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Cooperative Learning in Mathematics

A HANDBOOK FOR TEACHERS

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Introduction and Overview

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Small-group cooperative learning provides an alternative to both traditional whole-class expository instruction and individual instruction systems. The procedures described in this volume are realistic, practical strategies for using small groups in mathematics teaching and learning. These methods can be applied with all age levels of students, all levels of the mathematics curriculum from elementary school through graduate school, and all major topic areas in mathematics.

Systematic and frequent use of small-group procedures has a profound positive impact upon the classroom climate; the classroom becomes a community of learners, actively working together in small groups to enhance each person's mathematical knowledge, proficiency, and enjoyment. Frequent use of small groups also has an enlivening and invigorating impact on the professional lives of mathematics teachers.

To avoid confusion, I would like to clarify a point of terminology. During the late 1960s, the pioneering workers in this field tended to use terms such as *small-group learning* or *small-group teaching*. In the 1980s the term *cooperative learning* became more prevalent. Cooperative learning involves more than just putting students together in small groups and giving them a task. It also involves very careful thought and attention to various aspects of the group process, as will be explained. We have chosen the name *cooperative learning* for this volume. However, at times we will talk about "small-group learning," "small-group teaching," or "group work." No distinction in meaning is intended.

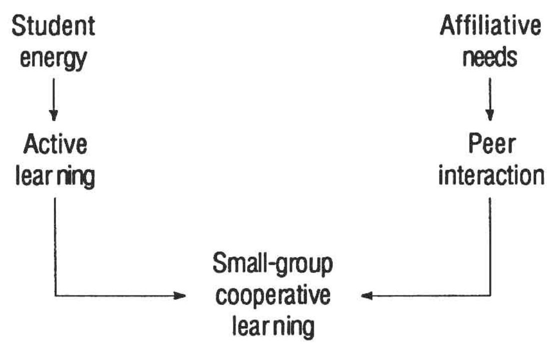
This handbook is the first comprehensive work devoted to small-group cooperative learning methods in mathematics. It is designed for all those who wish to expand their repertoire of available instructional strategies in mathematics: classroom teachers, mathematics professors, teacher educators, mathematics supervisors, staff developers, curriculum specialists, and researchers.

This volume is truly a cooperative and interdisciplinary effort. A variety of perspectives on small-group cooperative learning in mathematics are provided by a set of expert practitioners, staff developers, and researchers. The set of authors includes professionals who work with a diversity of age levels and who assume varying responsibilities. Whereas the majority of the authors are mathematics educators or classroom teachers, there are representatives from such fields as pure mathematics, teacher education, special education, staff development, educational research, social psychology, and anthropology. Several of the authors have 15 to 20 years of experience with cooperative learning.

This introduction has been designed to set the stage for the viewpoints that follow. It provides a rationale for the use of cooperative learning, a brief description of cooperative learning procedures in mathematics, a quick overview of research outcomes, and a set of key questions to consider when reading chapters 1-12.

Rationale

Since authors of various chapters also address this topic, the case for cooperative learning methods is stated only briefly here. Young people have tremendous energy, yet school learning situations often require students to sit quietly and listen passively. The teacher must then exert strong control to keep the students quiet and on the task at hand; this takes an inordinate amount of time away from instruction and learning. Instead, why not mobilize students' energy levels by engaging them actively in the learning process? Moreover, human beings have strong affiliative needs for contact and communication with others. Indeed, many students are motivated to come to school in order to be with their friends; they have a strong need to be accepted, to belong, and sometimes to influence others (Glasser, 1986). Yet school "discipline" is often designed to prevent students from talking to one another in class. In contrast, by setting up learning situations that foster peer interactions, the teacher meets a basic human need for affiliation and uses the peer group as a constructive force to enhance academic learning. How can active engagement in learning be combined with peer interaction? By letting students work together in small cooperative groups. This argument is summarized in the following diagram.



As demonstrated above, cooperative learning makes use of basic characteristics of human nature. Hence, it is not likely to be just another passing fad on the educational scene.

Cooperative Learning in Mathematics Instruction

Why does cooperative learning have a place in mathematics instruction? The learning of mathematics is often viewed as an isolated, individualistic matter. One sits alone with paper, pencil, and perhaps calculator or computer and struggles to understand the material or solve the assigned problems. This process can often be lonely and frustrating. Perhaps it is not surprising that many students and adults are afraid of mathematics. In contemporary language, they are troubled by math avoidance or math anxiety. They often believe that only a few talented individuals can compete successfully in the mathematical realm, whereas most of humanity is fit only for a life of mathematical mediocrity or incompetence.

Small-group cooperative learning addresses these problems in several ways.

1. Small groups provide a social support mechanism for the learning of mathematics. Students have a chance to exchange ideas, to ask questions freely, to explain to one another, to clarify ideas and concepts, to help one another understand the ideas in a meaningful way, and to express feelings about their learning. This is part of the social dimension of learning mathematics.

2. Small-group learning offers opportunities for success for all students in mathematics (and in general). Students within groups are not competing one against another to solve problems. The group interaction is designed to help all members learn the concepts and problem-solving strategies.

3. Unlike many other types of problems in life, school mathematics problems can actually be solved in reasonable lengths of time, such as a class period. Moreover, mathematics problems are ideally suited for group discussion in that they have solutions that can be objectively demonstrated. Students can persuade one another by the logic of their arguments.

4. Mathematics problems can often be solved by several different approaches. Students in groups can discuss the merits of different proposed solutions and perhaps learn several strategies for solving the same problem.

5. Students in groups can help one another master basic facts and necessary computational procedures. These can often be dealt with in the context of the more exciting aspects of mathematics learning through games, puzzles, or discussion of meaningful problems.

6. The field of mathematics is filled with exciting and challenging ideas that merit discussion. One learns by talking, listening, explaining, and thinking with others, as well as by oneself. Buck (1962, p. 563) puts it this way:

   > *Let me remind you that* student-student *interactions are also important in learning, and that at the professional level, much*

> *mathematical research springs from discussions* between *mathematicians. Moreover, a test of understanding is often the ability to communicate to others; and this act itself is often the final and most crucial step in the learning process.*

7. The role of small groups in mathematical communication is addressed in the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (1989):

> *Teachers foster communication in mathematics by asking questions or posing problem situations that actively engage students. Small-group work, large-group discussions, and presentation of individual and group reports—both written and oral—provide an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies. Small groups provide a forum for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarizing discoveries in writing. Whole-class discussions enable students to pool and evaluate ideas; they provide opportunities for recording data, sharing solution strategies, summarizing collected data, inventing notations, hypothesizing, and constructing simple arguments.*

8. Mathematics offers many opportunities for creative thinking, for exploring open-ended situations, for making conjectures and testing them with data, for posing intriguing problems, and for solving nonroutine problems. Students in groups can often handle challenging situations that are well beyond the capabilities of individuals at that developmental stage. Individuals attempting to explore those same situations often make little progress and experience severe and unnecessary frustration.

There are many additional reasons for using cooperative learning both in general and in the field of mathematics. Further elaboration is presented by the various authors in this volume.

Classroom Procedures

The following brief discussion of instructional procedures is expanded upon in later chapters. A class period might begin with a meeting of the entire class to provide an overall perspective. This may include a teacher presentation of new material, class discussion, posing problems or questions for investigation, and clarifying directions for the group activities.

The class is then divided into small groups, usually with four members each. Each group has its own working space, which might include a flipchart or section of the chalkboard. Students work together cooperatively in each group to discuss mathematical ideas, solve problems, look for patterns and relationships in sets of data, make and test conjectures, and so on. Students actively exchange ideas with one another and help each other learn the material. The teacher takes an active role, circulating from group to group, providing assistance and encouragement, and asking thought-provoking questions as needed.

In each type of small-group learning, there are a number of leadership and management functions that must be performed. These are generally handled by the teacher, although some of them may be explicitly delegated to the students. The list of functions includes:

- Initiate group work
- Present guidelines for small-group operation
- Foster group norms of cooperation and mutual helpfulness
- Form groups
- Prepare and introduce new material in some form: orally to the entire class; orally to separate groups; via written materials—worksheets, activity packages, text materials, and special texts designed for groups
- Interact with small groups in various possible ways: observe groups, check solutions, give hints, clarify notation, ask and sometimes answer questions, give specific feedback, point out

errors, provide encouragement, reinforce social or group skills, help groups function, furnish overall classroom management

- Tie ideas together
- Make assignments of homework or in-class work
- Evaluate student performance

Each of these functions can be performed in various ways and to varying degrees, depending upon the model of small-group instruction in effect. Additional ideas, advocated by some of the authors, are as follows: structured positive interdependence, equal status interaction, assigned social roles, explicit processing of academic and social skills, perspective taking, and team building.

Research Outcomes

The outcomes of cooperative learning methods have generally been quite favorable. Reviews of research have been presented by Sharan (1980), Slavin (1980, 1983a, 1983b), and the Johnsons (1974, 1981, 1983). Reviews by Davidson (1985; 1989), and by Webb (1985; 1989) specifically address cooperative learning in mathematics.

Research has shown positive effects of cooperative learning in the following areas, which are described in more detail in later chapters:

- Academic achievement
- Self-esteem or self-confidence as a learner
- Intergroup relations, including cross-race and cross-cultural friendships
- Social acceptance of mainstreamed children
- Ability to use social skills (if these are taught)

Davidson (1989) reviewed more than 70 studies in mathematics comparing student achievement in cooperative learning versus whole-class traditional instruction. In more than forty percent of these studies, students in the small-group approaches significantly outscored the control students on individual mathematical performance measures. In only two studies did the control students perform better, and both these studies had design irregularities. This evidence might be reassuring to teachers who are concerned about the potential effects of cooperative learning methods on their students' achievement in mathematics.

The effects of cooperative learning of mathematical skills were consistently positive when there was a combination of individual accountability and some form of team recognition for commendable team achievement. The effects of small-group learning were nonnegative (that is, not significantly different from traditional instruction) if the teacher had no prior experience in small-group learning, was not aware of well-established methods, and did very little to foster group cooperation or interdependence.

Defining Characteristics

What are the defining characteristics (critical attributes) of small-group cooperative learning in mathematics? Our definition includes the following characteristics:

1. A mathematical task for group discussion and resolution (if possible)
2. Face-to-face interaction in small groups
3. An atmosphere of cooperation and mutual helpfulness within each group
4. Individual accountability

The authors represented in this book would generally agree on these first four points. However, certain authors would advocate including one or more of the following points:

5. Heterogeneous or random grouping
6. Explicit teaching of social skills
7. Structured mutual interdependence

To illustrate the range of differences among the authors, we have included a brief discussion of the controversial points 5, 6, and 7.

Point 5. There is a debate about the requirement of heterogeneous or random grouping. Most of the authors advocate teacher-selected heterogeneous groups based on mathematical performance, race/ethnicity, and gender. However, some authors advocate random grouping; others prefer student choice of group members. Most would agree that homogeneous groups consisting of all slow learners or all high achievers do not work well for either range of performance.

Point 6. Similarly, there is debate about the teaching of social skills: Should these skills be explicitly modeled, practiced, and discussed? Does this depend on the extent to which students are already well versed in social skills?

Point 7. Finally, there is controversy about the degree of structuring mutual interdependence. How much interdependence is necessary for a group activity to be considered truly cooperative? Perhaps there is a continuum involving greater or lesser degrees of interdependence. At one extreme are tasks that require input from all members—if one person withholds information, the task cannot be completed. At the opposite extreme are tasks that can be resolved by individuals—for example, individuals solving the same exercises and comparing results with their group members. Interdependence (called "positive interdependence" in the literature) can be structured in several ways, which are described in later chapters.

Key Questions

To assist the reader, we have identified a set of key questions to keep in mind while examining the various chapters. These are the questions most frequently asked by teachers interested in implementing small-group cooperative learning. Each author addresses many (but not all) of the questions from his or her own perspective. While there are many points of commonality, there are also differences, depending on the authors' viewpoints, on such variables as:

- The age level of the students
- The type of small-group process

- The mathematical goals of the model
- The beliefs about the nature of mathematics
- The philosophical basis for small-group learning
- The importance of nonmathematical goals, such as learning communication skills and fostering intergroup relations

We shall present two versions of the set of key questions. The first is a brief set of basic questions, suitable for an initial reading of the chapters. The second list is greatly expanded; it is designed to facilitate an in-depth study of the chapters.

Basic Set of Key Questions

1. What is the rationale for small-group cooperative learning in mathematics?
2. How does a teacher begin group work in mathematics?
3. What factors motivate students to learn and explore in small groups?
4. How are groups formed, and how frequently should group membership be changed?
5. How does one foster cooperative behavior among students?
6. What are appropriate leadership styles for the teacher?
7. What types of mathematical activities are most appropriate for small-group learning?
8. How frequently do group activities occur?
9. How are students held accountable and graded?
10. What management issues does group work raise for teachers, and how can these be handled?
11. What types of physical room arrangements are used with small groups?

12. How can group work be used in combination with other instructional methods?
13. How is group work adjusted to meet the needs of different types of students?
14. What are the outcomes of cooperative small-group learning in mathematics?
15. What do students and teachers perceive as the strengths and limitations of cooperative learning in mathematics?

Expanded Set of Key Questions

These key questions are arranged in categories to facilitate in-depth study.

Rationale

- What is the rationale for the use of cooperative learning in general?
- What is the specific rationale for small-group cooperative learning in mathematics?
- Are there different reasons for using different models of group work?

Beginning

- How does a teacher begin group work in mathematics?
- Does a teacher ease slowly into group work or attempt to implement it quickly on a full-scale level?
- How can initial student or teacher uncertainty about group work be handled?
- Will there be a transition period during which groups have not yet learned to function effectively? What can be done to ease this transition?