Lecture Notes in Physics

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Gravitation, Geometry and Relativistic Physics

Proceedings, Aussois, France 1984

Edited by Laboratoire "Gravitation et Cosmologie Relativistes", Université Pierre et Marie Curie et C.N.R.S., Institut Henri Poincaré, Paris

Springer-Verlag
Berlin Heidelberg New York Tokyo

Lecture Notes in Physics

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Proceedings of the "Journées Relativistes"
Held at Aussois, France. May 2–5, 1984

Edited by Laboratoire "Gravitation et Cosmologie
Relativistes", Université Pierre et Marie Curie et C.N.R.S.,
Institut Henri Poincaré, Paris

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

Editor

Laboratoire de Physique Théorique "Gravitation et Cosmologie Relativistes"
C.N.R.S./U.A. 769, Université Pierre et Marie Curie, Institut Henri Poincaré
11, rue Pierre et Marie Curie, F-75231 Paris Cedex 05, France

ISBN 3-540-13881-1 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-13881-1 Springer-Verlag New York Heidelberg Berlin Tokyo

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2153/3140-543210

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PREFACE

This year our laboratory[⁎] has organized the "Journées Relativistes" in Aussois[#] from May the 2nd to May the 5th and edited the following proceedings.

Twenty-five years ago the theoretical tools for relativistic gravitation used to look very specialized and the orders of magnitude of the effects too small for experimentation. Then the field was often thought of as rather isolated. Nowadays this opinion is no longer valid.

Since the early days the subject has exploded in different directions and merged into several topics related to almost all the fields in physics. This results in a scientific community which has no precise name but exists nevertheless. In this community, the researchers are more or less specialized but the community itself is not : on one hand, sophisticated structures and geometrical tools are studied and used in mathematics and theoretical physics ; on the other hand, technological progress and the paucity of deep empirical knowledge provide the experimentalists with a strong motivation whatever the difficulties are.

As the different possible topics cover a broad range of preoccupations, we chose to emphasize the physical points of view : theoretical and experimental physics, astrophysics and cosmology. Within this framework, the key words of the meeting were "synthesis" and "prospect".

As a synthesis our goal was i) to present the developments of the subject from the early Riemannian geometry until nowadays with physical, epistemological and historical points of view : geometry, general relativity, experimental gravitation ...

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[#] Meeting supported by the University Pierre et Marie Curie, the C.N.R.S. and the D.R.E.T.

ii) to summarize the situation of several important subjects concerning relativistic gravitation and related topics : thermal background radiation, gravitational lenses, inflationary universe ...

As prospects we chose to emphasize i) the diversity and the vitality of "geometrical physics", including relativistic gravitation and relativity : general relativity, supergravity, atomic physics, solid state physics ...

ii) the necessity of extra theoretical studies and clarifications in several fields where experiments and observations display a high accuracy : geodesy, atomic physics ...

iii) the fruitfulness of experimental gravitation (and especially of gravitational wave detection experiments) which was the starting point of recent discussions and works on quantum non-demolition[†], squeezed states, addition of laser fields, high performance interferometers ...

If conclusions were to be drawn from the meeting, on one hand I would put forward that besides the traditional problems (e.g. quantum gravity, gravitational fields from given sources, early universes ...) there exists an expanding field of preoccupations in "geometrical physics" related to very different theoretical, observational and experimental topics. On the other hand, I would especially emphasize that several precise theoretical questions, originating from the increasing accuracy of experiments and observations, have been asked during this meeting. They provide theoreticians with subjects for reflection and require answers in the near future.

We all especially acknowledge F. Allix and C. Trecul for their material organization of the meeting and C. Trecul for her help in the elaboration of the following proceedings.

September 1984

Ph. TOURRENC
Directeur du laboratoire

Unfortunately we could not include in these proceedings the paper of W. Unruh because it did not arrive on time.

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I - Introduction

In preparing this review of research in general relativity over the past 35 years, I have been impressed by how much in fact has been accomplished. As a result I have had to make a severe selection of material in order to avoid being entirely trivial. Some of you undoubtedly would have made other choices. My remarks are divided into five sections which are titled : Gravitational Radiation, Conservation Laws, Blackholes, Quantum Gravity, and Predictions. The uneven emphasis of these areas results in part from my own experience and in part from what I believe have been important accomplishments.

II - Gravitational Radiation

It is perhaps surprising to most people in the audience to realize that as late as 1957, at the Chapel Hill Conference, H. Bondi and T. Gold argued that gravitational radiation could not exist. Their arguments were tied to the steady state cosmology which at that time still had a few more years of life. What is particularly interesting is that within a year Bondi, Pirani, and I. Robinson[1] published their historic paper giving an exact plane wave solution and within a second year Bondi was lecturing about gravitational radiation in asymptotically flat space-times although the detailed paper[2] was not published until 1962. The plane wave solution which is based on earlier work by Einstein and Rosen[3], may be written in the form

$$ds^2 = e^{2A(u)}\, du\,(du+2dx) - u^2\left(e^{2B(u)}dy^2 + e^{-2B(u)}dz^2\right) . \qquad (1)$$

Satisfaction of the Einstein equations implies

$$2A' = uB'^2 \qquad (2)$$

while the vanishing of the Riemann tensor implies

$$B'' + 2u^{-1}B' - uB'^3 = 0 \,. \qquad (3)$$

If one attempts to cover the manifold with a single coordinate system, the metric of Eq. (1) exhibits a nasty singularity at u = 0. However, following the very important work of Lichnerowicz and the people around him[4], Bondi, Pirani and Robinson define a non-singular solution using three coordinate patches (Fig.1). The flatness condition, Eq. (3) is satisfied everywhere except in the cross-hatched region where Eq. (2) holds, but not Eq. (3). In region II and III, which overlap with I,

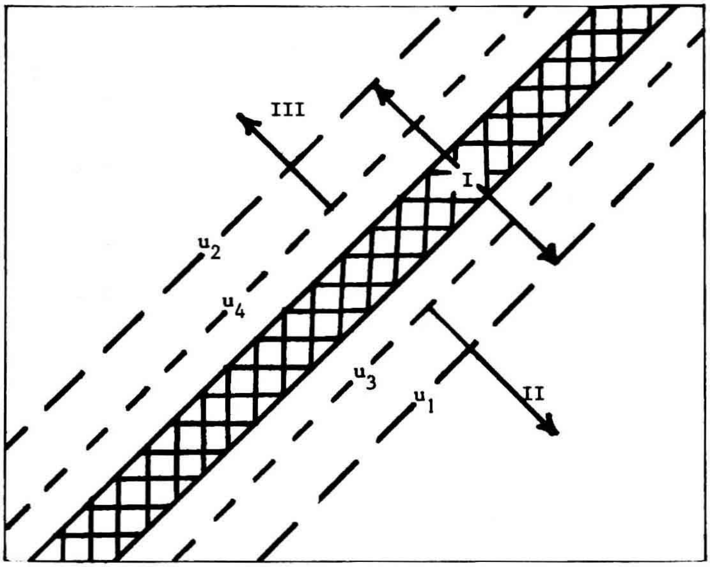

Fig. 1. The three coordinate patches for the plane wave solution. The curvature tensor R_{abcd} is different from zero only in the cross hatched region in patch I : ($0 < u_1 < u < u_2$). The metric takes the standard Minkowski form in patch II ($u < u_3$) and patch III ($u > u_4$).

coordinates can be chosen so that $g_{ab} = \eta_{ab}$, the Minkowski metric. This solution has a five parameter symmetry group : rigid translations in the y-z plane ; rigid translation along x, and a two parameter group of null rotations which leaves du unchanged.

I have presented this example in order to emphasiez two things :

1) At that time there were very few exact solutions known and

2) the methods for looking for exact solutions were relatively undeveloped.

The development of methods for studying exact solutions come from three directions :

1) The Petrov classification introduced to physicists by Pirani[5],

2) The analysis of the geometry of timelike and null congruences by Ehlers, Kundt, and Sachs[6], and

3) The systemitizing of the use of symmetries begun by Ehlers and Kundt[6].

Now, of course, there is a whole book devoted to exact solutions[7] and in England MacCallum is creating a catalog of exact solutions on a computer.

The Bondi analysis[2] of asymptotically flat space-times did not attempt to construct exact solutions. Rather he proceeded physically to ask whether the structure of the Einstein equations was such that they allowed behavior far from matter which one could identify with the radiation of gravitational energy. Indeed, he assumed that in future null directions far from matter space-time was sufficiently close to Minkowski space that there exist nice null surfaces which are like null cones. He introduced an asymptotic coordinate system based on these outgoing "null cones", the null geodesics generate the cones and a foliation of sphere-like 2-surfaces :

$$ds^2 = e^{2b}du^2 + 2e^{2b}du\,dr - r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du) ,$$

$$\phi = 1 - 2m(u,\theta,\varphi)/r ,$$

$$h_{AB} = \begin{pmatrix} 1 + (\sigma^\circ + \bar{\sigma}^\circ)/2r & \sin\theta\,(\sigma^\circ - \bar{\sigma}^\circ)/2ir \\ \sin\theta\,(\sigma^\circ - \bar{\sigma}^\circ)/2ir & 1 - (\sigma^\circ + \bar{\sigma}^\circ)/2r \end{pmatrix} + \cdots . \qquad (4)$$

Note that the 1/r part of h_{AB} has the form of a shear tensor. The important result which Bondi found is that the total mass

$$M = \frac{1}{4\pi}\oint m(u,\theta,\varphi)\,\sin\theta\,d\theta\,d\varphi \qquad (5)$$

is a non-increasing function of time :

$$\dot{M} = -\frac{1}{4\pi}\oint \dot{\sigma}^\circ \dot{\bar{\sigma}}^\circ \sin\theta\,d\theta\,d\varphi . \qquad (6)$$

His analysis was limited to axial symmetry. It was extended and made more rigorous by Sachs[8] and Newman-Penrose[9] by use of tetrad components and spin coefficients.

Anyone who has read the Sachs paper of 1962 knows that it required a major effort to analyze the Einstein equation $G_{ab} = 0$. Newman and Penrose had the brilliant idea that by considering the components of the Weyl tensor as independent field variables, the Bianchi identities become field equations. This yields a quasi-linear system of equations which can be studied in a transparent fashion. Another important technical innovation is the spinor analysis developed by Roger

Penrose[10,11]. In a natural way, the introduction of a spinor basis splits the Weyl tensor and the rotation coefficients into self-dual and anti-self-dual parts, and this gives one better control of the calculation. The same decomposition, of course, can be carried out with tetrads alone, but later I shall discuss a more fundamental use of spinors.

The analysis by Bondi, Sachs and Newman-Penrose depended on taking limits $r \to \infty$. While their results are physically and intuitively satisfying, it was not clear to what extent they depended on the specific coordinate system adopted. It was not easy to study how energy momentum or angular momentum depended on the particular foliation of null surfaces u = constant. Also one had an asymptotic geometry, but one did not have a geometry at infinity ; thus, one could have an asymptotic symmetry group, but not a symmetry group. In other words, the geometrical structures which might have physical content could not be easily studied because they had no home.

The difficulty was over come by Penrose[11] with the introduction of a space-time with boundary $(\hat{M}, \hat{g}_{ab})$ with the properties :

1) in the interior of $\hat{M}$, $\hat{g}_{ab} = \Omega^2 g_{ab}$;

2) on the boundary, $\partial\hat{M}$, $\Omega = 0$, $\nabla_a \Omega = n_a \neq 0$;

3) on $\Omega = 0$, $K_{abcd} = \Omega^{-1} C_{abcd}$ exists ;

4) $R_{ab} = 0 \Rightarrow (\nabla_a \Omega)(\nabla_b \Omega)\, g^{ab}\big|_{\Omega=0} = 0$.

Property (3) implies that $\Omega \sim 1/r$ and (4) tells us that the boundary $\mathcal{I}$ is a null surface. Therefore future null infinity $\mathcal{I}^+$ has a singular induced metric g_{mn} such that $g_{mn} n^n = 0$. The restriction to $\mathcal{I}^+$ of $n^a = g^{ab} \nabla_b \Omega$ is tangent to the null generators of $\mathcal{I}^+$.

One can show that for all asymptotically flat space-times g_{mn} and n^n define a universal structure which is independent of the particular physical space-time as long as the conditions of asymptotic flatness are satisfied[12]. The asymptotic symmetries of the physical space-time can be defined in terms of this universal structure :

$$\mathcal{L}_{\xi^a} g_{mn} = 2k\, g_{mn} \, , \qquad \mathcal{L}_{\xi^a} n^m = -k\, n^m \quad . \tag{7}$$

Under action of the mapping, $\mathcal{I}^+$ undergoes a conformal transformation which, because g_{mn} is singular, is a six parameter group isomorphic to the Lorentz transformations. In addition there is an infinite dimensional abelian group, the supertranslations. These are non-rigid translations along the generators of

$\mathscr{I}^+$. The translations are constant along each generator but may vary continuously and differentiably from one generator to the next. Thus the symmetry group G is a semi-direct product of the conformal transformations and the supertranslations. The supertranslations form an invariant subgroup and the factor group is isomorphic to the Lorentz group. There is a four parameter invariant subgroup which defines the rigid translations. These are important in defining the Bondi energy-momentum.

If one only has a universal structure on $\mathscr{I}^+$, where does the physics come in ? First of all, the requirement that $\Omega^{-1} C_{abcd}$ has a limit gives rise to the Sachs peeling theorem[13]. That is, it tells us that in the physical space the radiative part of the Weyl tensor falls off as $1/r$, the component associated with the mass falls off as $1/r^3$, and that associated with the quadrupole moment, as $1/r^5$. Furthermore, one observes that because the metric becomes singular, the connection on $\mathscr{I}^+$ is not uniquely defined by the universal structure. However, the connection in the physical space-time induces a connection on $\mathscr{I}^+$. The difference between the induced connection and the connection defined only by $D_j g_{mn} = 0$ is of the form $\Delta \Gamma_{mn}{}^{j} = \gamma_{mn} n^j$ where γ_{mn} is a symmetric tensor. One can show that γ_{mn} contains a part N_{mn} which is conformally invariant and satisfies the algebraic conditions $N_{mn} n^n = N_{mn} g^{mn} = 0$ where g^{mn} is any quasi-inverse of g_{mn} $(g_{mj} g^{jk} g_{kn} = g_{mn})$. N_{mn} depends only on $\dot{\sigma}^0$ (see Eq.(4)) and therefore is the rate of change of shear tensor.

II - Conservation Laws

From Noether's theorem, we know the diffeomorphisms of general relativity lead to differential identities among the field equations which in turn lead to conservation laws. Very early in the history of general relativity one understood that there are problems with energy. For example, there is a coordinate system for the Schwarzschild solution in which the Einstein pseudo-tensor vanishes everywhere, yet there exists a surface integral which defines the total energy as the mass. A local energy density is still elusive, but in asymptotically flat spaces-times invariant expressions for the energy-momentum and, in part, for the angular momentum have been constructed[15].

In a manner similar in spirit to the conformal completion at null infinity, one can study the geometry and structures defined on the hyperboloid of space-like directions at space-like infinity. If one assumes that the "magnetic" part of the Weyl tensor, that is, that part which results from rotational motions of the mass, falls off as $1/R^4$ instead of $1/R^3$ as is true of the "electric" part, then the asymptotic symmetry group is just the Poincaré group - the super translations can be eliminated. Therefore, one can write down invariant integrals for energy-momentum and angular momentum which are constants of the motion and have the usual properties of such quantities in Lorentz covariant theories[16]. At null infinity, the situation is not as good. First of all, when gravitational radiation is present one cannot have constants of the motion. None the less, using the Komar expression of the conservation laws in terms of a vector field ξ^μ,

$$U^\mu = 2\sqrt{-g}\,\nabla_\nu(\nabla^\nu\xi^\mu - \nabla^\mu\xi^\nu)\,, \quad U^\mu{}_{,\mu} \equiv 0 \tag{8}$$

(with the additional condition $\nabla_\nu \xi^\nu = 0$ one can use the translation subgroup of the supertranslations to define energy-momentum as a 2-dimensional surface integral on $\mathcal{I}^+$ and a flux integral to define the change in energy-momentum if the surface of integration is distorted[15]. However, a similar construction for angular momentum has not been constructed. One can write down an angular momentum integral but its behaviour under distorsions of the two-surface in general will not vanish even in Minkowski space. It is my understanding that some progress has been made in terms of a suggestion by Roger Penrose[17], but I do not know the details.

Perhaps the most important question which has only fairly recently been settled is the question of the positivity of energy in general relativity. Actually there are two related questions : Given $T_{\mu\nu}t^\mu t^\nu \geq 0$ for all time-like vectors t^μ , $t^\rho t_\rho > 0$,

1) is ADM energy defined at spatial infinity necessarily non-negative, and

2) in the presence of gravitational radiation is the Bondi energy defined on $\mathcal{I}^+$ non-negative ?

The answer to both questions is in the affirmative.

The first question was answered definitively in 1979 by Schoen and Yau[18] who used rather delicate theorems about minimal surfaces to prove the theorem. This was followed by a beautiful, relatively simple proof by Edward Witten[19]. The argument can be put in the following form :

Define $D_a := \nabla_a - t_a t\cdot\nabla =: D_{AA'}$ where t_a is the unit normal to a space-like 3-surface Σ . Consider $t^{BA'}\bar{\xi}^{B'}(D_{AA'}D_{BB'} - D_{BB'}D_{AA'})\xi^A$ and use the Witten equation $D_{AA'}\xi^A = 0$.

Then we find

$$-D_m\,(t^{AA'}\bar{\xi}_{A'}D^m\xi^A) = -\,t^{AA'}(D_m\xi_A)(D^m\bar{\xi}_{A'}) + 4\pi T_{ab}\,t^a k^b, \tag{9}$$

where $k^a = \sigma^a{}_{AA'}\,\xi^A\bar{\xi}^{A'}$. One can show that there exists a unique spinor which is a constant at spatial infinity and which satisfies the Witten equation. Then we find that with the positive energy condition, $T_{ab}t^a k^b \geq 0$, the right hand side is positive or zero. Thus,

$$I = -\oint_{\partial\Sigma}(t^{AA'}\bar{\xi}_{A'}\,D^m\xi_A)\;dS_m \geq 0 \quad . \tag{10}$$

One can show further that

$$I = P_a k^a \tag{11}$$

where P_a is the ADM four-momentum. This argument can be modified to show that the Bondi mass at null infinity is likewise positive or zero[20,21].

The importance of Witten's proof goes beyond the theorem itself. While spinors have been used extensively in general relativity, in every other case when spinors have been used to discuss the Einstein equations, one could equally well have used tetrad vectors. For the first time spinors have an intrinsic role for which tetrads cannot be substituted.

III - Black holes

Certainly the most fascinating objects of study in general relativity are the black holes. They were very poorly understood until recently. Some of us heard a lecture by K.C. Wali[22] who described Chandrasekhar's difficulty in having his theory of white dwarfs accepted. Although the theory of white dwarfs does not involve general relativity, it very definitely involves gravitation. Eddington clearly understood that the implications of Chandrasekhar's theory was that a sufficiently massive star could collapse to a singularity. He felt this was absurd and thereby delayed acceptance of the theory of white dwarfs among astronomers and astrophysicists.

In 1939 Oppenheimer and Snyder[23] calculated the spherically symmetric collapse of pressure free dust using the Einstein equations. They showed that there is nothing in the Einstein equations which would stop the collapse and the formation of the horizon associated with the Schwarzschild solution. However, this result was not exploited until the 50's when John Wheeler and his students[24] began looking at the collapse of various stellar models. Most of this work used spherically symmetric distributions, but the models did take into account nuclear forces. Their results showed that cold stars - after the completion of nuclear burning - less than 1.4 M could reach equilibrium as white dwarfs. Stars more massive would pass through the white dwarf stage to another equilibrium position for neutron stars. The upper limit for a neutron star depends on the assumptions made concerning the equation of state. It is estimated to be $1.5.\ M_\odot \lesssim M \lesssim 5M_\odot$. Stars much more massive than this are known and while there is no explicit proof that such stars cannot lose enough mass to fall below this limit, there is also no proof then they can and always will. Furthermore, the collapse of matter below the Schwarzschild radius does not require an exotic equation of state. If a globular cluster of $10^8\ M_\odot$ collapses, the mean density in the volume $V_s = (4\pi/3)\, R_s^3$ is that of water, $\rho = 1\text{gm/cm}^3$.

In 1963 R.P. Kerr constructed the axi-symmetric stationary solution for a rotating mass. A year and a half later the solution including electric charge was constructed. These solutions also exhibited a horizon inside of which there is no escape to time-like infinity. In 1967 Werner Israel[26] proved that a static space-time with a smooth spherically symmetric horizon was necessarily Schwarzschild and in 1975, after considerable work by Brandon Carter[27] and others, David Robinson[28] completed the proof that the Kerr solution was the unique stationary axi-symmetric solution. Israel himself extended his proof to include charge, but the proof that the charged Kerr or Kerr-Newman solution is unique was published only last year by P. Mazur[29] in Poland and G. Bunting[30] in Australia. However, John Wheeler had been saying since the late 60's that "Black holes have no hair" by which he meant that

1) Kerr-Newman is the unique (physically important) stationary axi-symmetric solution ;

2) A collapsing star will radiate away its quadrupole and higher moments and settle down in an equilibrium state which is Kerr or Kerr-Newman and therefore depends only on the three parameters M, J, Q.

While there was general belief that symmetry was not important in the collapse of a massive star, before 1965 there was no geometrical characterization of the properties of the Schwarzschild (or Kerr) singularity which would allow one to study this question. There was, of course, the Raychaudhuri equation which follows from the Einstein equations with an irrotational perfect fluid as a source :

$$\frac{d\theta}{ds} = -\sigma_{ab}\sigma^{ab} - \frac{1}{3}\theta^2 - 4\pi\kappa(\rho+3p) , \tag{12}$$

θ is the divergence and σ_{ab} the shear tensor for the flow lines, while ρ and p are the density and pressure of the fluid, respectively. If $(\rho + 3p) \geq 0$, the right hand side is negative definite and the divergence necessarily decreases. Furthermore, from $\nabla_b T^{ab} = 0$ one finds

$$\frac{d\rho}{ds} = -(\rho + p)\theta \tag{13}$$

so that if θ becomes negative, ρ necessarily increases monotónically. However, this result is local and does not give the global information needed to characterize the horizon.

The first global theorem on singularities is due to R. Penrose[31]. This begins with the missing link - the characterization of the essential property of the horizon : the existence of a trapped surface. Penrose defines a trapped surface to be a closed space-like 2-surface such that the family of orthogonal outgoing null geodesics as well as the family of orthogonal ingoing null geodesics is converging so then one can expect the causal future of the trapped surface to be bounded. That is effectively what the Penrose theorem proves. More precisely, he shows that if

1) There exists a global Cauchy surface,

2) A positive energy condition is satisfied, $R_{ab}k^a k^b \geq 0$ for all null vectors