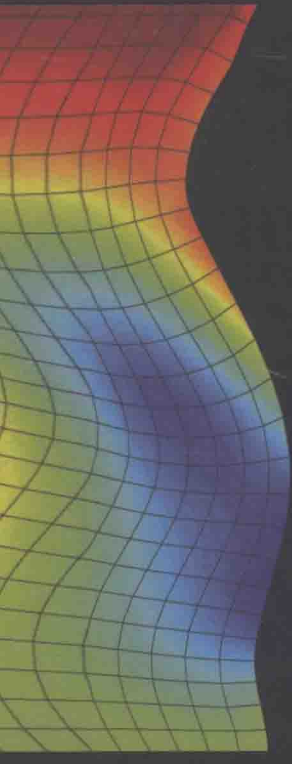
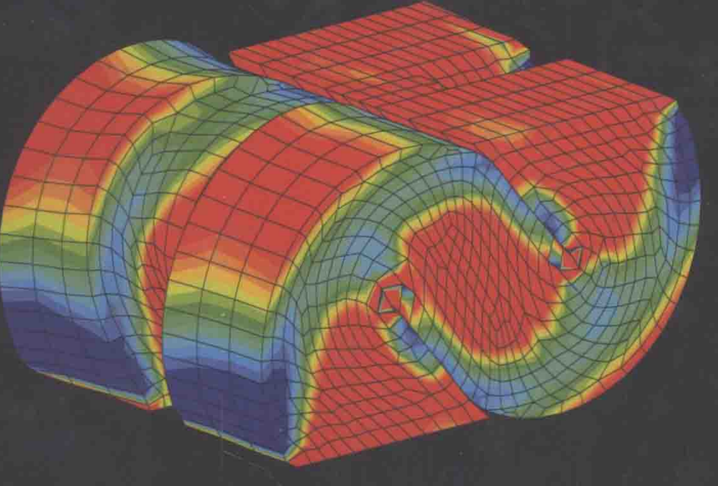


Constitutive Models for Rubber IV

Edited by P.-E. Austrell & L. Kari



PROCEEDINGS OF THE 4TH EUROPEAN CONFERENCE FOR CONSTITUTIVE MODELS FOR RUBBER, ECCMR 2005, STOCKHOLM, SWEDEN, 27–29 JUNE 2005

Constitutive Models for Rubber IV

Edited by

Per-Erik Austrell
Division of Structural Mechanics, Lund Institute of Technology, Lund University, Lund, Sweden

Leif Kari
The Marcus Wallenberg Laboratory for Sound and Vibration Research, Royal Institute of Technology, Stockholm, Sweden

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CONSTITUTIVE MODELS FOR RUBBER IV

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Foreword

These are the Proceedings of the Fourth Conference on Constitutive Modelling of Rubber, ECCMR: the European Conference on Constitutive Models for Rubber. This biennial conference is hosted in 2005 in Stockholm, Sweden, with an international array of delegates. It is with pleasure that we have seen the conference flourish since its inauguration in 1999, attracting an increasing number of scientists who are keen to expand their knowledge on the intriguing subject of modelling rubber behaviour.

Whereas the last conference in London in 2003 placed an emphasis on dynamic properties and the Fletcher-Gent effect, this year's conference focuses on fracture and fatigue. Fatigue is an important and highly relevant topic in the industrial context and a challenge to the scientific community. It is crucial to be able accurately to predict rubber fatigue, especially, for example, in the automotive industry. Much progress has recently been made in this field, and this is demonstrated in this collection of papers. The second largest topic in these Proceedings is the established topic of hyperelastic modelling which continues to engage many scientists.

The editors would like to express their special gratitude to James Busfield and Alan Muhr who organised the very successful London conference two years ago. Their help and assistance, their willingness to share their experience and access to valuable documentation have greatly facilitated our work.

The review process was performed by the members of the scientific committee. The editors would like to thank: K. Akutagawa, D. Besdo, R.de Borst, P. Buckley, J. Busfield, P. Charrier, A. Dorfmann, M. Enelund, A.N. Gent, G.A. Holzapfel, M. Kaliske, M. Klüppel, W.V. Mars, A. Muhr, R.W. Ogden, S. Reese, M. Ristinmaa, D. Steigmann, A.G. Thomas, E. Verron, A. Wineman, O.H. Yeoh and A. Zdunek.

Finally we would like to express our gratitude to the conference sponsors: Abaqus Inc., Bridgestone, Comsol Inc., Lund University, MSC-Software, MTS Systems Norden AB, Queen Mary College University of London, Royal Institute of Technology (KTH), Trelleborg AB, and Wenner-Gren Foundations.

Per-Erik Austrell
Leif Kari
April 2005

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Foreword (Volume 1)

The extraordinary stress–strain behaviour of rubber has presented an opportunity for inventive engineers and a challenge for scientists since the mid-nineteenth century, and continues to do so today. Major branches of theory, such as the statistical theory of rubber elasticity and finite strain elasticity theory, have been spawned by the properties of rubber. Until recently, however, the theoretical framework for large deformations found little application among rubber engineers because the mathematics rapidly becomes intractable for all but the simplest components. The advent of affordable and powerful computers has changed all this, and brought the challenge of rubber to new sets of people – software engineers and desk-top, as opposed to empirical, designers.

The development of the statistical theory of rubber elasticity in the 1940s, of finite strain elasticity theory in the 1950s, and of convenient forms for the strain energy function in the 1970s, all focused on modelling the elastic characteristics of rubber. Although much literature has appeared in recent years following this theme, the Physics of Rubber Elasticity by L.R.G. Treloar (3rd Edition, Clarendon Press, Oxford, 1975) and the proceedings of a Discussion on Rubber Elasticity (Proc.Roy.Soc.London, 1976, A351, No. 1666, 295–406) remain very valuable reviews.

The treatment of rubber as a 'hyperelastic' material – that is, a material modelled by a strain-energy function for finite strains – was implemented into finite strain element analysis in the 1980s and is now widely available in commercial software packages.

However, only a few engineering elastomers – such as unfilled natural rubber and some grades of polyurethane – really conform to the "hyperelastic" ideal. Most other engineering elastomers incorporate "reinforcing" fillers, needed to confer adequate strength properties and also to improve processing characteristics and to enable adjustment of hardness over a wide range. The stress–strain characteristics of such filled elastomers depart significantly from elasticity. While ways of thinking about these departures – such as "dynamic-to-static ratio" of rubber springs – may have satisfied a previous generation of design engineers, there is now an opportunity to apply more sophisticated models.

One major current challenge is thus to model those aspects of the inelastic behaviour that are relevant to engineers, and to do this in such a way that the models are implementable in finite element analysis.

Although potentially the involvement of representatives of several disciplines should facilitate progress, this is only the case if they talk to each other. In practice, software engineers might rely on the literature and on desk-top designers as sources of information about rubber, and fail to achieve as good a balance of understanding as they could if they listened also to experimental rubber scientists and empirical designers. Applied mathematicians might develop phenomenological models which address issues of secondary interest to designers, or which misrepresent important aspects of the experimentally observed behaviour. Experimentalists might develop models without reference to the existing framework of continuum mechanics, resulting in internal inconsistencies and difficulty in implementation in software packages. The First European Conference on Constitutive Models for Rubber sprang from the idea of providing a forum for multi-disciplinary discussion, seeking to bring the fragmented strands of recent research together.

Within the UK a start has been made in this direction – through a workshop on Deformation Modelling for Solid Polymers (Oxford University, 1997) and a seminar on Finite Element Analysis of Elastomers (Institution of Mechanical Engineers, London, 1997). The proceedings of the latter are available as a publication of the same name (Professional Engineering Publications, London, 1999). Similarly, in Germany a workshop of Finite Element Analysis – Basics and Future Trends was organised by the Deutsche Institute für Kautschuk Technology (Hannover, 1998). The interest in these essentially national meetings suggested that further cross-fertilisation should be stimulated by providing a European forum for discussion.

The contributions to this Proceedings cover a wide range of subjects. Consistent with the analysis given above, relatively few authors chose to present hyperelastic models for rubber; however, readers interested in this topic will find ample references to earlier work. Several contributions address inelastic effects associated with filled elastomers – such as Mullins' effect and quasi-static hysteresis. For others – most obviously in processing uncured rubber – the interest is in modelling viscoelasticity. In addition to stress–strain behaviour, work is presented on frictional contact and on mechanical failure. Looking at the applications side, computational techniques are addressed and applied to a diverse range of components, including tyres, earthquake isolation bearings and intervertebral discs. Overall, the authors have achieved progress in a wide range of areas – including experimental results, theory and practical utility. They raise many questions as well, as one might expect from the first forum of this kind.

We would like to thank our colleagues on the Scientific Committee (R.W. Ogden, Chairman; D. Besdo, R.de Borst, K.N.G. Fuller, H.A. Mang, H. Menderez, G. Meschke and H. Rothert) and all the authors who have worked with us to produce this book.

Al Dorfmann
Alan H. Muhr
Vienna/Hertford, June 1999

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Prediction of fatigue crack initiation in rubber with the help of configurational mechanics

E. Verron
Institut de Recherche en Génie Civil et Mécanique (GeM)
École Centrale de Nantes, Nantes, France

ABSTRACT: The present paper is a first step towards the definition of a new multiaxial fracture criterion for rubber-like materials. Here, the end of life is defined by crack initiation. Motivated by recent experimental observations, it is assumed that elastomers contain a uniform distribution of voids. Then, in order to determine the energy release rate which opens voids, the framework of Mechanics in Physical Space is considered. More precisely, properties of the configurational stress tensor are thoroughly examined to derive the criterion. Predictions are compared with published experimental data and the efficiency of the theory is highlighted.

1 INTRODUCTION

Rubber parts are widely used in many applications such as tires, engine mounts, bumpers or shoes. For the development of new industrial products, both short- and long-term durabilities should be estimated. Depending on applications, crack nucleation or crack growth can be considered. As shown by Mars & Fatemi (2002), only few criteria are used for crack nucleation and their efficiency is questionable whereas crack growth approaches are well-established since the pioneering work of Rivlin & Thomas (1953).

The present paper focuses on the prediction of crack initiation in rubber for which most widely used criteria are the maximum principal stretch and the strain energy density. Nevertheless, none of them is applicable to multiaxial loading conditions (see Mars & Fatemi (2002) and the references herein). More recently, some authors consider that rubber contains intrinsic flaws which can be seen as small cracks that grow under cyclic loading conditions. It leads to the definition of the cracking energy density which represents the portion of the total energy that contributes to the growth of small cracks Mars (2002). For a given surface, it is defined as "the work performed by the tractions on the surface in deforming the surface". The aim of the present paper is to rationalize this approach using the theoretical framework of the Eshelbian mechanics.

In the next Section, principal works on the Eshelbian mechanics are recalled. Section 3 is devoted to the formulation of the new criterion. Then, some comparisons of the theory with published experimental data are presented in Section 4 to demonstrate the efficiency of the approach. Finally, concluding remarks are given in Section 5.

2 A BRIEF OVERVIEW OF CONFIGURATIONAL MECHANICS

The concept of configurational stress was first introduced by Eshelby (1951). The author studied forces that applied on defects and introduced the following "stress tensor"

$$\mathbf{\Sigma} = w\left(\mathrm{sym}\left(\nabla\mathbf{u}\right), \mathbf{x}\right)\mathbf{I} - \left(\nabla\mathbf{u}\right)^T \boldsymbol{\sigma}, \tag{1}$$

where w is the strain energy density which depends on both the symmetric part of the displacement gradient $\mathrm{sym}(\nabla\mathbf{u})$ and the position vector $\mathbf{x}$, and $\boldsymbol{\sigma}$ is the Cauchy stress tensor. In this equation, the superscript ^{T} stands for the transposition. The tensor Σ is classically referred as the configurational stress tensor. For a homogeneous body, i.e. without defects, and in absence of body forces, Σ satisfies a strict conservation law:

$$\mathrm{div}\,\mathbf{\Sigma} = \mathbf{0}. \tag{2}$$

Moreover, in the case of a body (or a part of a body) of volume V and boundary S which contains a defect, the force which acts on this defect is given by the integration of the configurational traction vector over S

$$\mathbf{G} = \oint_S \mathbf{\Sigma}\cdot\mathbf{N}\,dS, \tag{3}$$