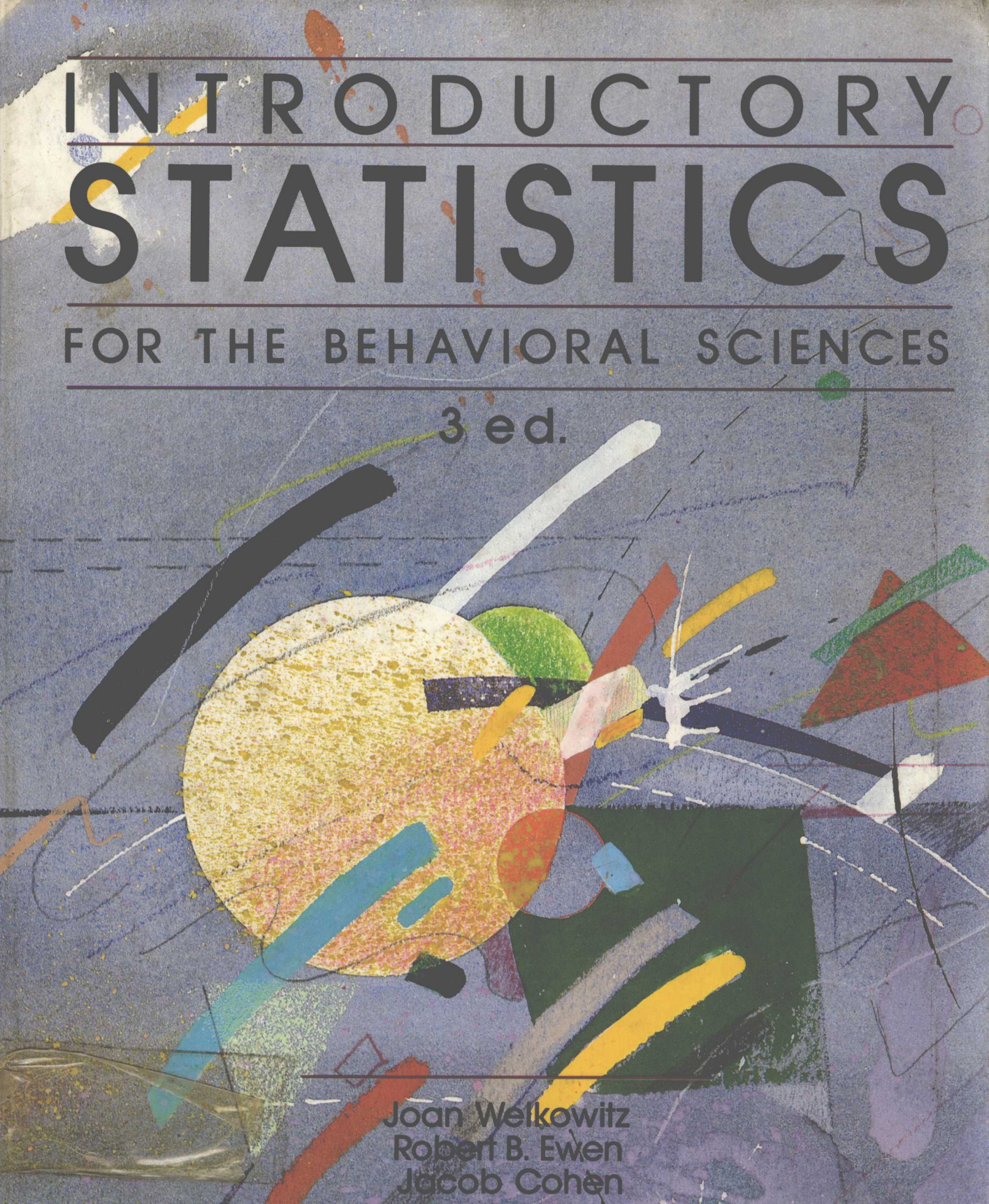
INTRODUCTORY STATISTICS

FOR THE BEHAVIORAL SCIENCES

3 ed.



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INTRODUCTORY STATISTICS FOR THE BEHAVIORAL SCIENCES

THIRD EDITION

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Dedicated to
our students—past, present, and future—
to Walter, Julie, Larry, David, Sara, and Ray
and to Pat and Erika

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IMPORTANT TEXTUAL FORMULAS

Measures of Central Tendency

$$\bar{X} = \frac{\Sigma X}{N}$$ mean (p. 46)

$$Mdn = LRL + \frac{pN - SFB}{f} \cdot h$$ median (p. 53)

Measures of Variability

$$\sigma^2 = \frac{\Sigma(X - \bar{X})^2}{N} = \frac{1}{N}\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]$$ population variance (p. 64)

$$s^2 = \frac{\Sigma(X - \bar{X})^2}{N - 1} = \frac{1}{N - 1}\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]$$ population variance estimate (p. 64)

Transformed Scores

$$Z = \frac{X - \bar{X}}{\sigma}$$ (p. 73)

$$T = 10Z + 50$$ (p. 76)

$$\text{SAT} = 100Z + 500$$ (p. 77)

$$PR = L\% + \left(\frac{score - LRL}{H} \cdot I\%\right)$$ percentile rank (p. 36)

$$Score_p = LRL + \left(\frac{pN - SFB}{f} \cdot h\right)$$ score at the pth percentile (p. 39)

Probability

$$P(\text{event}) = \frac{\text{number of ways the specified event can occur}}{\text{total number of possible events}} \quad \text{(p. 95)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \text{(p. 97)}$$

$$P(A \text{ and then } B) = P(A) \times P(B) \quad \text{(p. 98)}$$

Inferences about the Mean of one Population

$t = \frac{\bar{X} - \mu}{s_{\bar{X}}}, df = N - 1$	statistical test for mean of a population	(p. 136)
$s_{\bar{X}} = \frac{s}{\sqrt{N}}$	estimated standard error of the mean	(p. 123)
$\bar{X} - ts_{\bar{X}} \leq \mu \leq \bar{X} + ts_{\bar{X}}$	confidence interval for μ	(p. 141)
$\delta = \gamma\sqrt{N}$	index for analysis of power determination	(p. 219)
$\gamma = \frac{\mu_1 - \mu_2}{\sigma}$	effect size	(p. 219)
$N = \left(\frac{\delta}{\gamma}\right)^2$	sample size determination given power, alpha, and effect size	(p. 223)

Inferences about the Proportion of One Population

$z = \frac{p - \pi}{\sqrt{\pi(1 - \pi)N}}$	statistical test for population proportion	(p. 144)
$p - z\sigma_p \leq \pi \leq p + z\sigma_p$	confidence interval for population proportion, π	(p. 145)
$\delta = \gamma\sqrt{N}$ where $\gamma = \frac{\pi_1 - \pi_0}{\sqrt{\pi_0(1 - \pi_0)}}$	index for analysis of power determination	(p. 224)
$N = \left(\frac{\delta}{\gamma}\right)^2$	sample size determination given alpha, power, and effect size	(p. 225)

Inferences about the Means of Two Independent Populations

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}},\ df = N_1 + N_2 - 2$$ statistical test for the difference between two independent means (p. 159)

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}$$ estimated standard error of the difference (p. 158)

$$[(\bar{X}_1 - \bar{X}_2) - ts_{\bar{X}_1 - \bar{X}_2}] \leq \mu_1 - \mu_2 \leq [(\bar{X}_1 - \bar{X}_2) + ts_{\bar{X}_1 - \bar{X}_2}]$$ confidence interval for $\mu_1 - \mu_2$ (p. 161)

$$\delta = \gamma\sqrt{\frac{N}{2}}$$ index for analysis of power determination (p. 229)

$$\gamma = \frac{\theta}{\sigma}$$ effect size (p. 223)

$$N = 2\left(\frac{\delta}{\gamma}\right)^2$$ sample size determination given power, alpha, and effect size (p. 231)

$$r_{pb} = \sqrt{\frac{t^2}{t^2 + df}}$$ measure of strength of association (p. 210)

Inferences about the Means of Two Dependent Populations

$$t = \frac{\Sigma D}{\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N - 1}}},\ df = (\text{number of pairs} - 1)$$ statistical test for the difference between two means (matched samples) (p. 166)

Linear Correlation and Prediction

$$r_{XY} = 1 - \frac{1}{2}\frac{\Sigma(Z_X - Z_Y)^2}{N}$$ (p. 177)

$$= \frac{N\Sigma XY - \Sigma X\Sigma Y}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$ Pearson r (p. 179)

$$Y' = b_{YX}X + a_{YX}$$ regression line for predicting Y (p. 186)

$b_{YX} = r_{XY}\frac{\sigma_Y}{\sigma_X} = \frac{N\Sigma XY - \Sigma X\Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$	slope for predicting Y (regression constant)	(p. 189)
$a_{YX} = \bar{Y} - b_{YX}\bar{X}$	Y intercept	(p. 189)
$\sigma_{Y'} = \sqrt{\frac{\Sigma(Y - Y')^2}{N}} = \sigma_Y\sqrt{1 - r_{XY}^2}$	standard error of estimate	(p. 195)
$\delta = \gamma\sqrt{N - 1}$	index for analysis of power determination	(p. 226)
$\gamma = \rho_1$	effect size	(p. 226)
$N = \left(\frac{\delta}{\gamma}\right)^2 + 1$	sample size determination given power, alpha, and effect size	(p. 227)

Other Correlational Techniques

$r_s = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$	Spearman rank-order correlation coefficient	(p. 204)
$r_{pb} = \frac{\bar{Y}_1 - \bar{Y}_0}{\sigma_Y}\sqrt{pq}$	point biserial correlation coefficient	(p. 206)

Analysis of Variance

$SS_T = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$	total sum of squares	(p. 241)
$SS_B = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_2} + \cdots + \frac{(\Sigma X_k)^2}{N_k} - \frac{(\Sigma X)^2}{N}$	sums of squares between groups	(p. 242)
$SS_W = SS_T - SS_B$	sums of squares within groups	(p. 244)
$df_B = k - 1$		(p. 244)
$df_W = N - k$		(p. 244)
$MS_{\text{any effect}} = \frac{SS_{\text{that effect}}}{df_{\text{that effect}}}$	mean square	(p. 245)
$F_{\text{any effect}} = \frac{MS_{\text{that effect}}}{MS_{\text{error}}}$	F ratio	(p. 245)
$t_{\text{protected}} = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{MS_W\left(\frac{1}{N_i} + \frac{1}{N_j}\right)}}$	multiple comparisons: the protected t test	(p. 250)

$\varepsilon = \sqrt{\frac{df_B\,(F-1)}{df_B\,F + df_W}}$	measure of strength of association, epsilon	(p. 252)

Chi Square

$\chi^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$	chi square	(p. 276)
$\phi = \sqrt{\frac{\chi^2}{N}}$	index of strength of association for 2 × 2 table	(p. 285)
$C = \sqrt{\frac{\chi^2}{N + \chi^2}}$	contingency coefficient; index of strength of association for tables larger than 2 × 2	(p. 287)
Cramér's $\phi = \sqrt{\frac{\chi^2}{N(k-1)}}$	index of strength of association for tables larger than 2 × 2, free of dependence on table size	(p. 288)

Nonparametric and Distribution-Free Methods

$z = \frac{T_1 - T_E}{\sigma_T}$ where $\sigma_T = \sqrt{\frac{N_1N_2(N+1)}{12}}$	Rank–Sum test	(p. 298)
$r_G = \frac{2(\bar{R}_1 - \bar{R}_2)}{N}$	Glass rank biserial correlation: measure of strength of association	(p. 300)
$H = \frac{12SS_B}{N(N+1)}$ where $SS_B = \frac{T_1^2}{N_1} + \frac{T_2^2}{N_2} + \cdots + \frac{T_k^2}{N_k} - \frac{N(N+1)^2}{4}$	Kruskal–Wallis H test for differences among locations of two or more independent samples	(p. 302)
$\varepsilon_R = \sqrt{\frac{H-k+1}{N-k}}$	measure of strength of association for H test	(p. 304)
$z = \frac{T_1 - T_E}{\sigma_{T_M}}$ where $\sigma_{T_M} = \sqrt{\frac{(2N+1)T_E}{6}}$	Wilcoxon test for difference between location of two matched samples	(p. 307)
$r_C = \frac{4(T_1 - T_E)}{N(N+1)}$	matched pairs rank biserial correlation: measure of strength	(p. 309)

PREFACE

This book represents the efforts of three authors who have jointly accumulated many years of experience in statistical procedures through teaching and research efforts. Our purpose has been to introduce and explain statistical concepts and principles clearly and in a highly readable fashion, assuming minimal mathematical sophistication but at the same time avoiding a "cookbook" approach to methodology.

We have attempted to present a broader outlook on hypothesis testing than is customary by devoting an entire chapter to the much neglected concepts of statistical power and the probability of a Type II error. To our knowledge, this is the first time that power tables that can easily be used by beginning students of statistics have been included in an introductory statistics textbook. As another important extension of conventional tests of significance, the conversion of t values and other such results of significance tests to correlational-type indices which express *strength* of relationship have also been included. Also, special time-saving procedures for hand calculation which have become outmoded by the ready availability of electronic calculators and computers, such as the computation of means and standard deviations from grouped frequency distributions, have been omitted.

Throughout the text, the robustness of parametric procedures has been emphasized. However, recognizing the fact that nonparametric tests are widely used, we have included a chapter on this subject. Hopefully, we have also included enough information so that those who use such techniques will be aware of their disadvantages and use them wisely. We have also included a section, within the chapter on analysis of variance, on multiple comparisons. Fisher's LSD method is presented as an extremely useful, though fairly simple comparison method.

Statistics is a complicated subject for many students, and a major goal of this edition has therefore been to improve and clarify our method of presentation. To this end, the text, tables, and figures have been reset and

restyled. Another chapter has been added on graphic techniques in recognition of the increasing focus on knowing and being able to describe your data before proceeding with specific statistical tests. Chapter summaries have been expanded by including "Reminder" sections so that students will have this information available for ready reference. And answers to selected problems have been included at the end of the accompanying "Workbook," so that students may check their work upon completion in at least some instances and gain immediate feedback.

ACKNOWLEDGMENTS

Thanks are due to the many encouraging friends and relatives; to colleagues who made many useful comments on our first editions; and most of all to our students who have provided invaluable feedback to both the textbook chapters and the workbook.

We wish to thank Dr. Carol S. Furchner, Dr. Seymour Giniger, and Dr. Joseph A. Trzasko for their most helpful comments and suggestions. We are also indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Oliver & Boyd, Edinburgh, for permission to reprint Tables C, D, and G from their book *Statistical Tables for Biological, Agricultural, and Medical Research,* and to the Iowa State University Press, Drs. J. W. Dunlap and A. K. Kurtz, the Houghton Mifflin Company and *Psychometrika,* Dr. A. L. Edwards, the Holt, Rinehart, and Winston Company, and the *Annals of Mathematical Statistics* for permission to reproduce the other statistical tables in the Appendix.

GLOSSARY OF SYMBOLS

Numbers in parentheses indicate the chapter in which the symbol first appears.

a_{YX}	Y intercept of linear regression line for predicting Y from X (12)
α	criterion (or level) of significance; probability of Type I error (10)
b_{YX}	slope of linear regression line for predicting Y from X (12)
β	probability of Type II error (10)
$1 - \beta$	power (14)
C	contingency coefficient (17)
cf	cumulative frequency (2)
χ^2	chi square (17)
D	difference between two scores or ranks (11)
$\overline{D}$	mean of the Ds (11)
df	degrees of freedom (10)
df_B	degrees of freedom between groups (15)
df_W	degrees of freedom within groups (15)
df_1	degrees of freedom for factor 1 (16)
df_2	degrees of freedom for factor 2 (16)
$df_{1 \times 2}$	degrees of freedom for interaction (16)
δ	delta (14)
ε	epsilon (15)
f	frequency (2)
f_e	expected frequency (17)
f_m	number of negative difference scores (18)
f_o	observed frequency (17)

f_p	number of positive difference scores (18)
F	statistic following the F distribution (15)
γ	effect size, gamma (14)
G Mdn	grand median (18)
h	interval size (3)
H	statistic following the Kruskal—Wallis test (18)
$H\%$	percent of subjects in all intervals higher than the critical one (3)
H_0	null hypothesis (10)
H_1	alternative hypothesis (10)
i	case number (1)
$I\%$	percent of subjects in the critical interval (3)
k	a constant (1)
k	number of groups (or the last group) (15)
$L\%$	percent of subjects in all intervals below the critical one (3)
LRL	lower real limit (3)
LSD	Fisher protected t test (15)
Mdn	median (4)
MS	mean square (15)
MS_B	mean square between groups (15)
MS_W	mean square within groups (15)
MS_1	mean square for factor 1 (16)
MS_2	mean square for factor 2 (16)
$MS_{1\times2}$	mean square for interaction (16)
μ	population mean (4)
N	number of subjects or observations (1)
N_G	number of observations or subjects in group G (15)
π	hypothetical population proportion (10)
ρ	observed sample proportion (10)
$P(A)$	probability of event A (8)
PR	percentile rank (3)
ϕ	phi coefficient (17)
r_C	matched pairs rank biserial correlation coefficient (18)
r_G	Glass rank biserial correlation coefficient (18)
r_{pb}	point-biserial correlation coefficient (13)
r_s	Spearman rank-order correlation coefficient (13)
r_{XY}	sample Pearson correlation coefficient between X and Y (12)
$\bar{R}$	mean of a set of ranks (18)

ρ_{XY}	population correlation coefficient between X and Y (12)
s	sample standard deviation (5)
s^2	population variance estimate (5)
$s_D{}^2$	variance of the Ds (11)
s^2_{pooled}	pooled variance (11)
$s_{\bar{X}}$	standard error of the mean (10)
$s_{\bar{X}_1 - \bar{X}_2}$	standard error of the difference (11)
$Score_p$	score corresponding to the pth percentile (3)
SFB	sum of frequencies below the critical interval (3)
SS	sum of squares (15)
SS_{T}	total sum of squares (15)
SS_{B}	sum of squares between groups (15)
SS_{W}	sum of squares within groups (15)
SS_1	sum of squares for factor 1 (16)
SS_2	sum of squares for factor 2 (16)
$SS_{1\times 2}$	sum of squares for interaction (16)
Σ	sum or add up (1)
σ	standard deviation (5)
σ^2	variance (5)
σ_p	standard error of a sample proportion (10)
σ_T	standard error of the ranks of independent samples (18)
$\sigma_{T_{\text{M}}}$	standard error of the ranks of matched samples (18)
$\sigma_{\bar{X}}$	standard error of the mean when σ is known (10)
$\sigma_{Y'}$	standard error of estimate for predicting Y (12)
t	statistic following the t distribution (10)
T	T score (6)
T_{E}	expected sum of the ranks (18)
T_i	sum of ranks in group i (18)
θ	theta (14)
x	deviation score (4)
X'	predicted X score (12)
$\bar{X}$	sample mean (4)
$\bar{X}_G$	mean of group G (15)
Y'	predicted Y score (12)
z	standard score based on a normal distribution (9)
Z	standard score (6)

CONTENTS