Engineering Mathematics

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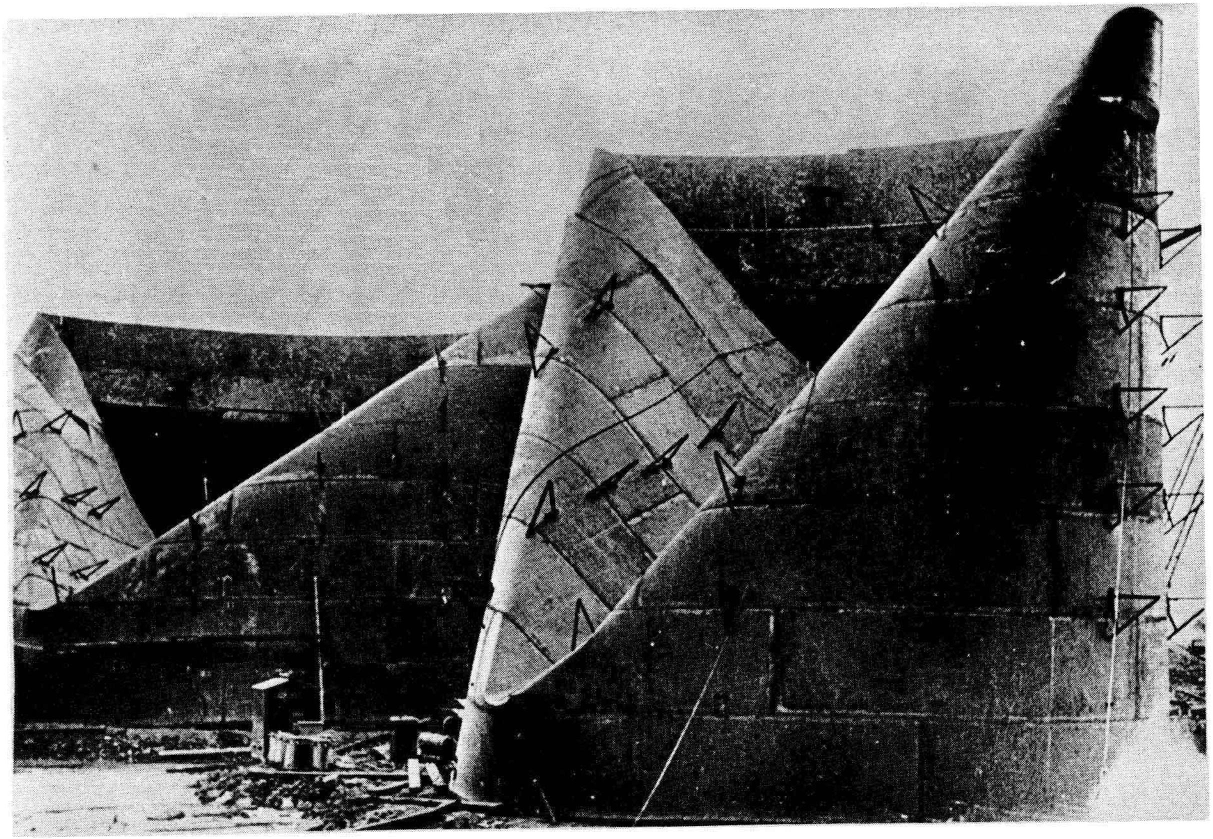
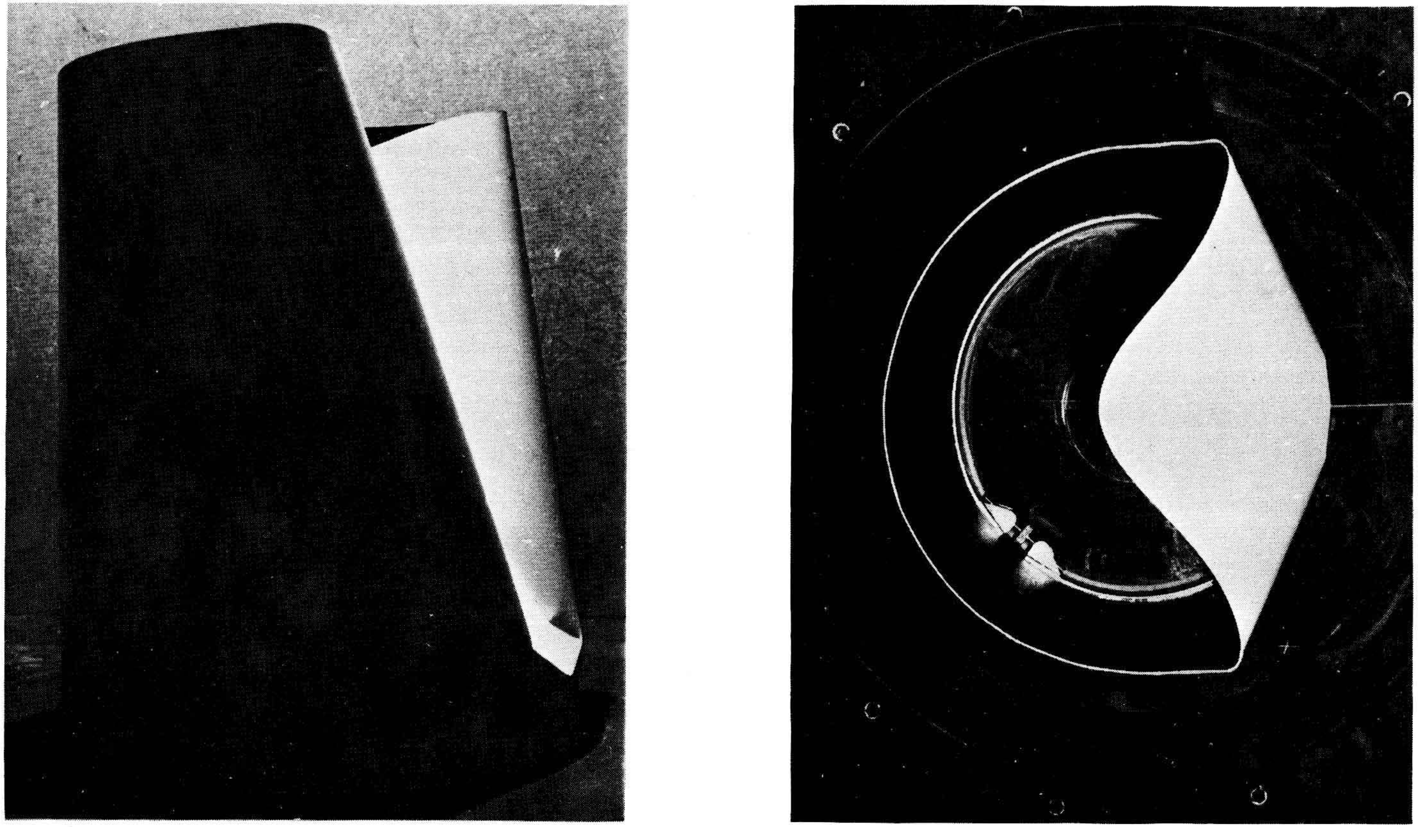


Wind-induced collapse of oil storage tanks at Haydock, Lancashire, England, in 1967



Photographs showing the wind tunnel simulation of the full-scale collapse of the shells in the previous photograph

Preface

It is well accepted that a good mathematical grounding is essential for all engineers and scientists. This book is designed to provide this grounding, starting from a fairly elementary level and is aimed at first year undergraduate science and engineering students in universities, polytechnics and colleges in all parts of the world. It would also be useful for students preparing for the Council of Engineering Institutions examinations in mathematics at Part 1 standard.

The basic concept of the book is that it should provide a motivation for the student. Thus, wherever possible, a topic is introduced by considering a real example and formulating the mathematical model for the problem; its solution is considered by both analytical and numerical techniques. In this way, it is hoped to integrate the two approaches, whereas most previous texts have regarded the analytical and numerical methods as separate entities. As a consequence, students have failed to realise the possibilities of the different methods or that on occasions a combination of both analytical and numerical techniques is needed. Indeed, in most practical cases met by the engineer and scientist the desired answer is a set of numbers; even if the solution can be obtained completely analytically, the final process is to obtain discrete values from the analytical expression.

The authors believe that some proofs are necessary where basic principles are involved. However, in other cases where it is thought that the proof is too difficult for students at this stage, it has been omitted or only outlined. For the numerical techniques, the approach has been to form a heuristically derived algorithm to illustrate how it is used and a formal justification is given only in the simpler cases.

Where a computer approach is possible, a flow diagram is provided; in the Appendix a summary is given of Fortran IV together with some specimen programs of problems or techniques discussed in the text. Many books are devoted solely to the teaching of computer programming; we therefore do not attempt to cover this topic in detail. It is hoped that students will either have prior knowledge of programming or will be studying this in a parallel course.

Throughout the text there is a generous supply of worked examples which illustrate both the theory and its application. Supplementary problems are provided at the end of each section.

Finally, we sincerely hope that both students and lecturers read the Open Letters which set the ethos and philosophy for the book.

A debt of gratitude to the following is acknowledged with pleasure:

Staff and students of Loughborough University of Technology and other institutions who have participated in the development of this text.

Mr. J. Mountfield of Warrington, Lancashire, for permission to use his photograph of the collapsed oil storage tanks.

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The University of London and the Council of Engineering Institutions for permission to use questions from their past examination papers. (These are denoted by L.U. and C.E.I. respectively).

Mrs. J. Russell for typing the major part of the book.

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CHAPTER FOUR: GEOMETRY, VECTORS AND COMPLEX NUMBERS

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APPENDIX

Chapter Zero

Open Letters

OPEN LETTER TO STUDENTS

This is a book with a difference. It is different in that we do not seek simply to give you a grounding in those mathematical techniques you will need in your studies. Rather we hope that you will be encouraged to think mathematically. By this, we mean that after following through the text you will be able to look at some practical problem, think about forming the problem in mathematical terms, consider the possible ways of obtaining an answer to this mathematical problem and choose the most suitable way. Then you should be able to find an answer and furthermore interpret this answer in the context of the original problem. This is not to say that we do not place a great emphasis on building up your knowledge and skills in mathematical techniques. It is essential for you to be able to handle and manipulate mathematical formulae and equations: indeed, much of our book is devoted to the development of such mathematical ability. However, we regard this as only one part of the story and at the end of your study we hope that you will not have to ask the usual questions, *What use is all this mathematics?, Why do we have to study such abstract ideas?* and *How does this tie in with my other subjects?*

The whole scene is set in Chapter 1 and its sole purpose is to establish a way of thinking. For this reason there will appear to be little conventional mathematics in this chapter but we strongly plead with you to follow it closely. It will not be a waste of time we can assure you, and a careful study of the ideas involved will be repaid in that you will get a feeling for the relevance of the mathematical work and in that you will already be thinking along the right lines. Constant reference to this chapter will be made in the rest of the book and the whole development of the text hinges on a clear understanding of the principles expounded there.

We hope that you will enjoy using this book and wish you good luck in your studies.

OPEN LETTER TO TEACHERS

Perhaps you will have read the preface and the open letter to students. We should like to address a few remarks to you especially. This book is not for the lecturer who is content to approach engineering mathematics in the same way that he was taught. It tries to present an integrated study in two ways. Often the complaint has been made that mathematics is isolated from the engineering subjects and seems to bear little relevance; abstract ideas are studied with little attempt to link them to the engineering world. This book seeks, where possible, to introduce the techniques via practical examples. The second feature we want to emphasise is the welding together of numerical and analytical methods. We feel that the separation is artificial and we have tried to present a problem-oriented view in which the techniques that seem most suitable in a particular situation are employed; in any event, numerical methods, now well-established, often give a solution where the analytical techniques have failed and it is unrealistic to treat them as second-best. Indeed, some topics, for example, the behaviour of sequences, are better approached from a numerical view-point.

You will have noticed that we have asked the students to read carefully Chapter 1. We cannot emphasise the importance of this too strongly as, without so doing they will not really appreciate the flavour of the book. We are relying on your help in this matter. We hope that you will enjoy teaching your course along the lines we advocate; certainly we have found it a challenging and stimulating experience in the years we have taught via this approach.

Chapter One

Why Mathematics?

1.1 THE ROLE OF MATHEMATICS

In your previous mathematical work, the emphasis will have been placed most probably on the development of manipulative skills. The subject will have been broken down into fairly water-tight compartments and often it is not clear how one compartment impinges on the next. Some of the techniques studied may have appeared to be 'tricks' and this impression is encouraged by the fact that the numbers chosen are usually those that make the answers come out easily and exactly. This is not necessarily a criticism of the way you have learned mathematics up to this stage: in order to proceed it was necessary that you should have mastered many basic techniques and acquired much background information; this is the case in any other discipline, be it science, history, geography or languages.

However, the role of mathematics in the study of scientific or engineering problems goes deeper and wider than this. In such problems it may be that one specific answer is required or, more generally, the nature of the relationship between two or more of the variable quantities involved is sought in order that certain deductions may be drawn.

Some Simple Experiments

To clarify ideas, let us discuss four simple experiments.

(i) A uniform beam is simply supported near its ends and a load placed near its centre; the beam will be deflected. We may seek the maximum load that the beam can support before breaking or we may be interested in the profile that the beam adopts. More likely we shall be interested in predicting these features for beams at the design stage.

(ii) A liquid that has been heated and removed from the source of heat will cool; we shall be interested in the rate at which it cools, with a possible view to predicting the time that elapses before a specified temperature is reached.

(iii) If a loaded spring is set in motion, the position of the weight varies about its equilibrium position, sometimes below and sometimes above. We might wish to know the greatest depth the load reaches or how long it will be before the amplitude of the oscillations decreases below a certain amount.

(iv) A simple electrical circuit is set up in which the potential difference across the ends of a conductor is varied and the current passing through it is measured. The object might be to study the relationship between the potential difference and the current or to predict the current which flows through the conductor for a non-observed potential difference.

We can examine this last example more closely. Figure 1.1 below shows a typical set of results from the experiment.

Figure 1.1

We call the potential difference V and the current i. The results suggest a straight line relationship of the form $V = V_0 + Ri$ where V_0 and R are constants. (The values of V_0 and R can be obtained graphically or by using a method employing directly the numerical values of the observations. The danger in using the graphical approach to fitting the straight line is that it is a subjective process and different people might obtain different results. However, the graphical approach does have one advantage sometimes in that a wrong observation can be eliminated from consideration.)

This relationship is the *mathematical model* for the physical situation and is an **empirical** formula, since it is based on experimental data and not on a background theory. However, this model can be used to predict values of i for non-observed values of V and vice-versa, and, provided the model gives reasonable predictions, it will suffice.

These predictions can be made graphically or by using a numerical formula, i.e. a formula which involves the specific numerical values of the observations.

There is little virtue in deducing a relationship $V = V_0 + Ri + 10^{-6}Ri^2$ if the extra term plays no part in the predictions, at least to the accuracy to which we are working. However, we must bear in mind that we can only safely predict *within* the range of observations – a process known as **interpolation**; the dangers attendant on predicting *outside* the range (**extrapolation**) are shown in Figure 1.2.