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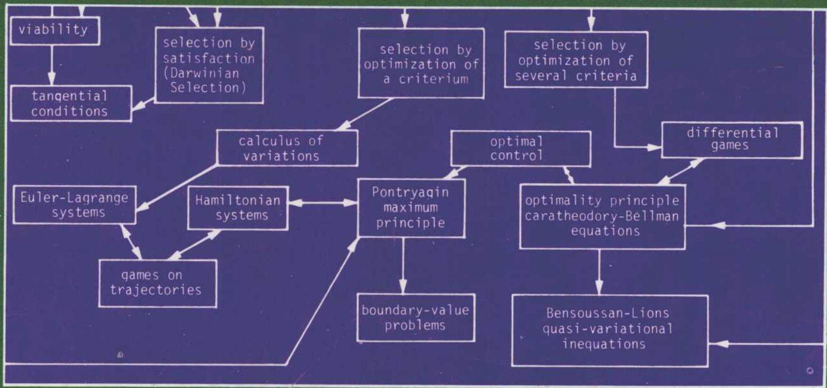

Mathematical Techniques of Optimization, Control and Decision

J. P. Aubin,
A. Bensoussan,
I. Ekeland,
editors

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PREFACE

This book is intended to be the first of a series. Its purpose is to provide reports on non-standard aspects of applied mathematics, centered mainly around decision making. The mathematical problems one is led to are quite varied, and so are the techniques used to solve them. This is why the contributions in the volume cover so broad a range, from ordinary to partial differential equations, from Banach space theory to game theory. But under this variety of method lies a unity of purpose, as J. P. Aubin explains in the general overview which opens this volume.

All the contributions were initiated as colloquium talks at the C.E.R.E.M.A.D.E. during the 1979 - 1980 academic year. We thank the contributors for making their work available for this volume.

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GENERAL OVERVIEW

GENERAL OVERVIEW

J. P. Aubin

C.E.R.E.M.A.D.E.
Université de Paris IX Dauphine

C.E.R.E.M.A.D.E. stands for "Centre de Recherches de Mathématiques de la Décision"--Research Center in the Mathematics of Decision Making. From its foundation in 1971, the C.E.R.E.M.A.D.E. has drawn most of its problems from the *organization sciences*, management, economics, biology, systems theory, and this is where the techniques developed at the C.E.R.E.M.A.D.E. find their most immediate applications. The study of large systems, and the mathematics of decision making, are recurrent features throughout this research.

To be sure, this area of research has acquired widespread recognition and importance since the new industrial revolution. But it can also claim great antiquity: Systems theory was born in the 17th century to cope with the problems posed by mechanics and physics. Indeed, at the same time, its connection with optimization already appeared, albeit in a mysterious way (Fermat's, later Maupertuis', principle of least action). This is also the time when Euler laid the foundations of the Calculus of Variations.

The fact that mathematical concepts and techniques developed to solve problems in 18th century mechanics and physics have acquired a great importance in contemporary economics and management is a great comfort to mathematicians, always sensitive to the structuring and unifying power of the instrument they yield. The Lagrange multipliers, for instance, which first appeared in mechanics, have long since travelled to many other fields of science.

On the other hand, the fact that these problems have attracted for so long the attention of so distinguished mathematicians is quite chastening. In modern times, it has taken nearly three quarters of a century before the economic problems raised by Cournot, Edgeworth, Pareto, Walras, were translated into mathematics and subsequently solved.

All this may help to understand why the mathematicians at C.E.R.E.M.A.D.E. have tried to look for motivations and applications in mechanics and engineering as well as in the organization sciences. Comparison between these problems is possible and fruitful, providing structuring, simplification, in short, a better understanding of both. The C.E.R.E.M.A.D.E. encourages scientists with different backgrounds to share ideas and problems, and to meet together the challenge of modern societies.

Now that the aim is clear, we will give some indication on the work that has been done. After an introductory chapter, to show the kind of problems which arise in organization sciences, we classify subsequent material under the two themes of systems theory and decision making, the final chapter showing the interplay between them.

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I. MATHEMATICS OF DECISION MAKING

1. The complexity of organization sciences.

Organization sciences, for instance economics and management, face us with very complex mathematical problems. One could go so far as to say that these problems are more complex than those arising in physical sciences. The range of phenomena encountered in the physical sciences can be formalized by means of a limited number of equations. Moreover, with a suitable interpretation of variables and unknowns, these equations remain valid for very different phenomena. In many situations from physics or mechanics, sufficiently representative models of the fundamental phenomena are realized with a restricted number of equations. Each of these equations has given rise to a whole body of literature, for certainly the physical sciences are not simple. Nevertheless we claim that they are less complex than the organization sciences.

This is a miracle of nature which astonished Einstein and which has no reason to be universal. Consider, for example, instead of the organization sciences, biology, which cannot by any means be dealt with by a limited number of equations. We have very few ideas about any mathematical laws which govern living organisms.

We think that the situation of organization sciences falls between that of the physical sciences and those of biology.

To support this argument let us consider two examples. First that of a market economy where one finds very numerous decision makers, with vastly different size and behavior. They reach decisions in real time, with uncertain knowledge of the future, thus entering into a complex interplay of alliances and conflicts. What is the outcome of this game? Is it possible to determine a macroeconomic behavior that can be predicted and perhaps controlled?

Next consider an example in management, that of a large multinational firm which decides its investments, capital, production, advertisement, personnel management, policy with respect to its competitors, etc. Is it possible to optimize these decisions?

In these two cases we face complex phenomena which allow no simple model.

2. Possibility and use of modelling.

One could say, of course, and there are those who do, that this very complexity implies that mathematics is powerless to account for these phenomena and that other techniques must be used. And as we have mentioned biology, one thinks naturally of experimental, clinical and empirical methods.

These are obviously very useful, just as they are in the physical sciences, where they in no way obviate the usefulness of quantitative or theoretical methods. There is no conflict between experimental and theoretical physicists. The place of each is recognized.

In biology, on the other hand, the experimental approach is largely predominant. Our claim is that the situation in the organization sciences is--or ought to be--more akin to the one in physics.

Take the example of a multinational firm. At each instant it can be represented by a certain number of variables: The quantities produced, the stocks, the supplies coming in; The loans, the short, medium and long term investments, the capital in various currencies, the demand with its risks, advertisement assets, marketing factors; The situation of the capital, work force, investments. There are also the state of the environment, the existence of competitors, of subcontractors, cartels, etc.

It is not at all impossible to imagine and even to formulate relations between these variables, which let us recall, are functions of time, involving uncertainties of various kinds. Starting from this analysis, decisions can be made in such a way as to optimize the given criteria. There may be several of these, even for a single decision maker, and they usually vary with time. Finally, let us not forget that each decision maker will take into account what he believes will be the behavior of others and the general trend.

What renders the situation complex is less the nature of the relations than their sheer number and, of course, the uncertainty and imprecision of the data. Despite all this, organizations and especially firms must all solve this kind of problem, and others which shall be presented in the following pages.

This initial observation explains why we feel that mathematical techniques in the organization sciences or as we call it, the mathematics of decision making, must be developed together with research of a more empirical nature.

3. Mathematical economics.

Let us begin with economics. This is first of all a science, which implies that the economist seeks to put forward fundamental laws without immediately concerning himself about action. It was, therefore, inevitable that mathematical economics, as developed under the impetus of Walras, Pareto, Arrow, Debreu, Scarf, may sometimes seem extremely abstract and far removed from applications, as was the case when mathematical physics was developed.

Purely mathematical models have often predicted the existence of planets or certain elementary particles long before empirical evidence could be reached. In a similar way, models of mathematical economics which are developed abstractly, enrich our knowledge and may be useful in the future. The notion of market equilibrium price defined years ago is beginning to be used in concrete computations, thanks to recent algorithms, particularly for agricultural products. I.I.A.S.A. (International Institute for Applied Systems Analysis) has set up a whole program of research in this area, which is becoming extremely useful for the planning divisions of countries or organizations such as the E.E.C. (European Economic Community), the O.E.C.D. (Organization for Economic Cooperation and Development), etc.

The development of mathematical economics has required fundamental research in mathematics which goes far beyond the techniques and methods used in physics. New mathematical theories have been created for that purpose.

But fundamental research in mathematics is by no means restricted to mathematical economics. Econometrics, large macroeconomic models and maquettes have required the development of completely original algorithms. A new and important phenomenon appears here--the conjunction of mathematics of decision making and computer science, with an aim to immediate applications.

4. Management systems.

As for management, it must be concerned with concrete problems while economics are allowed a more abstract outlook. I would liken economics to physics or biology, and management to engineering or medicine. It is essentially an applied science, but the problems which it raises are not simple. As we have seen, one encounters complex dynamical systems which are little understood. The problem is to make decisions in such a way as to satisfy or optimize several criteria, often within situations of conflict. The more we try to reflect reality, the more we feel the need to develop new approaches, more and more sophisticated, which therefore require an enormous effort of fundamental and applied research.

5. The need for theory.

Control theory, first deterministic, then stochastic, and finally adaptative, has laid a firm claim on this area of applications. Indeed, management systems can be considered deterministic, stochastic, or adaptive, according to the level of sophistication which is required. The ultimate aim is to manage optimality and in real time inventories or financial assets. There is another very important field of applications for control theory, namely engineering. There is constant interaction between these two areas, and ever more frequently problems occur with technological and economic sides, closely linked.

Game theory also is essential to the organization sciences. Let us mention, among more recent developments, differential games, leader-follower games, voting procedures, multi-criteria optimization, the theory of teams, etc. All are strongly motivated by management: Advertisement, pricing, expansion, all the main policies of a firm lead to such situations.

There is no need to elaborate on the need for research in the mathematics of forecasting, model identification, sensitivity analysis, decomposition methods, hierarchical analysis. All this research is both fundamental and applied. It is applied because it attempts to arrive rapidly at implementable algorithms and even computer software. It is fundamental because it leads to complex theoretical problems.

Nevertheless in the case of management, in contrast to mathematical economics, one does not attempt to axiomatize a phenomenon in order to extract its structure. Theory can be justified only insofar as it is necessary for solving concrete problems. The motivation must ever remain present. This is why research is directed toward data processing and algorithms. The degree of difficulty remains exactly the same.

6. The limits of mathematical techniques.

Certainly one may object that all this effort is of interest only if in the end we dispose of data to feed into the models. This is correct, but here again progress is very rapid, thanks to computer science and more particularly to the development of mini computers, very well adapted to data preparation, and with prices which make them accessible even for medium-sized firms. Progress is also due to highly mathematized techniques of data analysis.

To be sure, we do not claim, that management and economics will use in the near or distant future only mathematical or computer techniques. What is true is that the combination of these techniques, together with data banks, will allow us to deal with a few important but specific problems in the life of organizations. The leadership