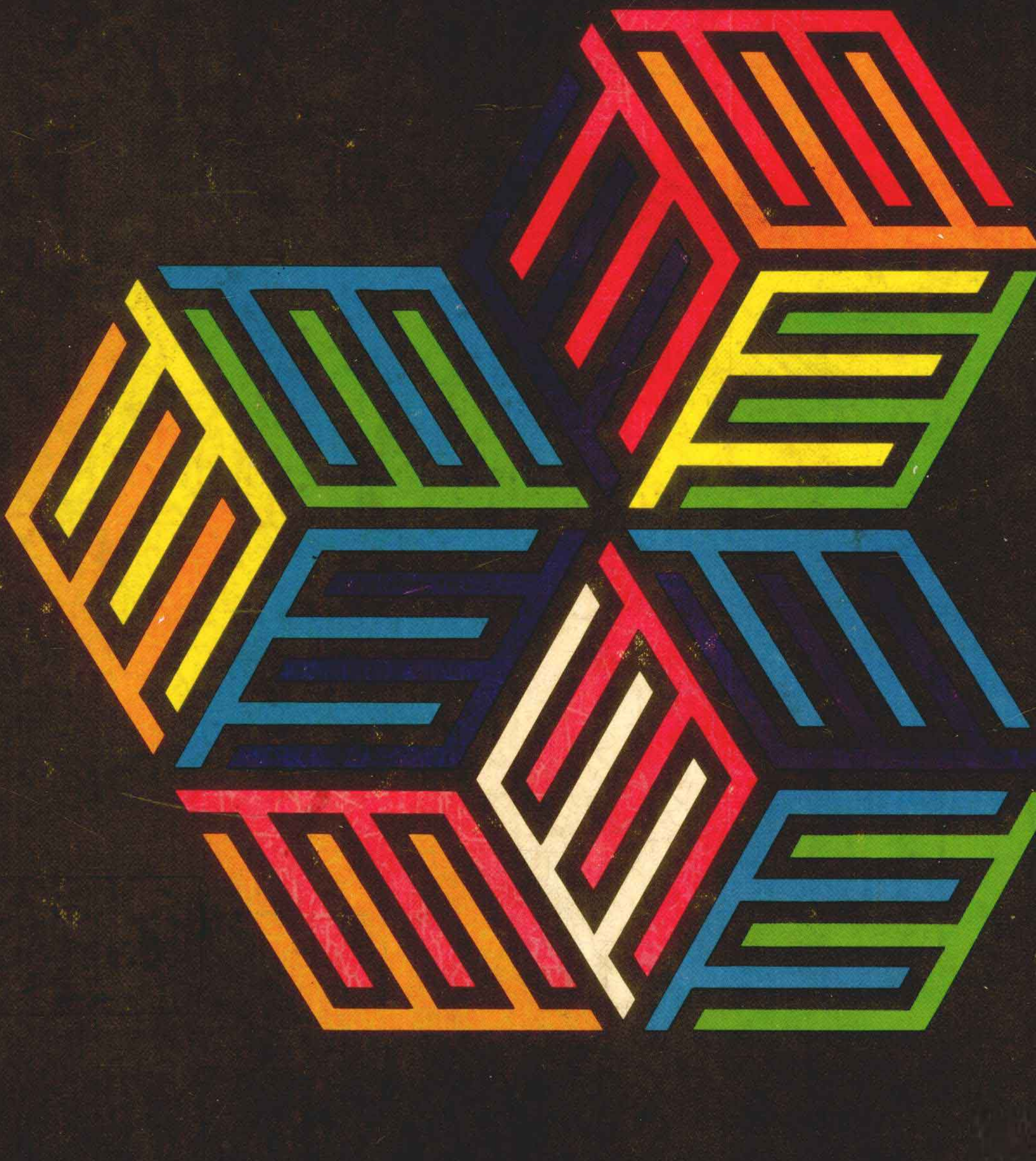
ELEMENTARY ALGEBRA

Thomas M. Green



Elementary Algebra

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Preface

This text is written for adults who typically have not been students of mathematics but now, for one reason or another, desire to gain some familiarity with the method and content of algebra.

There are many good reasons to study algebra, not the least of which is to prepare for more advanced mathematical study. Often, however, the need to prepare for advanced mathematics courses is not a consideration. Indeed, the major concern of some individuals is to merely enter the mainstream of mathematical thinking through an introductory course in algebra. Accordingly, particular care has been exercised by the author and editors to present a text that will provide a smooth transition from arithmetic to algebra and subsequently to more advanced mathematics if desired.

The first six chapters are organized to provide the reader who has little or no previous knowledge of algebra with a development that bridges the gap from the concrete examples of arithmetic to the more abstract or generalized aspects of algebra. Presented in these chapters is the necessary background and structure for the real numbers. The methods for solving simple equations are also developed in these chapters. Together with Chapters 7 and 8 the first six chapters provide a good introduction to algebra for those readers who do not intend to continue to a more advanced course in mathematics.

Chapters 9 through 13 develop algebraic skills in manipulating polynomials and other algebraic expressions. Quadratic equations, more advanced graphing techniques, and inequalities are covered. These topics are important for those readers preparing for more advanced mathematical study. Depending on the background and purposes of the reader, he or she may wish to spend less time on the first six chapters and concentrate on the latter half of the book.

Each chapter is subdivided into sections. Hence, Section 3.2 is the second section of Chapter 3. Each section is followed by a set of exercises and each chapter is followed by a set of review exercises for that chapter. The answers to the odd-numbered exercises in each set are found at the back of the book.

Each section is usually developed around one or two topics. "Illustrations" are used throughout to amplify the formal statements of certain concepts which are numbered and named as definitions, axioms, theorems, or merely properties. For instance, Theorem 3.2.4 refers to the fourth formal statement of some concept in Section 3.2. The illustrations relate the concept to some specific fact or statement that the reader will likely be familiar with, often a statement from arithmetic.

Although some of the properties of algebra are presented as definitions, axioms, or theorems and the proofs of some of the theorems are provided, it is not intended that this presentation be a formal axiomatic development of a mathematical system. Rather, the generalization of arithmetic is stressed. When a proof of a theorem is presented, it is to illustrate the various logical consequences of properties or assumptions that have been previously presented.

"Examples" are presented within the sections that detail the manipulative steps that are important to the development of the computational and problem-solving skills. The functional use of a second color aids in this development. All of the substantive methods and algebraic process for problem solving at the beginning level are presented in the examples. The reader wishing only to review could easily scan the examples of the pertinent chapter and try the review exercises.

The author wishes to thank and express his gratitude to the editors and staff of Macmillan Publishing Co., Inc., and to the many others who have contributed in one way or another to the ultimate development of the manuscript, not excluding the author's family and friends whose contributions were considerable.

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Finally, however, all liabilities in the manner of presentation must of necessity accrue to the author.

San Pablo, California T. M. G.

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Contents

1

Introduction to Algebra

1.1 What Is Algebra?

The answer to the question "What is algebra?" will reveal itself as you progress through this book. To begin to answer the question we should ask another question, "What was arithmetic?" From your experience with arithmetic many of you probably will find that you know more about algebra than you realize.

Arithmetic is, most likely, the first acquaintance most people have with mathematics, and so it gives us a common starting point. It is here that one learns that arithmetic applies to almost everything, to apples and unicorns, to sheep and dollars, and to ideas of the mind. The type of things does not matter; numbers apply to sets or collections of things just because they are things. Of all things it is true that two and two make four. In contrast to arithmetic, algebra is more concerned with the study of processes and methods than with particular answers to particular problems.

Algebra is, in the traditional sense, a *generalization of arithmetic*. This statement is full of implications. The generalizing of certain properties of arithmetic is accomplished in a symbolic manner by using letters for numbers, other symbols for the operations, and still other symbols for equality and inequality relationships.

The numbers most familiar to everyone are the *whole numbers,* that is,

$$0, 1, 2, 3, \ldots$$

The three dots after the last number indicate a continuation of the sequence of numbers. These are the numbers we use for counting purposes and it is the process of counting that provides a basis from which most of the rules or properties of arithmetic are derived.

For example: in arithmetic the equalities

$$2 + 2 + 2 = 3 \times 2 \qquad \text{and} \qquad 4 + 4 + 4 = 3 \times 4$$

are special cases of the "general" rule,

$$n + n + n = 3 \times n$$

where the letter n represents any number. Any other letter could have been used. Thus, it is also true that

$$a + a + a = 3 \times a$$

Instead of the expression "$3 \times n$," the expression "$3n$" is the notation commonly used in algebra for the product of 3 and some number n, or in other words "3 times n." Thus, we write

$$n + n + n = 3n$$

A whole number times any number is called a *multiple* of that number. Hence, $3n$ is a multiple of the number n, and $7a$ is a multiple of the number a.

Illustration 1 If $a = 5$, then $7a = 7 \times 5$, that is, $7a = 35$. The number 35 is a multiple of 5.

In Illustration 1 we *evaluate* $7a$ by *substituting* the number 5 for the number a and then performing the operation of multiplication indicated by the expression $7a$. The substitution process is a very important process in mathematics.

Example 1 If y is 4, find the value of the following expressions: (a) $5y$; (b) $3y + 4y$.

Solution: We substitute the number 4 for y.

(a) $5y = 5 \times 4$
$= 20$

(b) $3y + 4y = 3 \times 4 + 4 \times 4$
$= 12 + 16$
$= 28$

Generalizing statements and properties of arithmetic is a very important aspect of algebra. We symbolize properties of arithmetic by using letters to represent numbers.

Illustration 2 The following two statements are general assumptions that we make in arithmetic.

(i) If zero is added to any given number, the sum is always that given number. We can symbolize this general property of arithmetic by the equality

$$a + 0 = a$$

where a represents any number.

(ii) If two numbers are added, the order of the numbers does not affect the sum. We can symbolize this general property of arithmetic by the equality

$$x + y = y + x$$

where x and y represent any two numbers.

In Illustration 2 we have two equalities, each symbolizing a general property of arithmetic. Some equalities, like those in Illustration 2, are called *identities*. Identities can be *tested* by substituting arbitrary numbers for the letters. We must replace every occurrence of a given letter with the same number.

Illustration 3 The equality symbolizing the general rule in Illustration 2(ii) can be tested by substituting arbitrary numbers for x and y. Suppose we replace x by 2 and y by 7, then for the equality

$$x + y = y + x$$

we obtain,

$$2 + 7 = 7 + 2$$

Adding the numbers on the left side of the equality we obtain 9 and adding the numbers on the right side of the equality we obtain 9. Hence, we get

$$9 = 9$$

The equality is shown to be correct when x is 2 and y is 7.

Example 2 Test the general rule represented by the equality

$$a + a + a + a = 4a$$

by replacing a by 5.

Solution: Substituting 5 for a in the given equality we obtain

$$5 + 5 + 5 + 5 = 4 \times 5$$

Adding the numbers on the left side we obtain 20; multiplying the numbers on the right side we obtain 20. Hence, we get

$$20 = 20$$

The equality is shown to be correct when a is 5.

Example 3 Test the statement represented by the equality

$$3n + 5n = 8n$$

for a specific value of n.

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Solution: Suppose we replace n by 4. Then the given equality becomes

$$\begin{aligned} 3 \times 4 + 5 \times 4 &= 8 \times 4 \\ 12 + \quad 20 &= 32 \\ 32 &= 32 \end{aligned}$$

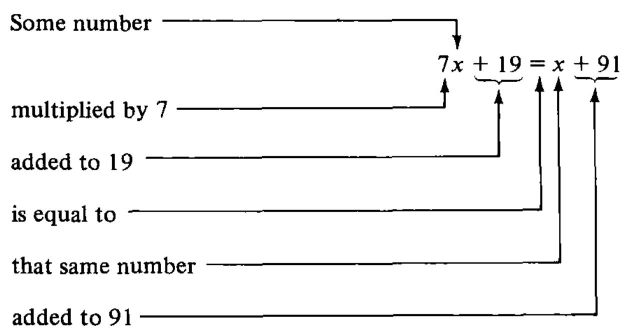
The equality is found to be valid when n is 4.

The use of symbols (letters) for numbers was introduced by François Vieta (1540–1603), a French lawyer and mathematician. This innovation was the one that gave real power and generality to the then infant science of algebra. For this reason Vieta is sometimes referred to as the "father of algebra."

The origin of the word algebra comes from the Arabic word "Al-Jabr" (a word employed by the Arabs, having to do with "solving equations") used in the title of a book on the subject by an Arabian mathematician, Musa-al-Khowarizmi (ca. 820 A.D.). This work found its way to Europe after the Dark Ages and its subject came to be known as "algebra." Algebra was then practically synonymous with the "science of solving *equations*." In its early development algebra was concerned with solving a problem, such as the following:

> "Some number multiplied by 7 and then added to 19 is equal to that same number added to 91."

To solve this problem we obtain an *equation*.



The expression $7x + 19$ is called the *left member* of the equation and the expression $x + 91$ is called the *right member* of the equation. Solving this equation is the process of finding the number represented by x so that the left and right members are equal.

Example 4 Write an equation that can be used to solve the following problem: some number multiplied by 5 and then added to 10 is equal to 24.

Solution: If x represents the unknown number, then we have the equation

$$5x + 10 = 24$$

In the process of solving equations we make use of the fact that most arithmetic operations can be "undone." Thus, for example if we have the equation

$$x + 5 = 8$$

we subtract the number five from both members of the equation and obtain

$$x + 5 - 5 = 8 - 5$$

or, equivalently,

$$x = 3$$

We have used the fact that subtraction is the *inverse* of addition. Subtracting a number is the *inverse* of adding that number. Adding a number is the *inverse* of subtracting that number. Multiplying by a number is the *inverse* of dividing by that number. Dividing by a number is the *inverse* of multiplying by that number if that number is not zero.

In Chapter 4 we study equations in greater detail. A few more examples will serve to illustrate how the concept of *inverse operations* is used in the process of solving equations.

Example 5 Solve the equation

$$x - 5 = 2$$

Solution: Because the inverse of "subtracting 5" is "adding 5," we add 5 to both members of the given equation and obtain

$$x - 5 + 5 = 2 + 5$$

or, equivalently,

$$x = 7$$

Example 6 Solve the equation

$$3x = 24$$

Solution: Because the inverse of "multiplying by 3" is "dividing by 3," we divide each member of the given equation by 3 and obtain

$$3x \div 3 = 24 \div 3$$

or, equivalently,

$$x = 8$$

An important characteristic of algebra is the logical-deductive nature of its development. Unlike most sciences, mathematics is primarily deductive rather than inductive. For instance, the experimental and natural sciences are inductive because we start from *specific* isolated facts and draw *general* results from many observations and experiments. However, in mathematics we start with a few general assumptions and from them deduce specific results.

Illustration 4 From the general assumption that the product of two numbers is zero only when at least one of the numbers is zero, we may deduce from the equation

$$3n = 0$$

the specific result

$$n = 0$$

Illustration 5 From the general assumption that the difference between two numbers is zero only when the numbers are equal, we may deduce from the equation

$$x - y = 0$$

the specific result

$$x = y$$

Furthermore, we may deduce from the equation

$$5 - x = 0$$

the specific result

$$x = 5$$

From some basic assumptions about whole numbers of arithmetic it is possible to prove by deduction many properties of numbers. One such property is the "summation property."

1.1.1 Summation Property The sum of all the whole numbers from 1 to any given whole number is equal to one half of the product of the given whole number and the next consecutive whole number. Using symbols, where n is the given whole number, we write

$$1 + 2 + 3 + \cdots + n = \frac{1}{2} \times n \times (n + 1) \qquad (1)$$

In equality (1) parentheses are used with $n + 1$ to indicate that $(n + 1)$ represents a single value. Thus, $\frac{1}{2} \times n \times (n + 1)$ represents the product of the three numbers $\frac{1}{2}$, n, and $(n + 1)$.

Example 7 Find the sum of all the whole numbers from 1 to 10.

Solution: From the Summation Property where n is 10 we have

$$\begin{aligned} 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 &= \frac{1}{2} \times 10 \times 11 \\ &= \frac{1}{2} \times 110 \\ &= 55 \end{aligned}$$

Example 8 Find the sum of all the whole numbers from 100 to 200.

Solution: First find the sum of all the whole numbers from 1 to 99 and then subtract this sum from the sum of all the whole numbers from 1 to 200.

The sum of all the whole numbers from 1 to 99 is $\frac{1}{2} \times 99 \times 100$.

$$\begin{aligned} \frac{1}{2} \times 99 \times 100 &= \frac{1}{2} \times 9900 \\ &= 4950 \end{aligned}$$

The sum of all the whole numbers from 1 to 200 is $\frac{1}{2} \times 200 \times 201$.

$$\begin{aligned} \frac{1}{2} \times 200 \times 201 &= \frac{1}{2} \times 40{,}200 \\ &= 20{,}100 \end{aligned}$$

We subtract and obtain

$$20{,}100 - 4950 = 15{,}150$$

Therefore, the sum of all the whole numbers from 100 to 200 is 15,150.

In answering our question "What is algebra?" we find equations and numbers as the objects that are studied. One set of numbers that we have already encountered is the set of whole numbers. Probably less familiar to you are certain other numbers known as the *integers,* which include the *negative integers*. For example, consider the temperature on a cold day. This measurement may be a number of degrees "below zero." If the scale on a thermometer is marked for temperatures above zero by 2, 4, 6, 8, and so on, some sort of designation is needed to mark the scale for temperatures below zero. Other examples are measurements of altitudes above and below sea level, profits and losses in financial matters, gains and losses in a game. The designations of "positive" and "negative" are used to denote numbers that in some sense are to be considered "opposites" of each other. Negative numbers are used for the purpose of measuring quantities oriented in a direction *opposite* to the positive direction.

Illustration 6 If "positive 10" denotes a credit of $10, then "negative 10" denotes a debit of $10. If "positive 5" denotes a distance of 5 miles east from some starting point then "negative 5" denotes a distance of 5 miles west from the starting point. If "positive 1" denotes a charge on a proton then "negative 1" denotes a charge on an electron.

The symbols used to denote the positive and negative designations are $+$ and $-$.

"positive 10" is denoted $^{+}10$

"negative 10" is denoted $^{-}10$

If a number is written without a positive or a negative symbol, it is considered to be a positive number. Thus,

$$10 = {}^{+}10$$

All of the whole numbers greater than zero are considered to be positive. Corresponding to each positive number is a negative number. For instance, corresponding to $^{+}10$ is $^{-}10$; these corresponding numbers are called "opposites" of each other.

Illustration 7 $^{+}10$ is the opposite of $^{-}10$ and $^{-}10$ is the opposite of $^{+}10$.

When positive and negative numbers are depicted on a scale or line, one point on the line is designated as the "zero point"; this point is a reference point called the *origin* and corresponds to the number zero. Then all of the points on one side of the origin correspond to the positive numbers and all those points on the other side or opposite side of the origin correspond to the negative numbers. Zero is neither a positive nor a negative number. Figure 1.1.1 shows a scale that depicts land elevations above and below sea level; the elevation of a point at sea level corresponds to the number zero and such a point serves as the origin for the scale. Figure 1.1.2 shows a scale on a thermometer that uses positive and negative numbers to denote Fahrenheit temperatures above and below zero degrees.

The collection of whole numbers together with the opposites of the positive whole numbers form a set of numbers called the *integers*.

1.1.2 Definition The collection of numbers 0, 1, 2, 3, . . . together with $^{-}1$, $^{-}2$, $^{-}3$, . . . is the set of *integers*.

In our study of negative numbers we will primarily concern ourselves with learning how these numbers behave when subjected to the fundamental operations of addition and multiplication and the inverses of these operations, subtraction and division.

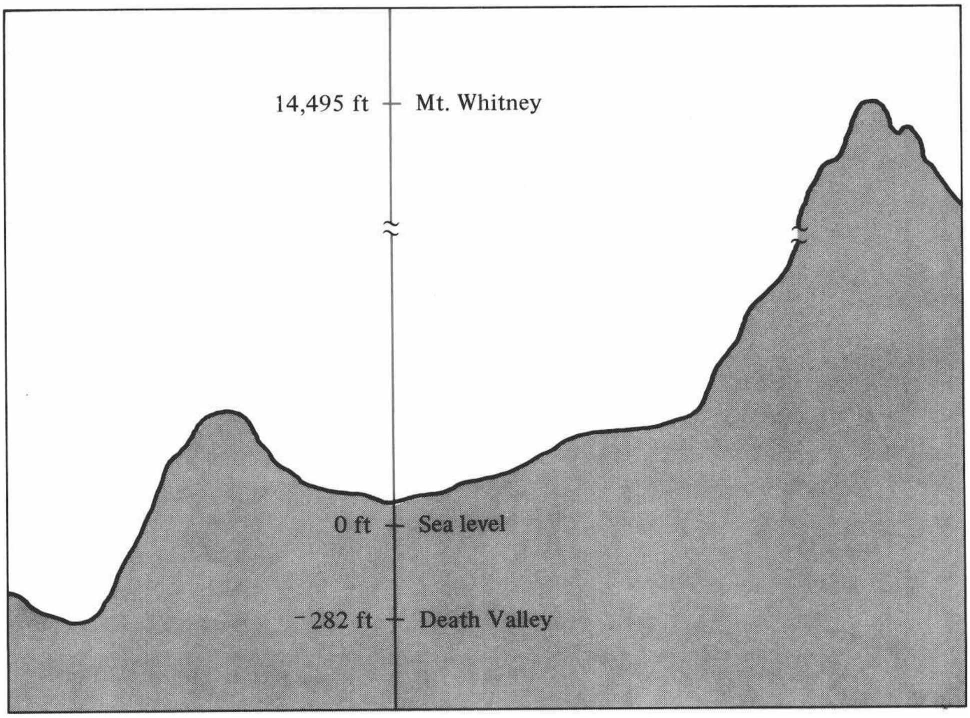
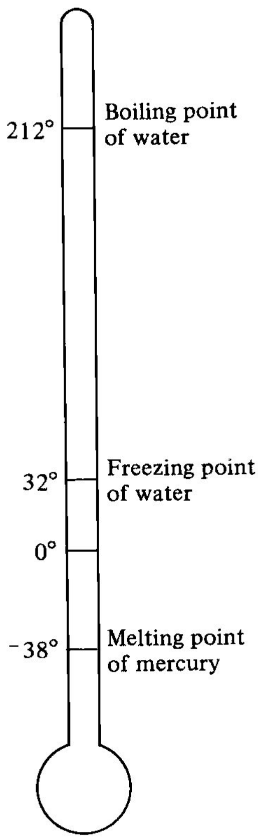


Figure 1.1.1



Fahrenheit Temperature Scale

Figure 1.1.2

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