FUZZY SETS AND THEIR APPLICATIONS TO COGNITIVE AND DECISION PROCESSES

EDITED BY

Lotfi A. Zadeh

King-Sun Fu

Kokichi Tanaka

Masamichi Shimura

FUZZY SETS AND THEIR APPLICATIONS TO COGNITIVE AND DECISION PROCESSES

EDITED BY

Lotfi A. Zadeh
University of California, Berkeley
Berkeley, California

King-Sun Fu
Purdue University
West Lafayette, Indiana

Kokichi Tanaka
Osaka University
Toyonaka, Osaka, Japan

Masamichi Shimura
Osaka University
Toyonaka, Osaka, Japan

ACADEMIC PRESS, INC.

(Harcourt Brace Jovanovich, Publishers)

Orlando San Diego New York London
Toronto Montreal Sydney Tokyo

COPYRIGHT © 1975, BY ACADEMIC PRESS, INC.
ALL RIGHTS RESERVED.
NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS, INC.
Orlando, Florida 32887

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

U. S.--Japan Seminar on Fuzzy Sets and Their Applications, University of California, Berkeley, 1974.
Fuzzy sets and their applications to cognitive and decision processes.

Bibliography: p.
Includes index.
1. Set theory--Congresses. 2. Pattern perception--Congresses. 3. Decision-making--Congresses. 4. Problem solving--Congresses. I. Zadeh, Lotfi Asker. II. Title.
QA248.U54 1974 511'.3 75-15772
ISBN 0-12-775260-9

PRINTED IN THE UNITED STATES OF AMERICA

85 86 87 88 9 8 7 6 5 4

CONTRIBUTORS

K. Asai, Department of Industrial Engineering, University of Osaka Prefecture, Sakai, Osaka 591, Japan

S. Y. Bang, Department of Computer Science, University of Texas, Austin, Texas

C. L. Chang, IBM Research Laboratory, San Jose, California 95193

S. S. L. Chang, Department of Electrical Sciences, State University of New York, Stony Brook, New York 11794

G. F. DePalma, Departments of Computer Sciences and Electrical Engineering, Northwestern University, Evanston, Illinois 60201

K. S. Fu, School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

T. Fukumura, Department of Information Science, Faculty of Engineering, Nagoya University, Nagoya 464, Japan

L. W. Fung, School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

J. A. Goguen, Computer Science Department, University of California, Los Angeles, California 90024

N. Honda, Research Institute of Electrical Communication, Tohoku University, Sendai 980, Japan

Y. Inagaki, Department of Information Science, Faculty of Engineering, Nagoya University, Nagoya 464, Japan

T. Kitagawa, Kyushu University, Fukuoka 812, Japan

M. Kochen, Mental Health Research Institute, University of Michigan, Ann Arbor, Michigan 48014

M. Kokawa, Automatic Control Laboratory, Nagoya University, Nagoya 464, Japan

M. Mizumoto, Department of Information and Computer Sciences, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan

K. Nakamura, Automatic Control Laboratory, Nagoya University, Nagoya 464, Japan

M. Nasu, Research Institute of Electrical Communication, Tohoku University Sendai 980, Japan

M. Oda, Automatic Control Laboratory, Nagoya University, Nagoya 464, Japan

T. Okuda, Department of Industrial Engineering, University of Osaka Prefecture, Sakai, Osaka 591, Japan

A. Rosenfeld, Computer Science Center, University of Maryland, College Park, Maryland 20742

M. Shimura, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan

M. Sugeno, Tokyo Institute of Technology, Tokyo 152, Japan

H. Tanaka, Department of Industrial Engineering, University of Osaka Prefecture, Sakai, Osaka 591, Japan

K. Tanaka, Department of Information and Computer Sciences, Faculty of Engineering Science, Osaka University, Toyonka, Osaka 560, Japan

T. Terano, Tokyo Institute of Technology, Tokyo 152, Japan

L. Uhr, Computer Sciences Department, The University of Wisconsin, Madison, Wisconsin 53706

C. K. Wong, Computer Sciences Department, IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

S. S. Yau, Departments of Computer Sciences and Electrical Engineering, Northwestern University, Evanston, Illinois 60201

R. T. Yeh, Department of Computer Science, University of Texas, Austin, Texas

L. A. Zadeh, Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, California 94720

此为试读，需要完整PDF请访问：www.ertongbook.com

PREFACE

The papers presented in this volume were contributed by participants in the U.S.–Japan Seminar on Fuzzy Sets and Their Applications, held at the University of California, Berkeley, in July 1974. These papers cover a broad spectrum of topics related to the theory of fuzzy sets, ranging from its mathematical aspects to applications in human cognition, communication, decision-making, and engineering systems analysis.

Basically, a fuzzy set is a class in which there may be a continuum of grades of membership as, say, in the class of *long* objects. Such sets underlie much of our ability to summarize, communicate, and make decisions under uncertainty or partial information. Indeed, fuzzy sets appear to play an essential role in human cognition, especially in relation to concept formation, pattern classification, and logical reasoning.

Since its inception about a decade ago, the theory of fuzzy sets has evolved in many directions, and is finding applications in a wide variety of fields in which the phenomena under study are too complex or too ill defined to be analyzed by conventional techniques. Thus, by providing a basis for a systematic approach to approximate reasoning, the theory of fuzzy sets may well have a substantial impact on scientific methodology in the years ahead, particularly in the realms of psychology, economics, law, medicine, decision analysis, information retrieval, and artificial intelligence.

The U.S.–Japan Seminar on Fuzzy Sets was sponsored by the U.S.–Japan Cooperative Science Program, with the joint support of the National Science Foundation and the Japan Society for the Promotion of Science. In organizing the seminar, the co-chairmen received considerable help from J.E. O'Connell and L. Trent of the National Science Foundation; the staff of the Japan Society for the Promotion of Science; and D. J. Angelakos and his staff at the University of California, Berkeley. As co-editors of this volume, we wish also to express our heartfelt appreciation to Terry Brown for her invaluable assistance

in the preparation of the manuscript, and to Academic Press for undertaking its publication.

For the convenience of the reader, a brief introduction to the theory of fuzzy sets is provided in the Appendix of the first paper in this volume. An up-to-date bibliography on fuzzy sets and their applications is included at the end of the volume.

CONTENTS

CALCULUS OF FUZZY RESTRICTIONS

L. A. Zadeh*
Department of Electrical Engineering
and Computer Sciences
University of California
Berkeley, California 94720

ABSTRACT

A fuzzy restriction may be visualized as an elastic constraint on the values that may be assigned to a variable. In terms of such restrictions, the meaning of a proposition of the form "x is P," where x is the name of an object and P is a fuzzy set, may be expressed as a relational assignment equation of the form R(A(x)) = P, where A(x) is an implied attribute of x, R is a fuzzy restriction on x, and P is the unary fuzzy relation which is assigned to R. For example, "Stella is young," where young is a fuzzy subset of the real line, translates into R(Age(Stella))= young.

The calculus of fuzzy restrictions is concerned, in the main, with (a) translation of propositions of various types into relational assignment equations, and (b) the study of transformations of fuzzy restrictions which are induced by linguistic modifiers, truth-functional modifiers, compositions, projections and other operations. An important application of the calculus of fuzzy restrictions relates to what might be called approximate reasoning, that is, a type of reasoning which is neither very exact nor very inexact. The

*This work was supported in part by the Naval Electronics Systems Command, Contract N00039-75-0034, The Army Research Office, Grant DAHC04-75-G-0056, and the National Science Foundation, Grant GK-43024X.

main ideas behind this application are outlined and illustrated by examples.

I. INTRODUCTION

During the past decade, the theory of fuzzy sets has developed in a variety of directions, finding applications in such diverse fields as taxonomy, topology, linguistics, automata theory, logic, control theory, game theory, information theory, psychology, pattern recognition, medicine, law, decision analysis, system theory and information retrieval.

A common thread that runs through most of the applications of the theory of fuzzy sets relates to the concept of a fuzzy restriction - that is, a fuzzy relation which acts as an elastic constraint on the values that may be assigned to a variable. Such restrictions appear to play an important role in human cognition, especially in situations involving concept formation, pattern recognition, and decision-making in fuzzy or uncertain environments.

As its name implies, the calculus of fuzzy restrictions is essentially a body of concepts and techniques for dealing with fuzzy restrictions in a systematic fashion. As such, it may be viewed as a branch of the theory of fuzzy relations, in which it plays a role somewhat analogous to that of the calculus of probabilities in probability theory. However, a more specific aim of the calculus of fuzzy restrictions is to furnish a conceptual basis for fuzzy logic and what might be called approximate reasoning [1], that is, a type of reasoning which is neither very exact nor very inexact. Such reasoning plays a basic role in human decision-making because it provides a way of dealing with problems which are too complex for precise solution. However, approximate reasoning is more

此为试读,需要完整PDF请访问: www.ertongbook.com

than a method of last recourse for coping with insurmountable complexities. It is, also, a way of simplifying the performance of tasks in which a high degree of precision is neither needed nor required. Such tasks pervade much of what we do on both conscious and subconscious levels.

What is a fuzzy restriction? To illustrate its meaning in an informal fashion, consider the following proposition (in which italicized words represent fuzzy concepts):

Tosi is young (1.1)

Ted has gray hair (1.2)

Sakti and Kapali are approximately equal in height. (1.3)

Starting with (1.1), let Age (Tosi) denote a numerically-valued variable which ranges over the interval [0,100]. With this interval regarded as our universe of discourse U, young may be interpreted as the label of a fuzzy subset[1] of U which is characterized by a compatibility function, μ_{young}, of the form shown in Fig. 1.1. Thus, the degree to which a numerical age, say u = 28, is compatible with the concept of young is 0.7, while the compatibilities of 30 and 35 with young are 0.5 and 0.2, respectively. (The age at which the compatibility takes the value 0.5 is the crossover point of young.) Equivalently, the function μ_{young} may be viewed as the membership function of the fuzzy set young, with the value of μ_{young} at u representing the grade of membership of u in young.

Since young is a fuzzy set with no sharply defined boundaries, the conventional interpretation of the proposition "Tosi is young," namely, "Tosi is a member of the class of young men," is not meaningful if membership in a set is

[1]A summary of the basic properties of fuzzy sets is presented in the Appendix.

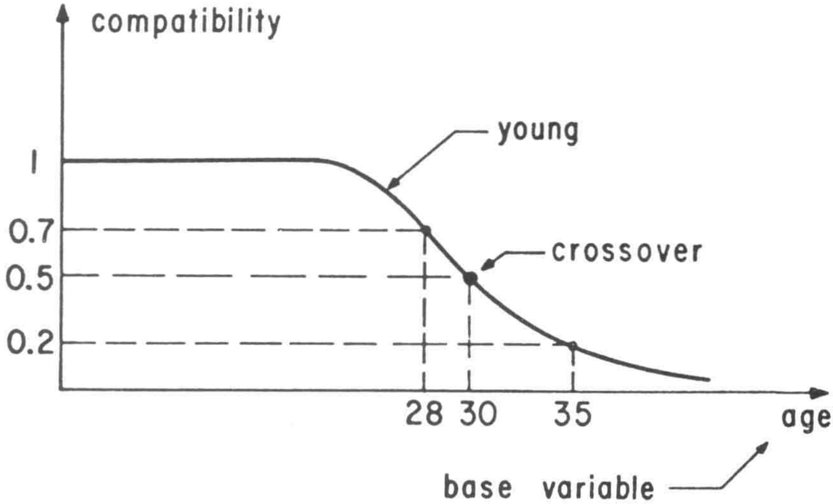

Figure 1.1 Compatibility Function of young.

interpreted in its usual mathematical sense. To circumvent this difficulty, we shall view (1.1) as an assertion of a restriction on the possible values of Tosi's age rather than as an assertion concerning the membership of Tosi in a class of individuals. Thus, on denoting the restriction on the age of Tosi by R(Age(Tosi)), (1.1) may be expressed as an assignment equation

R(Age(Tosi)) = young (1.4)

in which the fuzzy set young (or, equivalently, the unary fuzzy relation young) is assigned to the restriction on the variable Age(Tosi). In this instance, the restriction R(Age(Tosi)) is a fuzzy restriction by virtue of the fuzziness of the set young.

Using the same point of view, (1.2) may be expressed as

R(Color(Hair(Ted))) = gray (1.5)

Thus, in this case, the fuzzy set gray is assigned as a value to the fuzzy restriction on the variable Color(Hair(Ted)).

In the case of (1.1) and (1.2), the fuzzy restriction has the form of a fuzzy set or, equivalently, a unary fuzzy relation. In the case of (1.3), we have two variables to consider, namely, Height(Sakti) and Height(Kapali). Thus, in this instance, the assignment equation takes the form

R(Height(Sakti), Height(Kapali)) = approximately equal (1.6)

in which approximately equal is a binary fuzzy relation characterized by a compatibility matrix $\mu_{\text{approximately equal}}(u,v)$ such as shown in Table 1.2.

Table 1.2. Compatibility matrix of the fuzzy Relation approximately equal.

u \ v	5'6	5'8	5'10	6	6'2	6'4
5'6	1	0.8	0.6	0.2	0	0
5'8	0.8	1	0.9	0.7	0.3	0
5'10	0.6	0.9	1	0.9	0.7	0
6	0.2	0.7	0.9	1	0.9	0.8
6'2	0	0.3	0.7	0.9	1	0.9
6'4	0	0	0	0.8	0.9	1

Thus, if Sakti's height is 5'8 and Kapali's is 5'10, then the degree to which they are approximately equal is 0.9.

The restrictions involved in (1.1), (1.2) and (1.3) are unrelated in the sense that the restriction on the age of Tosi has no bearing on the color of Ted's hair or the height of Sakti and Kapali. More generally, however, the restrictions may be interrelated, as in the following example.

u is small (1.7)

u and v are approximately equal (1.8)

In terms of the fuzzy restrictions on u and v, (1.7) and (1.8) translate into the assignment equations

R(u) = small (1.9)

R(u,v) = approximately equal (1.10)

where R(u) and R(u,v) denote the restrictions on u and (u,v), respectively.

As will be shown in Section 2, from the knowledge of a fuzzy restriction on u and a fuzzy restriction on (u,v) we can deduce a fuzzy restriction on v. Thus, in the case of (1.9) and (1.10), we can assert that

$$R(v) = R(u) \circ R(u,v) = \underline{\text{small}} \circ \underline{\text{approximately equal}} \qquad (1.11)$$

where ∘ denotes the composition[2] of fuzzy relations.

The rule by which (1.11) is inferred from (1.9) and (1.10) is called the <u>compositional rule of inference</u>. As will be seen in the sequel, this rule is a special case of a more general method for deducing a fuzzy restriction on a variable from the knowledge of fuzzy restrictions on related variables.

In what follows, we shall outline some of the main ideas which form the basis for the calculus of fuzzy restrictions and sketch its application to approximate reasoning. For convenient reference, a summary of those aspects of the theory of fuzzy sets which are relevant to the calculus of fuzzy restrictions is presented in the Appendix.

2. CALCULUS OF FUZZY RESTRICTIONS

The point of departure for our discussion of the calculus of fuzzy restrictions is the paradigmatic proposition[1]

$$p \triangleq x \text{ is } P \qquad (2.1)$$

which is exemplified by

---

[2] If A is a unary fuzzy relation in U and B is a binary fuzzy relation in U x V, the membership function of the composition of A and B is expressed by $\mu_{A \circ B}(v) = \vee_u(\mu_A(u) \wedge \mu_B(u,v))$, where $\vee_u$ denotes the supremum over $u \in U$. A more detailed discussion of the composition of fuzzy relations may be found in [2] and [3].

[1] The symbol $\triangleq$ stands for "denotes" or "is defined to be."

x is a positive integer (2.2)

Soup is hot (2.3)

Elvira is blond (2.4)

If P is a label of a nonfuzzy set, e.g., $P \triangleq$ set of positive integers, then "x is P," may be interpreted as "x belongs to P," or, equivalently, as "x is a member of P." In (2.3) and (2.4), however, P is a label of a fuzzy set, i.e., $P \triangleq$ hot and $P \triangleq$ blond. In such cases, the interpretation of "x is P," will be assumed to be characterized by what will be referred to as a relational assignment equation. More specifically, we have

Definition 2.5 The meaning of the proposition

$$p \triangleq x \text{ is } P \tag{2.6}$$

where x is a name of an object (or a construct) and P is a label of a fuzzy subset of a universe of discourse U, is expressed by the relational assignment equation

$$R(A(x)) = P \tag{2.7}$$

where A is an implied attribute of x, i.e., an attribute which is implied by x and P; and R denotes a fuzzy restriction on A(x) to which the value P is assigned by (2.7). In other words, (2.7) implies that the attribute A(x) takes values in U and that R(A(x)) is a fuzzy restriction on the values that A(x) may take, with R(A(x)) equated to P by the relational assignment equation.

As an illustration, consider the proposition "Soup is hot." In this case, the implied attribute is Temperature and (2.3) becomes

$$R(\text{Temperature}(\text{Soup})) = \underline{\text{hot}} \tag{2.8}$$

with hot being a subset of the interval [0,212] defined by, say, a compatibility function of the form (see Appendix)

$$\mu_{\underline{\text{hot}}}(u) = S(u;\ 32,100,200) \tag{2.9}$$

Thus, if the temperature of the soup is u = 100°, then the degree to which it is compatible with the fuzzy restriction hot is 0.5, whereas the compatibility of 200° with hot is unity. It is in this sense that R(Temperature(Soup)) plays the role of a fuzzy restriction on the soup temperature which is assigned the value hot, with the compatibility function of hot serving to define the compatibilities of the numerical values of soup temperature with the fuzzy restriction hot.

In the case of (2.4), the implied attribute is Color(Hair), and the relational assignment equation takes the form

$$R(\text{Color}(\text{Hair}(\text{Elvira}))) = \underline{\text{blond}} \tag{2.10}$$

There are two important points that are brough out by this example. First, the implied attribute of x may have a nested structure, i.e., may be of the general form

$$A_k(A_{k-1}(\ldots A_2(A_1(x)) \ldots)); \tag{2.11}$$

and second, the fuzzy set which is assigned to the fuzzy restriction (i.e., blond) may not have a numerically-valued base variable, that is, the variable ranging over the universe of discourse U. In such cases, we shall assume that P is defined by exemplification, that is, by pointing to specific instances of x and indicating the degree (either numerical or linguistic) to which that instance is compatible with P. For example, we may have $\mu_{\underline{blond}}(\text{June}) = 0.2$, $\mu_{\underline{blond}}(\text{Jurata})$ = very high, etc. In this way, the fuzzy set blond is defined in an approximate fashion as a fuzzy subset of a universe of discourse comprised of a collection of individuals $U = \{x\}$, with the restriction R(x) playing the role of a fuzzy restriction on the values of x rather than on the values of an implied attribute A(x).[2] (In the sequel,

[2] A more detailed discussion of this and related issues may be found in [3], [4] and [5].

此为试读,需要完整PDF请访问: www.ertongbook.com