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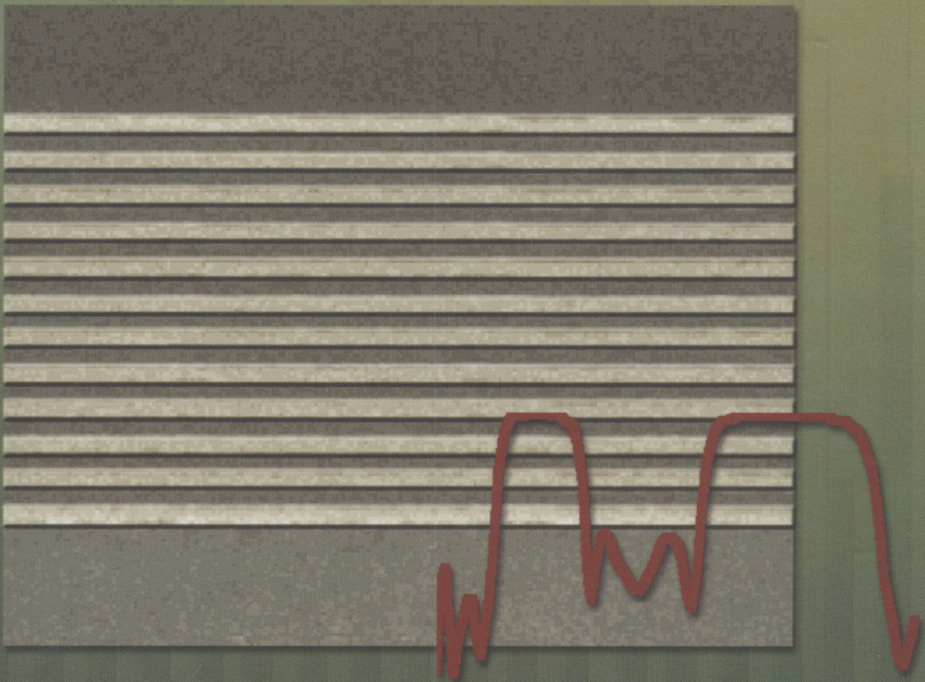
Modulated Thickness and Other Stopband Design Methods

Bruce E. Perilloux



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Introduction to the Series

Since its conception in 1989, the Tutorial Texts series has grown to more than 60 titles covering many diverse fields of science and engineering. When the series was started, the goal of the series was to provide a way to make the material presented in SPIE short courses available to those who could not attend, and to provide a reference text for those who could. Many of the texts in this series are generated from notes that were presented during these short courses. But as stand-alone documents, short course notes do not generally serve the student or reader well. Short course notes typically are developed on the assumption that supporting material will be presented verbally to complement the notes, which are generally written in summary form to highlight key technical topics and therefore are not intended as stand-alone documents. Additionally, the figures, tables, and other graphically formatted information accompanying the notes require the further explanation given during the instructor's lecture. Thus, by adding the appropriate detail presented during the lecture, the course material can be read and used independently in a tutorial fashion.

What separates the books in this series from other technical monographs and textbooks is the way in which the material is presented. To keep in line with the tutorial nature of the series, many of the topics presented in these texts are followed by detailed examples that further explain the concepts presented. Many pictures and illustrations are included with each text and, where appropriate, tabular reference data are also included.

The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, and micromachining. When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the processes and technologies under discussion.

It is my goal to maintain the style and quality of books in the series, and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

Arthur R. Weeks, Jr.
Invivo Research Inc. and University of Central Florida

Preface

This text is written for thin-film designers and students with advanced knowledge of multilayer, optical thin-film coatings. It focuses specifically on coatings that have high reflectance performance requirements in more than one spectral wavelength band or region. Many advanced optical systems that employ optical thin-film coatings rely on the performance attributes of multiple spectral bands. Several new analytical design methods that produce multiple stopbands as well as passbands are presented in this text. These analytical design methods produce discrete thin-film designs that will, in most cases, achieve specifications without any refinement of the design. If needed, the spectral performance of these analytical designs could be improved via common computer refinement algorithms. The theory of each design method in this text is presented along with design examples. Relatively basic exercises are provided for students as well as challenging ones for researchers.

This text does not attempt to cover the vast amount of material already published on thin films. The reader is expected to have a general knowledge of classical thin-film physics and designs. Several other texts cover the gamut of classical thin-film theory and designs, and additional information is readily available in several conference proceedings and many journal articles. Detailed derivations of Fresnel equations, complex refractive index, effective index, p- and s-polarization, phase, classical thin-film coating designs, etc., were intentionally omitted from this text. However, a brief introductory chapter on thin-film theory and design is included for completeness.

The coating designs that readers produce using the methods described in this text can be readily manufactured using common coating materials and process methods. In general, the layer thicknesses of these designs vary from a quarter-wave stack to a modulated profile. For some designs, process adjustments may be required for thin layers. However, thin-film manufacturing processes are not covered in this text since other texts and technical papers cover them.

The designs produced using the methods in this text can also be used as initial designs for computer refinement and synthesis algorithms. With some commercially available thin-film design software, some designs are produced with no starting design or specified layers; the algorithm adds layers of preselected materials until the desired spectral performance is achieved. Still, a good starting design helps to reduce the time or effort required to produce a final design that (1) achieves desired performance specifications; (2) is insensitive to layer thickness errors; and (3) can be manufactured. The design methods presented here are also expected to accomplish these tasks for some applications that require stopbands.

In general, the analytical design methods presented were developed using the following methodology. First, layer thicknesses of an arbitrary quarter-wave stack were modulated using various mathematical functions (e.g., sinusoidal). Next, a computer program was written to determine the existence of all stopbands produced from the modulated design. The resulting patterns of stopbands were evaluated graphically as functions of modulation parameters and spectral frequency.

Then, based on these graphical patterns of stopbands, analytical (linear) equations were tested by direct calculation of spectral performance to see if the stopbands could be reproduced analytically.

Several variations on the modulation of layer thickness are presented in this text, including an inhomogeneous rugate design. The last chapter presents a related, novel design method where one quarter-wave stack is linearly transformed into another. Here, empirical testing of layer thickness was used to develop general transform equations. A summary of each chapter and the appendixes follows:

Chapter 1 reviews the fundamental mathematics for thin-film design that applies to the proposed methods. Again, the objective here was to keep this chapter brief since this information can be found in many texts.

Chapter 2 introduces sinusoidal thickness modulation of quarter-wave stacks. First, stopbands and passbands are defined. Next, modulation parameters are assessed and many designs are evaluated. Then linear equations are determined that predict all possible stopbands. The last section evaluates the electric fields and the reflected differential phase shifts of some modulated designs.

Chapter 3 introduces discrete apodization of the modulated designs from Chapter 2. Two specific apodization functions are evaluated: amplitude modulation functions and Gaussian envelope functions. Linear equations are again determined that predict all possible stopbands.

Chapter 4 describes two variations of the modulation scheme from Chapter 3. First, chirped-modulation designs are evaluated for spectral performance. Next, a half-modulation is discussed where every other layer of a quarter-wave stack is modulated. Both of these methods are applied to dispersion-controlled mirrors used to produce femtosecond laser pulses. Two design examples and the limitations of these modulation schemes are covered.

Chapter 5 presents a novel, linear transform method that can be used to partially transform a given quarter-wave stack into a second quarter-wave stack. This transformation is accomplished by adjusting the individual layer thickness while the total thickness of the original quarter-wave stack remains constant. General transform equations are developed by direct numerical testing of the transform method. The purpose of this transform is to obtain, or achieve, some of the spectral properties of both quarter-wave stacks (i.e., stopbands).

The five appendixes provide some useful thin-film equations, the Chebychev polynomials used in Chapter 2, the FORTRAN source code used to determine all possible stopbands of modulated designs, several graphs of stopband positions, and a summary of the linear equations that predict stopband positions and the general linear transform equations.

Hopefully this text will provide readers with some new thin-film design tools, further insight to design methods, and inspiration for further research on thin-film design.

I would like to thank my family for their support of my research and writing of this text. I greatly appreciate several helpful discussions with Dr. Philip Baumeister, Dr. Angus Macleod, and Dennis Fischer on these modulation design methods. I also acknowledge the collaborative investigation of rugate versions of these modulated designs, and the rugate designs and calculations, provided by Dr. Pierre Verly at the National Research Council Canada. I would also like to thank the SPIE reviewers for suggesting several improvements to this text. I would like to thank Coherent, Inc. for supporting this work. Lastly, I would like to acknowledge my graduate professor, Dr. Rasheed M. A. Azzam, for his inspiration to continue my research of thin films.

Bruce E. Perilloux
September 2002

Definitions

Additive fractional units (S^+) The number of fractional parts of a quarter-wave layer(s) that combine to produce a new quarter-wave layer, as part of a new quarter-wave stack (see *fractional units*).

Amplitude-modulated TMD (AM-TMD) A variation of a thickness-modulated design where the modulation amplitude of the TMD is also modulated at a lower frequency.

Apodization Envelope function of refractive-index profiles of rugate filters used to suppress passband ripples; also used as envelope functions for layer thickness of TMDs.

Base period (TMD) The integer number of layers that have a nonrepeating thickness pattern (see *modulation period*).

Chirped TMD (C-TMD) A thickness modulated design where the modulation frequency varies or changes as a function of layer number.

Degenerate TMD Typically, a thickness modulated design that has no modulation (i.e., a quarter-wave stack).

Fractional units (U) The integer number of fractional parts that a quarter-wave thick layer must be subdivided into in order to transform a quarter-wave stack into a second stack (see *LOST equations*).

Gaussian envelope function TMD (G-TMD) A thickness-modulated design with the modulation amplitude determined by a Gaussian function.

Half-modulated TMD (H-TMD) A variation of a thickness-modulated design where the modulation is applied to every other layer; the layers that are not modulated all have the same optical thickness.

HWOT Half-wave optical thickness.

Linear optical stack transform (LOST) A linear transform of a quarter-wave stack where the layer thicknesses are adjusted to produce a second quarter-wave stack; a partial transform results in some properties from both quarter-wave stacks.

LOST equations Two general equations used to linearly transform the additive and subtractive fractional units of the quarter-wave layers of one quarter-wave stack into a second quarter-wave stack.

Modulation amplitude (k) The amplitude at which layer thicknesses are modulated (0–1).

Modulation frequency (f) The frequency at which layer thicknesses are modulated.

Modulation period (T) The number of layers of a modulated design that produces a unique layer pattern.

Quarter-wave stack transform See *linear optical stack transform.*

QWOT Quarter-wave optical thickness.

Rugate TMD An inhomogeneous design in which the continuously refractive index profile is determined using a Fourier transform of the desired transmission produced from a discrete-layered thickness-modulated design.

Subtractive fractional units (S^-) The number of fractional parts of a quarter-wave layer(s) that is subtracted from additive fractional units to produce a new quarter-wave layer, as part of a new quarter-wave stack (see *fractional units*).

Thickness-modulated design (TMD) A quarter-wave stack that has the optical thickness (QWOT) of its layers modulated by a periodic function, for example, a sinusoidal function.

Universal stopband equation (USE) A linear equation that is a function of modulation frequency, which predicts all possible stopbands for thickness-modulated designs.

Contents

Chapter 1

Introduction

The process of designing a thin-film coating can be approached from an analytical or a scientific basis, a numerical or an actual experience, or—with the capability of present thin-film design software—perhaps with no basis (knowledge) at all. Over many decades, several design, analysis, and other related books and technical articles have been published on this subject. As more optical systems have become multispectral with performance requirements at several wavelengths or wavelength regions, the required coatings have become more complex. A particular subset or type of coating design is typically used to highly reflect at several wavelengths and may also have high transmittance at others. Classical designs, such as bandpass, cavity, notch, minus filters, rugate, etc., are routinely used for these applications. This chapter is a brief review of well-established thin-film theory as it relates specifically to the above subset of coating designs.

1.1 Review of Mathematics for Thin-Film Design

The basic physics of optical thin films involves the interaction of one or more wavelengths of light and real media boundaries. Without these boundaries, Maxwell's equations describe properties of light waves that are typically modeled as planar or Gaussian. With boundary conditions, where different dielectric or conductive media are present, Maxwell's equations are solved accordingly to describe the reflected and transmitted waves. Specifically, the complex reflection coefficient is the amplitude ratio of the reflected to incident electric fields. From this coefficient, the reflectance (which is the ratio of the reflected wave's power to the incident wave's power) and the phase shift between the reflected and incident waves are determined. The rigorous mathematical development of the electromagnetic theory for propagating waves and interactions with boundaries for various materials is covered in other texts on electromagnetics[1] and optics.[2]

Beyond the fundamental electromagnetic concepts of propagating waves or beams and media and boundary conditions, single or multiple layers of so-called optical "thin" films have two or more boundaries with relatively small spacing or distance between the boundaries (i.e., this spacing is approximately on the order of the wavelength of the light source). These boundaries or interfaces cause a plurality of reflected waves that can interfere with each other in amplitude and phase. These reflected waves interfere, or add coherently, because the path differences within thin films are less than the coherence length, given by Eq. (1.1):[3]

$$l \approx \frac{\lambda^2}{\Delta\lambda}. \tag{1.1}$$

Here, l is the coherence length, λ is the wavelength of light, and $\Delta\lambda$ is the bandwidth. Over the short distances of thin films, most light sources are coherent and interference is observed. For larger distances (e.g., a millimeter) and for quasi-monochromatic light sources, interference can also be observed (the path length is still less than the coherence length). However, for path differences larger than the coherence length, multiple waves interfere with each other in intensity only (in an incoherent manner).[3] Still, the observation of interference effects depends on the detection conditions. This text investigates thin-film designs based on the optical interference of thin films, where an incident wave of light is multiplied into groups of reflected and transmitted waves.

The spectral performance of optical thin-film designs can be computer-modeled using any one of four methods or models (electric field matrix, characteristic matrix, reflection recursion, and admittance recursion).[4] For the purpose of this text, the characteristic matrix is used to model thin-film designs for the particular case of nonabsorbing media. The derivation of the characteristic matrix is not included because it is thoroughly covered in other thin-film texts. However, for completeness, definitions of characteristic matrix, admittance, and the calculation of reflectance are provided.[5]

From Ref. [5], the characteristic matrix for any isotropic and homogeneous thin-film layer is given by

$$\begin{bmatrix} \cos\delta_L & (i\sin\delta_L)/\eta_L \\ i\eta_L\sin\delta_L & \cos\delta_L \end{bmatrix}, \tag{1.2}$$

where the phase term δ_L is given by

$$\delta_L = \frac{2\pi N_L t_L}{\lambda}\cos(\theta_L). \tag{1.3}$$

N_L is the complex refractive index of the layer given by

$$N_L = n_L - ik_L, \tag{1.4}$$

where n_L and k_L are the refractive index and extinction coefficient, respectively, and t_L is the layer's physical thickness. Using Snell's law, the internal angle θ_L is readily found from the incident angle of the beam θ_o, from the complex refractive index of the incident medium N_o, and from N_L, where

$$N_o\sin(\theta_o) = N_L\sin(\theta_L). \tag{1.5}$$

The optical admittance of the layer, η_L, is dependent on the polarization of the incident beam and is given by

$$\text{s-polarization:}\quad \eta_L = N_L\cos(\theta_L) \tag{1.6}$$

and

$$\text{p-polarization:}\quad \eta_L = N_L/\cos(\theta_L). \tag{1.7}$$

It should be noted that the term for optical admittance of free space was omitted from Eqs. (1.6) and (1.7) because it cancels when solving for the admittance of the film-substrate optical system at the interface of the ambient medium and film. For reference, the optical admittance of free space Y_0 is the reciprocal of its optical impedance of free space Z_0 given by

$$Y_0 = \frac{1}{Z_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}, \tag{1.8}$$

where ε_0 and μ_0 are the permittivity and permeability of free space, respectively.

The general form of the optical system described has an ambient medium and thin-film layer(s) deposited onto a substrate as shown in Fig. 1.1. A monochromatic light wave or beam in the ambient medium impinges on the thin-film layer(s) and substrate and is subsequently reflected and transmitted. By solving for the electric and magnetic field vector amplitudes at the ambient-film (outer layer #1, see Fig. 1.1) interface, the optical admittance for the film-substrate system is determined from its characteristic matrix. From this film-substrate's admittance and that of the ambient medium, the reflectance and transmittance are determined.

As one could infer from Fig. 1.1, a small lateral shift of a reflected beam could occur. Departing from plane wave theory, for the special case of total internal reflection (TIR), a shift does occur. This shift is called the Goos-Hänchen shift.[6] However, this shift is typically ignored in thin-film modeling.

Without going through the derivation, the general form of the characteristic matrix for a multilayer film-substrate system is given by the matrix

$$\begin{bmatrix} E_A \\ H_A \end{bmatrix}, \tag{1.9}$$

where

$$\begin{bmatrix} E_A \\ H_A \end{bmatrix} = \left\{ \prod_{L=1}^{n} \begin{bmatrix} \cos\delta_L & (i\sin\delta_L)/\eta_L \\ i\eta_L \sin\delta_L & \cos\delta_L \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \eta_S \end{bmatrix} \tag{1.10}$$

and η_S is the admittance of the substrate medium given by

$$\eta_S = \frac{H_S}{E_S}. \tag{1.11}$$

As shown in Eq. (1.10), the corresponding multilayer thin film-system matrix is the product of the characteristic 2×2 matrices for each layer in the system. The subscript L denotes the Lth layer, where $L = 1$ is the adjacent layer to the ambient medium. A multiplicative factor is not shown in Eq. (1.10) because it is not required to calculate the optical admittance of the film-substrate system as defined below.