ADVANCES IN FUZZY SETS, POSSIBILITY THEORY, AND APPLICATIONS

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The Changes is a book
From which one may not hold aloof.
Its tao is forever changing—
Alteration, movement without rest,
Flowing through the six empty places,
Rising and sinking without fixed law,
Firm and yielding transform each other.
They cannot be confined within a rule,
It is only change that is at work here.

I Ching (Book of Changes)

PREFACE

Since its inception by Professor Lotfi Zadeh about 18 years ago, the theory of fuzzy sets has evolved in many directions, and is finding applications in a wide variety of fields in which the phenomena under study are too complex or too ill-defined to be analyzed by conventional techniques. Thus, by providing a basis for a systematic approach to approximate reasoning and inexact inference, the theory of fuzzy sets may well have a substantial impact on scientific methodology in the years ahead, particularly in the realms of psychology, economics, engineering, law, medicine, decision-analysis, information retrieval, and artificial intelligence.

This volume consists of 24 selected papers invited by the editor, Professor Paul P. Wang. These papers cover the theory and applications of fuzzy sets, almost equal in number. We are very fortunate to have Professor A. Kaufmann to contribute an overview paper of the advances in fuzzy sets. One special feature of this volume is the strong participation of Chinese researchers in this area. The fact is that Chinese mathematicians, scientists and engineers have made important contributions to the theory and applications of fuzzy sets through the past decade. However, not until the visit of Professor A. Kaufmann to China in 1974 and again in 1980, did the Western World become fully aware of the important work of Chinese researchers. Now, Professor Paul Wang has initiated the effort to document these important contributions in this volume and to expose them to the western researchers. A list of Chinese researchers in fuzzy set theory and applications is given in the appendix. The publication of this volume will certainly stimulate mutual exchanges of research ideas and results between Chinese and Western scientists and engineers working in the area of fuzzy sets and applications.

K. S. Fu
President, North American Fuzzy
Information Processing Society
Lafayette, Indiana

December 1982

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The recent opening of the People's Republic of China also opened for study the area of Chinese scientific advances. The researchers are grateful for this new opportunity resulting in a tremendous exchange of ideas which benefits both the Chinese and American people in addition to the universal world of science.

This editor would like to acknowledge and to express his appreciation for the financial assistance he has received for international travel (summer 1981) from the Josiah Charles Trent Memorial Foundation, Inc. The funds for this grant were given to Dr. and Mrs. James Semans through a trust provided by Mrs. Sarah P. Duke. For a project of this nature, it was necessary to take a second trip to China (summer 1982) in order to fully understand the scope of the fuzzy set research activities there. It is important to realize that the support of academic leaders; such as Mr. Zhu Jiusi, President of Huazhong University of Science and Technology, and Mr. Wang Pin-Yang, Chief Engineer of the Electric Power Science Research Institute, is extremely crucial for Chinese research activities to flourish. The appreciation of the value of international communication of my late Dean of Engineering, Dr. Aleksandar Vesic should also be recognized.

It is impossible to mention the names of all the researchers who have helped by offering suggestions about the content. Even more important, is their willingness in providing their best manuscripts to be included in this volume. However, I am profoundly grateful to Mr. Sam Earp of the E. E. Department at Duke University for many stimulating discussions in the course of putting these papers together. Dr. Enrigue H. Ruspini's assistance in preparing Chapter I which improves the overall presentation is also gratefully acknowledged. I also owe thanks to Mr. Wang Ju of U.C.L.A. and Academia Sinica for a portion of indexing work, and to Mr. Zhang Jinwen of Academia Sinica for providing much of the necessary information for the Appendix. I also want to thank Jane S. Culver for typing the manuscripts.

Finally, I am indebted to my chairman Dr. H. Craig Casey, Jr. for his support.

Paul P. Wang
April 1983 in Durham, N.C.

CONTENTS

FUZZY SET THEORY: PAST, PRESENT AND FUTURE

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In this volume twenty four previously unpublished papers have been selected to provide a reference work in the important area of fuzzy set theory and its applications. These contributions provide significant extensions of fuzzy set and possibility theories, as well as some novel applications of their concepts.

This initial chapter briefly reviews the rationale for the intoduction of the concept of fuzzy set and its subsequent theoretical development. Current trends in fuzzy set theory and its applications are then discussed. The discussion focuses particularly on the description of the characteristics of a large family of current technological problems that are well suited to treatment using fuzzy theoretical concepts. In this context certain recent criticisms to the applicability and technological necessity of the concept of fuzzy set are analyzed.

Each of the subsequent chapters of this volume is then briefly summarized. The chapter concludes with a brief assessment of the future role of fuzzy set technology in the solution of complex system analysis problems.

The concept of fuzzy set was introduced by L. A. Zadeh in 1965. Having dedicated his previous efforts to the study of theoretical and practical issues in system analysis and being already recognized as one of the leading contributors to modern control theory and system science, Professor Zadeh turned his attention in the 1960's to issues related to the inability of existing technology to make substantial contributions to the understanding of large complex systems. Observing that human capabilities for cognitive analysis of those systems were not well matched by the available analytical technology, he sought to identify those characteristics of cognitive analysis which were not found in existing formal concepts and techniques.

His analysis led him to two basic observations. First, humans had a capability to understand and analyze imprecise concepts which was not properly understood or emulated by existing analytical methods. Further, the current methodologies showed a concern for precise representation of certain system aspects that were not only irrelevant to the analysis goals but that were an actual impairment in reaching the system understanding objectives. The result of these basic considerations was his proposal to introduce a new concept, that of fuzzy set, as a basic notion in the representation and analysis of complex real world systems. This important proposal was a significant departure from the ideas supported by prevailing schools of epistemological thought that required precision as a sine qua non imposition on properly defined concepts.

The new notion was intended to formalize the idea of a class of real world entities where each such entity has a certain participation in the class capable of being measured along a continuous variable. Unlike probabilities which are primarily intended to represent a degree of knowledge about real entities, the degrees of membership defining the strength of participation of an entity in a class are a representation of the degree by which a proposition is partially true (in a certain specified sense). This basic epistemic difference between probabilistic concepts, derived from considerations about the uncertainty of propositions about the real world, and fuzzy set concepts, closely related to multivalued logic treatments of issues of imprecision in the definition of entities, properties and attributes, lead to the introduction of new formal methodologies that emphasized use of different quantitative relations between the degrees of membership to related classes (i.e., maximum and minimum operations instead of the real addition and product operations which are customary in standard probability calculus). Further, as pointed out by Gaines (1978), the new formalisms preserved a most desirable characteristic of the two-valued logic formulas they extended, that of strong truth-functionality. The increased computational ability gained as a consequence of the simpler relations linking the degrees of membership of related classes have also been a major reason for the appeal of the new methods over traditional probabilistic approaches which were also questionable on simple epistemic grounds.

In spite of the clear philosophical and methodological differences between fuzzy sets and random functions these distinctions remain to this day one of the major objections to the use of the new concept among its uninformed detractors.[1]

Since 1965 Professor Zadeh has continued to argue vigorously, together with an increasing number of researchers worldwide, for the use of fuzzy set concepts in problems related to the understanding and control of large systems.

His research efforts have been centered around two closely related system analysis issues. The first of these issues is that of the development of mechanisms for the representation and manipulation of the meaning of linguistic utterances about the state of the real world. The second is the use of these representations as constraints in the possible states of a real system being analyzed. This use of fuzzy theoretical concepts as a basis for a theory of possibility, was first proposed by Professor Zadeh in 1980 in the context of considerations about the nature of cognitive processes used in approximate reasoning. Again the emphasis was on the essential tradeoff between gains in the understanding of complex systems at the expense of unneeded precision. Now the scope had been enlarged, however, to allow its application of the formalization of important logical deductive processes. The result has been a theory which avoids the counterintuitive contradictions of other approaches and that can be effectively applied to practical problems.

The initial introduction of the concept of fuzzy set in 1965 was followed by numerous theoretical and practical developments in the ensuing years. The continued interest in fuzzy set theory and the vitality of the field are well evidenced by the contributions presented in this volume. These include both theoretical and applied results produced by researchers in the United States, Europe, Japan and the Far East. We are particularly pleased to devote nearly one third of this edited volume to papers from the People's Republic of China. Fuzzy set theory is one of those recently evolved technologies that has generated a great deal of interest among Chinese researchers. Today there are numerous research laboratories throughout China devoted exclusively to research on fuzzy set and possibility theories and their applications. Researchers outside China have been keenly aware of the scope of the research work being performed in the PRC and vice-versa. This awareness has recently been manifested in the desire,

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[1]It is only fair to add that this state of confusion has not been helped by some authors which have embraced the use of fuzzy theoretical concepts without in many cases fully understanding the real nature of the conceptual differences between random functions and fuzzy sets.

strongly expressed in both sides, for a continuing exchange of research contributions between Chinese, American and European institutions. We are particularly pleased to provide, through this volume, an opportunity to continue and further this exchange.[2]

The contributions included in this paper also reflect the diversity of current research efforts in fuzzy set theory. Before summarizing the individual contents of each contribution we proceed to discuss general characteristics of the state of the art in fuzzy set technology as evidenced by the present collection of contributions. In the context of this discussion we will also comment on certain recent criticisms to theoretical and applied results involving use of fuzzy set concepts.

Most of the initial work in the development and application of the concept of fuzzy set was theoretical in nature. While the rationale for the introduction of the concept was the practical need to understand and control a class of complex real world systems it was necessary at first to provide a solid formal framework that extended all mathematical constructs used for the modeling and representation of system characteristics. Further it was also required to develop a most needed understanding of the distinctions between the new epistemic approach and other existing approaches (notably those based on the concept of probability) in order to recognize their relative scope of applicability.

It is also natural that, in the past, the mathematical theory of fuzzy sets has seen a greater development than its applications. The concept of fuzzy set is a generalization of one of the most basic notion in Mathematics, that of set. Since, from this simple basis, a complex and rich framework of mathematical theories has been developed, it can be reasonably expected that a nontrivial conceptual extension at the very foundation of this formidable formal apparatus will trigger a series of generalizations of derived notions and entities.

Most of these generalizations are not, as sometimes criticized, trivial exercises in the enhancement of existing notions which are not mandated by any practical need. Multivalued logic researchers, prior to the introduction of the concept of fuzzy set, were well acquainted with the difficulties inherent in extending basic concepts as the result of enhancing the range of possible truth values of formal propositions. These problems are of three types.

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[2]In order to help this exchange the Appendix to this volume includes compiled information about the names, addresses and areas of research interest of Chinese scientists.

First, multiple extensions are possible, each resulting in different formal properties. Second, in most cases, all such extensions lose some of the desirable properties of the conventional concept being extended thus making more complex, not more trivial, the choice of a generalization approach. The well known problems associated with the production of a fuzzy set theory that preserves all the algebraic properties of conventional set theory are the simplest example of this basic difficulty. Finally, in spite of common misconceptions, these extensions are the consequence of a perceived practical need not appropriately met using conventional entities. The choice of the extension approach that best suits the particular needs of an applied problem poses additional investigative problems which, in almost every case, are not obvious.

Clearly, straightforward extensions produced solely on theoretical feasibility considerations have been made and will undoubtedly continue to be proposed in the context of fuzzy set theory. The ultimate validity of those extensions is essentially an issue in the philosophy of mathematics which is independent of the need for the notion of fuzzy set. In this regard we will not expect even the most ardent detractor of the theory of fuzzy sets to single our this theory as unique among mathematical frameworks in its production of results for the sake of theoretical richness and elegance.

At present, however, the technological trend in fuzzy set technology is one of change and transition. This transition, initiated a few years, ago has seen an increased interest on the applicability of the theoretical notions derived in the past to current important problems, notably in the areas of pattern recognition, decision analysis and approximate deductive reasoning. Broadly speaking, the most successful applications have been in areas where either alternative approaches (such as probability theory) have failed to provide appropriate results, or, more generally, in problems where the complexity and size of the underlying physical system was very large.

Criticisms about the overall applicability of fuzzy set technology, on the other hand, have not been directed towards the advantages to be gained by use of fuzzy set concepts in those applications. Rather, most critics have confined their primary attention to the applicability of fuzzy set theory to those systems currently studied through well defined mathematical parametric models. A common confusion between the actual physical systems being modeled and their mathematical counterparts has helped to promote this misunderstanding. When this dichotomy is ignored by confusion of object system and model, the applicability of a methodology intended to deal with imprecision and ill-definition is necessarily limited. Attempts to apply such a methodology are frequently inappropriate or, at best, result in the rediscovery or confirmation of known results rather than in the advancement of the frontiers of knowledge.

The applicability of fuzzy set theory is better assessed in the context of those problems not adequately addressed by current parametric models. These models commonly presuppose identifiability and measurability of all relevant parameters or characteristics of a physical system. Many systems of current technological interest are characterized by ill-defined and difficult to observe parameters (as is the case, for example, with medical diagnosis) or by the need to control their behavior so as to attain certain imprecise goals (as in the case with the economy of a country). It is for these systems that fuzzy set theory has the greatest applicability and it is their context that such applicability should be judged. Current experience provides strong evidence of the usefulness of fuzzy set approaches in these cases.

This utility is particularly evident in areas of current technological concern where probabilistic models are inadequate. Among the relevant problems of importance, those associated with the automation of approximate reasoning must be singled out for their importance. While the problems associated with the use of conventional probabilistic concepts for deductive inference purposes do not differ substantially from those that have been apparent since the inception of probability theory, these shortcomings are particularly evident in applications involving large, complex systems. These shortcomings are of three types: the conflicts between an objective (frequentist) and a subjective approach, the identification and manipulation of a large number of parameters (e.g., the myriad of conditional and marginal probabilities required to generate certainty factors in approximate reasoning models), and the interpretation of analytical results. These difficulties are fundamental in that physical reality is required to conform to one of several logically conflicting philosophies, problems in the derivation of appropriate parameterizations are not even addressed by the theory, and experimental results are interpreted in questionable ways.

A frequentist approach pretends to an objectivity that is clearly inapplicable when the model parameters, such as expectations, cannot be derived by experimentation. Subjective Bayesian approaches make use of subjective probabilities and are therefore vulnerable to criticism because the fundamental tenet of the scientific method, that data should determine conclusions, is implicitly ignored. It is also the case that statistical methods are often predicted on a parameterization that is arbitrary or unwieldy. The approach is axiomatic in that an adequate parameterization for a problem must be available ab initio. This is not true in general for complex systems, as evidenced by the interest in nonparametric methods.

Nonparametric methods are, however, heuristic and flawed in that they suffer from the same interpretative defects as parametric statistics.

The theory of fuzzy sets allows a holistic approach to problems of the type discussed above. Experimental results may be interpreted in a meaningful and useful manner due to the close connections between fuzzy set theory and natural languages.

In fuzzy set approaches, subjectivity does not necessarily determine or even influence experimental outcomes as in subjective Bayesian approaches. Rather, subjectivity is treated in a logical manner as only those qualities of data that are relevant to the solution of a problem are abstracted and used in assigning degrees of membership.

In our first presentation in this volume, Kautmann describes the increasing impact of fuzzy set theory using a comprehensive survey of papers and books published in 1980. Many theoretical and application-oriented details are included. A concise overview of progress in the theory and application of fuzzy sets is also given in Chapter II.

As we have previously emphasized introduction of the concept of fuzzy set as a formal extension of one of the most basic notions in Mathematics has led to the development of fuzzy counterparts of established mathematical disciplines. One important branch, fuzzy topology, has interested researchers since 1968. Pu and Liu have been invited to report on current research in this particular area in the People's Republic of China. In Chapter III Pu and Liu provide a detailed account of "who is doing what and how". Furthermore, relationships between their research activities and those of fuzzy topology researchers outside China have also been described in detail.

Liu devotes half of the discussions of Chapter IV to discussions of fuzzy convex sets. At the very outset of the development of Fuzzy set theory, Zadeh paid special attention to the investigation of this topic. Two important theorems, the separation theorem and the theorem on the shadows of fuzzy convex sets have been since extended and refined by Lowen and Zadeh, respectively.

Under additional assumptions, Liu is able to provide additional insights on the nature of shadows of fuzzy convex sets. Also in this chapter readers will find new results obtained considering fuzzy convex sets in Euclidean spaces which are extendable to linear spaces over real or complex fields.

In Chapter V, Cerruti characterizes compactness and ultracompactness of fuzzy topological spaces by means of a nearness relation between points of X and of an admissible Y of X. Cerruti proposes admissible extensions as a useful tool to extend classical topology concepts into fuzzy topology.

In the next chapter Huang and Tong show some invariant properties of fuzzy events under five operators, proceeding then to the development of fuzzy probability and possibility spaces. Also presented in Chapter VI are some sufficient and necessary conditions for fuzzy independence and noninteraction between fuzzy events.

A novel algebraic system, designated as SLOPE, is proposed by Cao. He is able to show in Chapter VII that a module over a SLOPE is a generalization of the fuzzy subsets. Several important theorems related to the structure of a slope module and the slope matrices can also be found in that chapter.

Wang discusses in Chapter VIII some problems concerning fuzzy statistics and random subsets. The rain space, a framework treating the fuzzy subsets as random subsets, is introduced by Wang. In this space, the measurability of random subsets is equivalent to strong measurability. Applying the graph of random subsets, he gives a correspondence theorem which combines falling subsets and measurable fuzzy subsets.

An extension of previous theoretical results on the important subject of fuzzy relationships is presented by Bouchon and Cohen. In Chapter IX, they study fuzzy binary relations R defined on a finite set E and associated with a distance relation d = 1-R. They also give metrical properties dealing with spheres and cliques in E. These properties are then used to construct partitions of E. In addition, Bouchon and Cohen derive combinatorial results for the number of fuzzy relations and partitions generalizing classical enumeration coefficients. All these results were derived assuming the membership functions take values in a finite set.

In Chapter X, Zhang presents the proofs of 23 lemmas, 3 basic lemmas and 7 theorems concerning fuzzy set structures with strong implications. Fuzzy set structures with strong implications have some very interesting logical characteristics - it is weaker than the normal fuzzy set structures, but a little stronger than some fuzzy set structures.

On a different note, Turksen discusses inference regions for fuzzy propositions in Chapter XI. Inference regions defined by upper and lower bounds are determined for fuzzy relational propositions. He shows that certain current fuzzy reasoning models produce results within these bounds.

Two types of grammars, fuzzy tree grammar and fuzzy forest grammar, are studied by Chu in the next chapter. The basic concept of a fuzzy tree and its properties are explained via graph theory. The fuzzy string grammar, defined by Lee and Zadeh, is then extended to the fuzzy tree grammar. Fuzzy tree grammar is also a generalization of the ordinary tree grammar as proposed by Brainerd. The fuzzy forest grammar, on the other hand, is the result of a further extension of the above development. Chu concludes that there are two interesting connections between a fuzzy forest and a binary tree: a fuzzy forest language and an n-fold fuzzy language proposed by Mizumoto.

Chapter XIII discusses the use of fuzzy production rules for decision making. The advantages of this formalism with respect to classical decision theoretic methodologies are presented by Lesmo, Saitta and Torasso. They also point out the uniformity of representation which allows the designer to easily implement an interface with the external user. An aspect of the formalism, which is analyzed in detail, concerns its suitability for developing methods for automatic learning of fuzzy production rules. An efficient algorithm is also outlined and an example of its use is also reported in this chapter by the authors.

Whalen and Schott employ the methodologies of fuzzy logic, production systems and quasi-natural language to construct a prototype decision support system. The system combines the expert's rules with the user's knowledge of the current environment to deduce suggested decisions for user evaluation. These suggestions are presented in natural-language form. Utilization of fuzzy concepts is essential for the development of their system.

To obtain computer understanding of a scene, imprecise information from several sources must be combined in reasonable ways. In their paper, Jain and Haynes discuss how elements of fuzzy set theory can be used to represent imprecise information and to combine this information. Their paper closes with a discussion of a system under design which exploits the inherent fuzziness of vision processes and the dynamism of the data at every level.

The paper by Henri Prade indicates how the field of information processing may be extended and enriched by incorporating fuzzy set methodologies into a natural language processor. The information itself may be descriptive or qualitative, with a natural interpretation in terms of membership grades. The types of processing that are considered include fuzzy decision procedures and fuzzy guidance procedures. The latter utilizes imprecise directions and distances, with landmarks as checkpoints. First, the notion of a fuzzy instruction, where the operands, operator, or results are fuzzy entities, is developed. The semantic pattern matching is considered as an information retrieval device, and finally there is a preliminary discussion of the idea of fuzzy data types.