideally, such a text should start from first principles of fluid mechanics and mechanics of continua, should show the passage from the microscopic to the macroscopic level of treatment, should emphasize the special features of porous media, establish the macroscopic theory and then show how it is applied to cases of practical interest.

It is rather surprising that in spite of its importance in many fields of practical interest, such as petroleum engineering, ground water hydrology, agricultural engineering and soil mechanics, so small number of treatises is available on fluids in porous media. This circumstance is even more surprising in view of the vast amount of literature published on the subject in a number of scientific and engineering journals. Although dynamics of fluids in porous media could become an interesting interdisciplinary course serving several departments, I believe that the relatively small number of courses offered by universities on the subject is due in part to lack of a suitable textbook. To overcome this lack I prepared notes for my own classes, which I present here in the form of a book, hoping that it will serve others in a similar situation.

The book is designed primarily for advanced undergraduate students and for graduates in fields such as ground water hydrology, soil mechanics, soil physics, drainage and irrigation engineering, sanitary engineering, petroleum engineering and chemical engineering, where flow through porous media plays a fundamental role. The book, I hope, will also serve the needs of scientists and engineers already active in these fields, who require a sound theoretical basis for their work. The emphasis in this book is on understanding the microscopic phenomena occurring in porous media and on their macroscopic description. The reader is led to grasp the meanings of the various parameters and coefficients appearing in the macroscopic descriptions of problems of flow through porous media, and their actual determination, as well as the limitations and approximations inherent in their description. In each case, the objective is to achieve a clear formulation of the flow problem considered and a complete mathematical statement of it in terms of partial differential equations and a set of initial and boundary conditions. Once a flow problem is stated properly in mathematical terms, three methods of solution are possible in principle:- analytic solution, numerical solution aided by high speed digital computers and solution by means of laboratory models and analogs. All three tools are described in this book. Typical examples of analytic solutions are scattered throughout the book, but no attempt is made to present a collection of a large number of solved problems. The principles of the numerical method of solution are presented, and a detailed description is given of laboratory models and analogs, their scaling and applications.

Mathematics is employed extensively and the reader is expected to have a good background in advanced engineering mathematics, including such subjects as vector analysis, Cartesian tensor analysis, partial differential equations and elements of the theory of functions.

No attempt is made to give a complete citation of all published literature or to indicate the first author on a particular subject. References selected for citation are those I think represent a more important point of view, are more appropriate from the educational point of view or are more readily available for the average reader.

Obviously a single book, even of this size, cannot include everything related to the subject treated. Although we consider porous media in general, the discussion is limited to media with relatively large pores, thus excluding clays and media with micropores or colloidal-size particles. Similarly, chemical and electrochemical surface phenomena are excluded. The discussion is restricted to Newtonian fluids.

With these objectives and limitations in mind, the book starts with examples of two important porous media: the ground water aquifer and the oil reservoir. An attempt is made to define porous media, and the continuum approach is introduced as a tool for treating phenomena in porous media. This requires the definition of a "representative elementary volume" based on the definition of porosity. Chapter 2 includes a summary of some important fluid and porous media properties. In chapter 3, the concepts of pressure and piezometric head are introduced. Chapter 4 starts with the definition of velocities and fluxes in a fluid continuum. Then the equations of conservation of mass, momentum and energy in a fluid continuum are presented, and using a porous medium conceptual model these equations are averaged to obtain the basic equations that describe flow through porous media: the equations of volume and mass conservation, including the equation of mass conservation of a species in solution (also called the equation of hydrodynamic dispersion), and the motion equation for the general case of an anisotropic medium and inhomogeneous fluid. Although the basic equations of motion and of mass conservation are developed from first principles in chapter 4, chapters 5 and 6 return to these topics, discussing them from a different point of view, perhaps more suitable for the reader who is less versed in fluid mechanics. Chapter 5 presents the equation of motion, starting from its original one-dimensional form (as suggested by Darcy on the basis of experiments), and extending it to three-dimensional flow, compressible fluids and anisotropic media. This chapter also contains a review of theoretical derivations of Darcy's law. My objectives in presenting this and similar reviews is to indicate research methods, such as the use of conceptual and statistical models. A section on the motion equation at high Reynolds numbers is also included.

In chapter 6, the control volume approach is introduced as a general tool for developing mass conservation equations. Special attention is devoted to deformable media. Also included in this chapter is the stream function and its relationship to the piezometric head. Once the continuity or mass conservation equations have been established, the next natural step is to consider the initial and boundary conditions. These are discussed in detail in chapter 7. Special attention is given to the phreatic surface boundary condition and to its description in the hodograph plane. The second part of this chapter contains a discussion on various analytic and numerical solution techniques.

Upon reaching this point, the reader should be able to state a problem of flow through porous media in terms of an appropriate partial differential equation and a set of initial and boundary conditions. He should also know the major methods of solution (analog solutions are discussed in chapter 11).

Chapter 8 deals with the problem of flow in unconfined aquifers. This is a problem often encountered in ground water hydrology and in drainage. The Dupuit assumptions are explained and employed to derive the continuity equations for unconfined flow. The hodograph method, as a tool for solving two-dimensional, steady phreatic flow problems, is discussed in detail with many examples. Several linearization techniques and solutions of the nonlinear equation of unconfined flow are also presented in this chapter.

In chapter 9 the discussion, hitherto confined to single-phase flow, is extended to polyphase flow in porous media, a topic of special interest in petroleum engineering. Starting from the fundamental concepts of saturation, capillary pressure and relative permeability, the motion and continuity equations are established. The case of unsaturated flow as treated by soil physicists is presented as a special case of flow of immiscible fluids, where one of the fluids—the air—is stationary and at constant pressure. Special cases of interest, dealing with infiltration into soils, are considered in more detail. A new concept is introduced: that of an abrupt interface as an approximation replacing the actual transition zone that occurs between two fluids, whether miscible or immiscible. A detailed discussion is presented on the coastal interface, of great interest to ground water hydrologists.

Chapter 10 deals with hydrodynamic dispersion. Again, although the fundamental equation is developed from first principles in chapter 4, a review of several other theories leading to this equation is presented. Special attention is given to the coefficient of dispersion and its relationship to matrix and flow characteristics. A section on heat and mass transfer completes the discussion on hydrodynamic dispersion.

Chapter 11 presents the use of models and analogs, both as research tools and as tools for solving boundary value problems. Following the presentation of a general method for deriving analog scales, a detailed description is given of the sand box model, the electric analogs of various types, the Hele–Shaw analogs and the membrane analog. Recommendations for application are indicated in each case.

In brief, this is the subject matter I have chosen to cover in this book. I have made an effort to present the information in such a way as to require a minimum of supplementary material, except for those who wish to dig more deeply into the subject. A large number of problems and exercises is included in this book.

I should like to express my appreciation to the many individuals who, through their comments and criticism, have contributed to the completion of this book. Special thanks are due to Dr. Y. Bachmat, Dr. C. Braester, Mr. E. A. Hefez and E. Goldshlager, for the help they have given me in reading, discussing and constructively criticizing the draft. Thanks are also due to the Department of Civil Engineering at M.I.T., and especially to Professor C.L. Miller, head of the department, Professor A. T. Ippen and Professor D. R. F. Harleman, who made it possible for me to write a large part of this book while spending a most fruitful year as a visiting professor at M.I.T.

The heaviest burden involved in writing this book was borne by my wife, Siona, who had to put up with the many inconveniences that are unavoidable when one is engaged in writing a book. For her constant encouragement to me throughout the various stages of writing, my hearty gratitude.

I realize that an attempt to represent a systematic account of a theory, such as I have made here, is bound to have defects. I will accept with gratitude all readers' suggestions directed toward the improvement of this book.

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