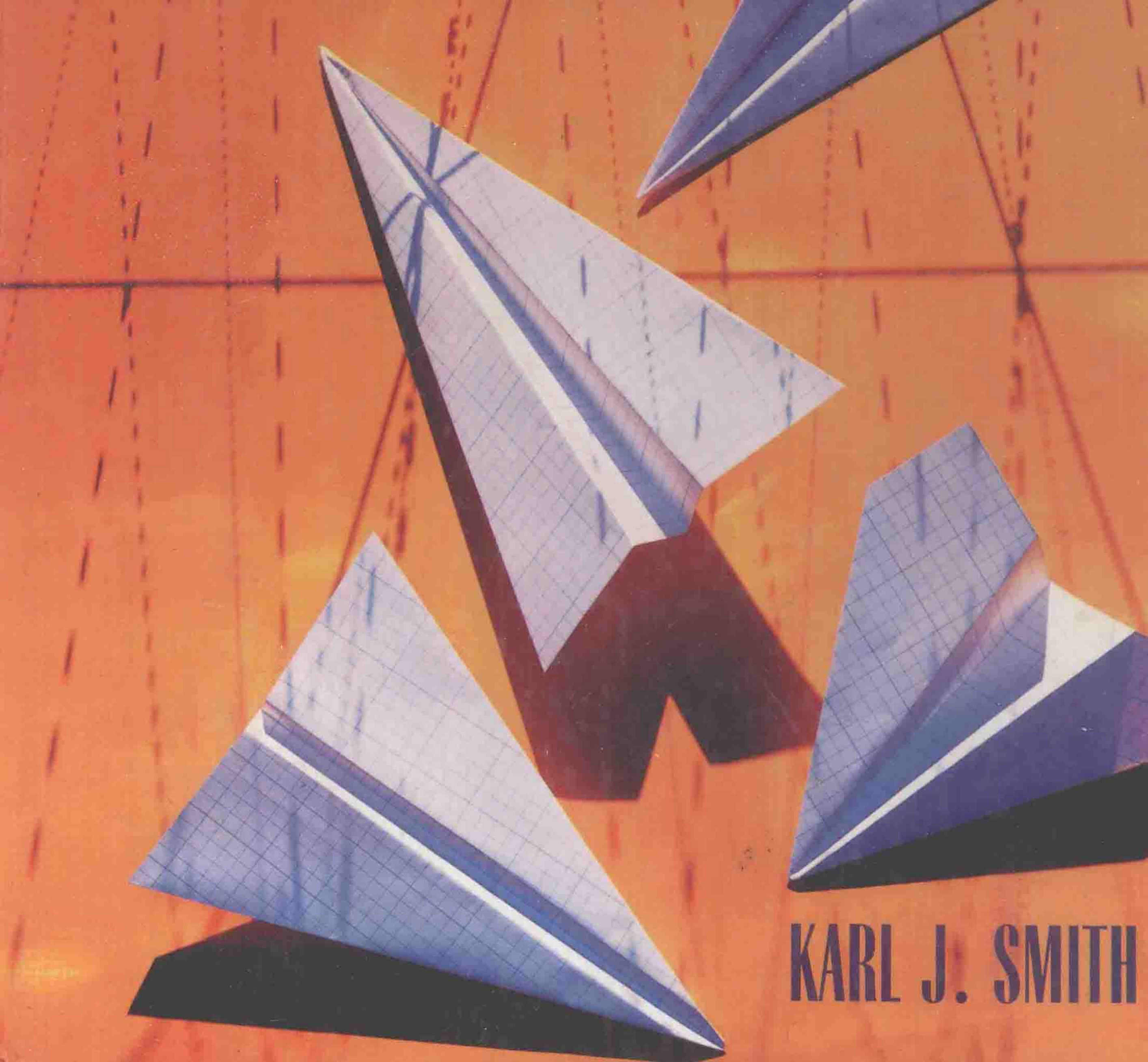
COLLEGE ALGEBRA

WITH GRAPHING AND PROBLEM SOLVING



KARL J. SMITH

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WITH GRAPHING AND PROBLEM SOLVING

Karl J. Smith, Ph.D

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Karl Smith has gained a reputation as a master teacher and guest lecturer. He has been a department chairperson and is past president of the American Mathematical Association of Two-Year Colleges. He has served on numerous boards, including the Conference Board of Mathematical Sciences (which serves as a liaison among the National Academy of Sciences, the federal government, and the mathematical community in the United States), the Council of Scientific Society Presidents, the Board of the California Mathematics Council for Community Colleges, and was invited to participate in the National Summit of Mathematics Education. He received his B.A. and M.A. from the University of California at Los Angeles, and his Ph.D. from Southeastern University. In his spare time he enjoys running and swimming.

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PREFACE

In the last decade a revolution in the teaching of mathematics has taken place. As President of the American Mathematical Association of Two-Year Colleges, I spoke to many people from schools all over the country, and I also had the opportunity to visit many campuses. As we talked about trends and changes in curriculum, several things became apparent to me:

- All students who earn a college degree should have at least one college-level mathematics course.
- Students should be exposed to the new technologies of the calculator, graphing calculator, and computer.
- Students should be given the opportunity for mathematical exposition, both individually and in small groups.
- The most common mathematics course required for general education is college algebra, but the content for this course is dictated by existing college algebra textbooks, and consequently many course outlines are not satisfactory.
    a. Instructors complained that too much time was spent on review.
    b. Instructors complained that students were not prepared for their courses, especially for those courses satisfying a graduation requirement.
    c. Instructors said they want a course that not only would prepare their students for more advanced study in mathematics, but at the same time would teach appropriate mathematics to students whose only exposure to mathematics is this one course.

You will find this to be a very different college algebra book; look at the Table of Contents. I developed the material to enable instructors to offer a *new* college algebra course within the existing curriculum framework, while at the same time adding a new and lively dimension to the course. I arranged the topics so that they would be consistent with the new technology available to most students—namely, the graphing calculator—even though it is not required that a student own such a calculator.

A major theme is mathematical modeling and problem solving. Chapter 3 introduces these topics, which not only include the "usual word problems" of algebra, but Chapter 3 also builds a procedure that can be used for problem solving in a more general context than the classroom. Since true problem solving, in the real world, is based on the principles of mathematical modeling, and in the spirit of the new NCTM *Standards*, you will find that I have presented both routine and nonroutine problems.

Just as I needed to rethink the content of the typical college algebra textbook, I believe the instructor needs to rethink what is taught. For example, many reviewers made the comment that they liked the chapter on combinatorics and probability, but that they do not have time for this topic. Lack of time and the fact that most people skip this important topic are the reasons it is included at the *end* of most college algebra books. However, if the course is to serve a general college population, one goal must be mathematical literacy. John Paulos, in his book *Innumeracy*, includes probability as one of the necessary ingredients of being able to function intelligently in our world today. Even though the chapter on probability still remains optional, I have placed probability midway through the book to emphasize its importance, and have included the binomial theorem in this chapter. In the past, I have found it difficult to motivate students into caring about expanding $(t + h)^4$, but in the context of combinatorics and probability, I am easily able to motivate this most important mathematical result.

Another example of rethinking what is taught in college algebra comes from the mathematics of finance. To function successfully in our society, it is important to understand how to handle finances, but the only place in mathematics we usually discuss annuities, sinking funds, and amortization is in *finite mathematics*, which is *not* a mainstream mathematics course. I believe one of the most important things I can do in college algebra is to give my students the ability to make intelligent financial decisions in their lives. For this reason, you will notice that the chapter on sequences and series includes the mathematics of finance. I have presented the formulas for annuities, sinking funds, and amortization in a nonthreatening, unified way *without* the uncomfortable specialized notation often associated with these topics.

"But where do I get time for all of these new topics?" you might ask. It is typical to start a college algebra course with a lengthy review of intermediate algebra. You will notice this book begins with polynomial functions. I believe that the necessary topics from intermediate algebra can be reviewed within the context of the development of college algebra topics. The benefit is additional teaching time with those topics that are properly included in the course.

A final example of rethinking the content of the course is to consider the role of complex numbers. It is typical to introduce complex numbers early so that they can be used in solving quadratic equations; they are then ignored for most of the rest of the course. In the real world, as well as in a first calculus course, the usual domain is the set of real numbers, and I believe it is far more realistic to also restrict the domain to be the set of real numbers. In this book, when I say that an answer does not exist it is to be understood that I mean it does not exist *in the set of real numbers*. I have included complex numbers in Appendix B and have used them only in the section on the Fundamental Theorem of Algebra. The use of complex numbers is optional in the section on solving polynomial equations, and I give answers in both the domain of real numbers and in the domain of complex numbers.

In April 1991, I attended the National Summit on Mathematics Assessment; the background paper for participants called for a curriculum in which "goals for student performance are shifted from a narrow focus on routine skills to the development of broad-based mathematical power." It is in this spirit of developing **broad-based mathematical power** that I have written this college algebra textbook. I have included several features which should help students understand the material:

- ***In Other Words*** boxes to put mathematical terminology into language easily understood by the student.
- ⊗ ***Hazard warnings*** to alert the student to common mistakes and pitfalls. ⊗
- ***Calculator Comments*** to use the new technology not only of scientific calculators, but also of the newer graphing and matrix calculators.
- ***What is wrong?*** problems to test understanding of common mistakes.
- ***Historical Notes*** and ***Profiles*** to show that mathematics is alive and the result of both history and current research.
- ***Group Research Projects*** and ***Individual Research Projects*** to encourage students to solve nonroutine problems by working in groups and individually. A 1992 report by the National Coalition of Girls' Schools states, "Cooperation and sharing is a much more female way to learn, rather than competing. . . . Frequently there's more than one way to solve a problem and putting several heads together gives people the kind of time they might need to solve the problem."* Students should be encouraged to communicate about mathematics and to describe quantitative situations in an expository paper. They should also be encouraged to do outside reading, so there is a suggested book report at the end of each chapter. The titles of the recommended books are also found in the Table of Contents. Some instructors have told me they are intimidated by the individual and group research projects. They say they have not read some of the recommended books, and add that some of the projects are too open-ended. I include open-ended projects in the spirit of the new teaching models that say the teacher does not need to be the one in front "imparting knowledge," but rather a participant and coach in the learning process. My advice to these instructors is to assign some of these readings and projects, and then just *stand back*. Whenever I have done this, I have been amazed at the quality and quantity of work some of my students will do—and sometimes from students who otherwise do not excel in a classroom setting.

A Note About Calculator Usage

Some reviewers were enthusiastic about this book, but said, "I could not use the book because only a small number of my students have a graphing calculator." In developing this book with my classes, I did not require that my students have a graphing calculator, and it was important to me to write a book that does *not* require that students have a graphing calculator. However, I *do* use a graphing calculator to enhance understanding of the exposition, and I encourage students to purchase a graphing calculator, if possible. To spend $60 for a tool that is so useful to their education is not an unreasonable expenditure. Since many will not have a graphing calculator, I use the

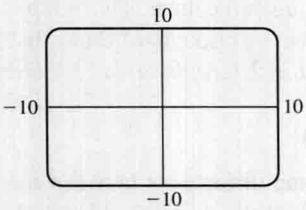
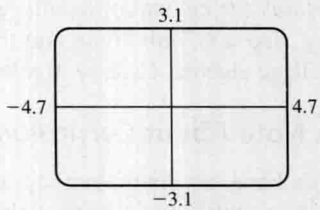
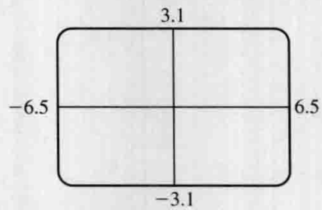
*Whitney Ransome, co-director of the National Coalition of Girls' Schools. The quotation is from a national study that explored sexual bias in schools, and was reported on March 7, 1992, at the annual meeting of the National Association of Independent Schools.

Calculator Comment boxes to show what would be seen if the example were worked on a graphing calculator. Therefore, students should look at those boxes even if they do not own a graphing calculator. In developing this material, I found that a calculator approach helped even those who do not own a graphing calculator.

A graph on a graphing calculator does not show the scale, so I have followed this convention with the calculator art in the book. However, note that the standard scale I have used for all calculator graphs (unless otherwise noted) is for $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. On the TI-81, TI-82, and TI-85, the [ZOOM] key refers to these x and y limitations as *standard*. If you have a calculator, you can see the actual coordinates by using the [TRACE] key. One of my major criticisms about the calculator-based mathematics material that I have seen published or presented at mathematics conferences is that the author of the material gets bogged down interpreting the precision of the calculator and the inputting of information into the calculator. Consequently, I lose track of the mathematics I'm trying to learn. Knowing about "pixel" size and how my calculator works does not enhance my *mathematical* understanding. Just as I am able to use my car to get to work without understanding the principles of the internal combustion engine, I believe we can use a calculator to advance our mathematical goals without understanding the details of "how" the calculations are accomplished. I do, however, believe it is important to discuss how to interpret calculator output. For example, we need to recognize 2.409 −28 as a number in scientific notation, 2.409×10^{-28}, which for most practical purposes is 0. Similarly, an output of 0.33333333 or 3.1415927 should be recognized as $\frac{1}{3}$ or π, respectively. Or, at a slightly higher level, if we are tracing a point on a curve and obtain successive coordinates (11.982187, −.00239876) and (12.0498731, .0498710), we need to discuss the appropriate mathematics to be able to conclude that the x-intercept is probably (12, 0). On the other hand, many real-world models do not have "nice" or rational intercepts and these problems are, from a practical standpoint, impossible to solve without the new technology. In such a case the [ZOOM] key can be used to approximate the intercept to any reasonable degree of accuracy. In other words, I believe the calculator to be an invaluable *tool* in understanding mathematics, but its use is not the ultimate goal of the material of this book.

The notation I introduce in the book is consistent with that used by graphing calculators. I have attempted to make this book as calculator-independent as possible, and for the most part do not include calculator keys showing which keys to press, but instead expect the student to know how to use his or her own calculator. Even though the technology changes faster than textbooks, I have included the table on the facing page, comparing the leading brands of graphing calculators at the time of this edition.

I also have resisted the temptation to insert a "calculator logo" on certain problems. Even though many problems unique to this book were designed with the calculator in mind, I believe the student should consider the calculator and the graphing calculator as useful *tools* for *any* problem. I did not design problems with "ugly answers" just to fit some artificial logo. On the other hand, the entire *organization* of the book was obviously influenced by the new technology, which once again seems to argue against using a logo for only some of the problems.

TEXT NOTATION	TI-81 OR TI-82 / TI-85	CASIO f_x SERIES	HP-48sx
Matrices			
[A]	[A]/[A]		[A]
RowSwap	RowSwap / rSwap		Mth MATRIX
Row+	Row+ / rAdd		*Will not do*
*Row	*Row / multR		*elementary row*
*Row+	*Row+ / mRAdd		*operations*
Graphing			
$y = 5x^3 + 4x^2 - 3x + 1$	Y= 5X^3+4X^2−3X+1 GRAPH	Mode 2 EXE 5X^3+4X^2−3X+1 SHIFT Cls EXE Mode 2 EXE SHIFT INS GRAPH Mode 1 Prog 0 EXE	5*X^3+4*X^2−3*X+1 ENTER SOLVE STEQ PLOT PLOTR ERASE DRAW
Default Window	ZOOM 6	RANGE SHIFT DEL RANGE	PLOT PLOTR NXT RESET
D: [−10, 10] R: [−10, 10]			
TRACE To display coordinates of a point	TRACE	SHIFT TRACE	COORD *Does not trace the curve, but floats freely*
ZOOM			
To see more detail:	ZOOM 2 ENTER	SHIFT +	ZOOM XY .5 ENTER
To see a larger picture:	ZOOM 3 ENTER	SHIFT −	ZOOM XY .2 ENTER
To set region to see: (i.e., define a box)	ZOOM 1	SHIFT ZOOM BOX	Z-BOX
To modify domain and range:	RANGE ENTER / WINDOW ENTER	Range	PLOT PLOTR
To set "square" window:	ZOOM 5 ENTER	*Not available*	PLOT PLOTR NXT RESET

And finally, a recommendation: If you are using a graphing calculator I strongly recommend that you cover Sections 5.2 (Parametric Equations) and 5.3 (Circle Function) since these sections are used extensively in Chapter 8 when graphing curves that are not functions.

A Note About the Title

I spent a great many hours trying to decide on an appropriate title for this book. I did not want to call it simply *College Algebra* because I believe it to be a departure from the existing college algebra books. I wanted to find a title that would communicate an approach that includes graphing calculators, but at the same time does not require them. I also wanted to communicate that the book is appropriate not only for those continuing in mathematics, but also for those needing a mathematics course to satisfy their general education requirements. One of the clearest presentations of algebra that I have ever read is a book called *Elements of Algebra* by Leonhard Euler, translated from the French notes by M. Bernoulli. The fifth edition of the book was printed in London in 1840. The book does not contain one graph, even though Euler has been called the Supreme Geometer. The use of a graph to enhance the development *is* new. Contemporary college algebra books *do* include a great many graphs, but they are still held to certain sections and are not used to *develop* an understanding of other algebraic topics. The availability of the graphing calculator has provided a unique opportunity to *rethink* the content of a college algebra course. This edition is a result of that rethinking, and it is with honor and tribute to Leonhard Euler that I call this development of college algebra, *College Algebra with Graphing and Problem Solving*.

A Note About Curriculum

You have, no doubt, noticed some differences between the Table of Contents of this book and the required curriculum at your school. What is the proper role of a textbook in curriculum development? Should books reflect new trends and technologies, or should they reflect existing classroom procedures and course requirements? Should new approaches be molded to fit old curriculum guides or should new curriculum guides be written to reflect new approaches?

For example, notice that I have included in Chapter 5 a section on parametric equations and a section on circle functions. You might say, "Why introduce trigonometry in a college algebra course? Certainly, you cannot hope to do justice to trigonometry with only two sections." I respond by saying that I am not introducing trigonometry in an algebra course; I am introducing two functions, defined by a unit circle—ideas that certainly reinforce the function concept. These nonalgebraic functions serve as an excellent source of ideas *about functions*, which strengthen algebraic understanding of the logarithmic and exponential functions, the conic sections, and inverse functions. Students taking this course will fit into one of two categories: either they will take trigonometry (or have taken it, or are taking it concurrently), or they will not take trigonometry. If they do not take trigonometry, then an exposure to the cosine and sine functions is a worthwhile one, since they will no doubt encounter these functions even if they never take another course. (At the very least, they will know the meaning of these keys on their calculators.) If they *do* take trigonometry, this introduction will strengthen their understanding of the cosine and sine *functions* of θ. I do not consider the introduction of these functions an advanced topic, and have found that when introduced as coordinates of a point on a unit circle, the sine and cosine are quite helpful in teaching the *notion* of inverse functions. I believe that the introduction provided in this book will greatly simplify the difficult topic of inverse functions when it is presented in a trigonometry course.

With the emerging technologies (whether you use graphing calculators or not), a topic of increasing importance is that of parametric equations. If you use this book, you should note that I introduce the idea of a parameter early (in Section 2.2) with the parametric form of the equation of a line. Parametric equations, in general, are introduced in Section 5.2, and are then used in Section 5.3 (circle functions), Chapter 8 (parametrization of the conic sections), and in Chapter 9 (solutions of dependent systems).

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