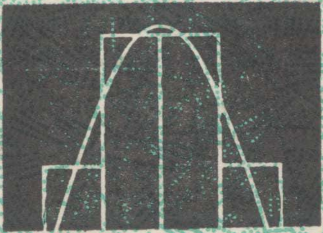
Probability and Random Processes for Electrical Engineering



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Preface

This book is intended for introductory courses on probability theory and random processes taught in electrical and computer engineering programs. Chapters 1 through 5 are designed for a one-semester junior/senior level course on probability theory. Chapters 6 through 8 are designed for a one-semester senior or first-year graduate course on random processes including Markov chains. Chapters 6, 8, and 9 can be used for a one-semester introduction to queueing theory.

The complexity of the systems encountered in electrical and computer engineering calls for an understanding of probability concepts and a facility in the use of probability tools from an increasing number of B.S. degree graduates. The introductory course should therefore teach the student not only the basic theoretical concepts, but also how to solve problems that arise in engineering practice. This course requires that the student develop problem-solving skills and understand how to make the transition from a real problem to a probability model for that problem.

Probability and Random Processes for Electrical Engineering presents a carefully motivated, accessible, and interesting introduction to probability. It is designed to allow the instructor maximum flexibility in the selection of topics. In addition to the standard topics taught in introductory courses on probability, random variables, and random processes, the book includes sections on modelling, basic statistical techniques, computer simulation, reliability, and concise but relatively complete introductions to Markov chains and queueing theory.

Relevance to Engineering Practice

A major problem in introductory probability courses is in motivating the students. One needs to show the student the relevance of the material to engineering practice. This book addresses this problem in Chapter 1 by discussing the role of probability models in engineering design. Applications from various areas of electrical and computer engineering are used to show how averages and relative frequencies provide the right tools for handling the design of systems that involve randomness. These application areas are used in examples throughout the text.

From Problems to Probability Models

How the transition is made from real problems to probability models is shown in several ways. First, important concepts usually are motivated by presenting real data or computer simulation data. Second, sections on basic statistical techniques have been integrated into the text. These sections show how statistical methods provide the link between theory and the real world. Finally, the important random variables and random processes are developed using model-building arguments that range from the simple to the complex. For example, in Chapters 2 and 3 we proceed from coin tossing to Bernoulli trials, and then to the binomial and geometric distributions, and finally via limiting arguments to the Poisson, exponential, and Gaussian distributions.

Examples and Problems

Numerous examples are included in every section. Examples are used to demonstrate analytical and problem-solving techniques, to motivate concepts using simplified cases, and to illustrate applications. The book includes over 550 problems, which are identified by section to help the instructor in selecting homework problems. Answers to selected problems are included at the end of the book, and a solutions manual is available to the instructor.

Computer Methods

The development of an intuition for randomness can be aided by the use of computer exercises. Simple computer programs can be used to generate random numbers and then random variables of various types. The resulting data can then be analyzed using the statistical methods introduced in the book. The sections on computer methods have been integrated into the text rather than isolated in a separate chapter because I feel that the learning of basic probability concepts will be assisted by performing the computer exercises. It should be noted that the computer methods introduced in Sections 2.7, 3.11, and 4.9 do not necessarily require entirely new lectures. The transformation method in Section 3.11 can be incorporated into the discussion on functions of a random variable, and similarly the material in Section 4.9 can be incorporated into the discussion on transformations of random vectors.

Random Variables and Continuous-time Random Processes

Discrete-time random processes provide a crucial "bridge" in going from random variables to continuous-time random processes. Care is taken in the first five chapters to lay the proper groundwork for this transition. Thus sequences of dependent experiments are discussed in Chapter 2 as a preview of Markov chains. In Chapter 4 I emphasize how a joint distribution generates a consistent family of marginal distributions. Chapter 5 introduces sequences of independent identically distributed (iid) random variables, and Chapter 6 considers the sum of an iid sequence to produce important examples of random processes. Throughout Chapters 6 and 7, a concise development of the concepts is achieved by developing discrete-time and continuous-time results in parallel.

Markov Chains and Queueing Theory

Markov chains and queueing theory have become essential tools in communication network and computer system modelling. In the introductory course on probability only a few changes need to be made to accommodate these new requirements. The treatment of conditional probability and conditional expectation needs to be modified, and the Poisson and gamma random variables need to be given greater prominence. In an introductory course on random processes a new balance needs to be struck between the traditional discussion of wide-sense stationary processes and linear systems, and the discussion of Markov chains and queueing theory. The "optimum" balance between these two needs will surely vary from instructor to instructor, so I have provided more material than can be covered in one semester in order to give the instructor leeway in striking a balance.

Suggested Syllabuses

The first five chapters form the basis for a one-semester introduction to probability. In addition to the optional sections on computer methods, these chapters also contain optional sections on combinatorics, reliability, confidence intervals, and basic results from renewal theory. In a one-semester course, it is possible to provide an introduction to random processes by omitting all the starred sections in the first five chapters and covering instead the first part of Chapter 6. The material in the first five chapters has been used at the University of Toronto in an introductory junior-level required course for electrical engineers.

A one-semester course on random processes with Markov chains can be taught using Chapters 6 through 8. A quick introduction to Markov chains and queueing theory is possible by covering only the first three sections of Chapter 8 and then proceeding to the first few sections in Chapter 9. A one-semester introduction to queueing theory can be taught from Chapters 6, 8, and 9.

Acknowledgments

I would like to acknowledge the help of several individuals. During his brief but brilliant career, my ex-student and colleague Gilbert Williams exemplified the engineer who can transform the insights that result from probabilistic reasoning into tangible improvements in the performance of

complex systems. The origin of many of the ideas in this book lie in discussions I had with Gil over the years.

During the initial phases of this project I received excellent advice on how to write a book from Paul Shields, Adel Sedra, and Safwat Zaky. At Addison-Wesley, my sponsoring editors, Barbara Rifkind and Tom Robbins, enthusiastically supported this project from the beginning and it was a pleasure to work with Bette Aaronson, the production supervisor. In addition, I would like to thank the many students who read the manuscript, especially Renos Melas, Massoud Khansari, and Peter Lau

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Toronto A. L-G

Contents

Chapter 1

Chapter 2

Chapter 3

Chapter 4

Chapter 5

Chapter 6

Chapter 7

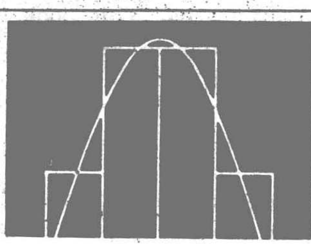
Chapter 8

Chapter 9

Appendixes

CHAPTER 1

Probability Models in Electrical and Computer Engineering



Designers today must often build systems that work in a chaotic environment:

- A large computer system must satisfy the diverse and fluctuating processing demands of the community it serves.
- Communication networks must be continually ready to meet the irregular demands of the customer for voice and data "information pipelines."
- Communication systems must provide continuous and error-free communication over channels that are subject to interference and noise.
- Word recognition systems must decode speaker inputs with high reliability.

Probability models are one of the tools that enable the designer to make sense out of the chaos and to successfully build systems that are efficient, reliable, and cost-effective. This book is an introduction to the theory underlying probability models as well as to the basic techniques used in the development of such models.

This chapter introduces probability models and shows how they differ from the deterministic models that are pervasive in engineering. The key properties of the notion of probability are developed, and various examples from electrical and computer engineering, where probability models play a key role, are presented. Section 1.6 gives an overview of the book.

1.1

MATHEMATICAL MODELS AS TOOLS IN ANALYSIS AND DESIGN

The design or modification of any complex system involves the making of choices from various feasible alternatives. Choices are made on the basis of criteria such as cost, reliability, and performance. The quantitative evaluation of these criteria is seldom made through the actual implementation and experimental evaluation of the alternative configurations. Instead, decisions are made based on estimates that are obtained using models of the alternatives.

A **model** is an approximate representation of a physical situation. A model attempts to explain observed behavior using a set of simple and understandable rules. These rules can be used to predict the outcome of experiments involving the given physical situation. A useful model explains all relevant aspects of a given situation. Such models can therefore be used instead of experiments to answer questions regarding the given situation. Models therefore allow the engineer to avoid the costs of experimentation, namely, labor, equipment, and time.

Mathematical models are used when the observational phenomenon has measurable properties. A mathematical model consists of a set of assumptions about how a system or physical process works. These assumptions are stated in the form of mathematical relations involving the important parameters and variables of the system. The conditions under which an experiment involving the system is carried out determine the "givens" in the mathematical relations, and the solution of these relations allows us to predict the measurements that would be obtained if the experiment were performed.

Mathematical models are used extensively by engineers in guiding system design and modification decisions. Intuition and rules-of-thumb are not always reliable in predicting the performance of complex and novel systems, and experimentation is not possible during the initial phases of a system design. Furthermore, the cost of extensive experimentation in existing systems frequently proves to be prohibitive. The availability of adequate models for the components of a complex system combined with a knowledge of their interactions allows the scientist and engineer to develop an overall mathematical model for the system. It is then possible to quickly and inexpensively answer questions about the performance of complex systems. Indeed computer programs for obtaining the solution of mathematical models form the basis of many computer-aided analysis and design systems.

In order to be useful, a model must fit the facts of a given situation. Therefore the process of developing and validating a model necessarily consists of a series of experiments and model modifications as shown in Fig. 1.1. Each experiment investigates a certain aspect of the phenomenon under investigation and involves the taking of observations and measurements under a specified set of conditions. The model is used to predict the outcome of the experiment, and these predictions are compared with the actual observations that result when the experiment is carried out. If there is a significant discrepancy, the model is then modified to account for it. The modeling process continues until the investigator is satisfied that the behavior of all relevant aspects of the phenomenon can be predicted to within a desired accuracy. It should be emphasized that the decision of when to stop the modeling process depends on the immediate objectives of the investigator. Thus a model that is adequate for one application may prove to be completely inadequate in another setting.

The predictions of a mathematical model should be treated as hypothetical until the model has been validated through a comparison with experimental measurements. A dilemma arises in a system design situation: The model cannot be validated experimentally because the real system does not exist. Computer simulation models play a useful role in this situation by presenting an alternative means of predicting system behavior, and thus as a means of checking the predictions made by a

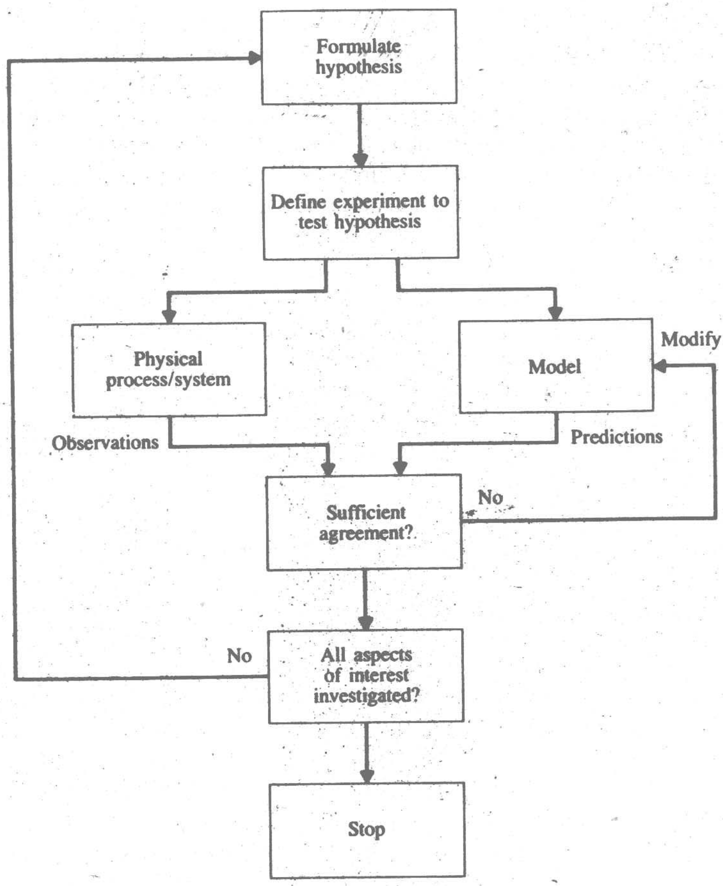


FIGURE 1.1 The modeling process.

mathematical model. A **computer simulation model** consists of a computer program that simulates or mimics the dynamics of a system. Incorporated into the program are instructions that "measure" the relevant performance parameters. In general, simulation models are capable of representing systems in greater detail than mathematical models. However, they tend to be less flexible and usually require more computation time than mathematical models.

In the following two sections we discuss the two basic types of mathematical models, deterministic models and probability models.