A TEXTBOOK OF
SOUND

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[For B.Sc. (Pass, Honours and Subsidiary), Engineering Students of Indian Universities and IAS Examinations]

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PREFACE TO THE SECOND EDITION

The present edition has been completely revised. Keeping in view the suggestions received from teachers and students, many chapters have been strengthened by adding new topics—What propagates in wave motion? Velocity of sound and frequency in Chapters Four and Five respectively. The new topics like silence zones, theory of resonator and dependence of the frequency of resonator on the size and shape of the mouth of the resonator have been added in Chapter Six. Chapters Seven and Ten have been improved. Many new solved numerical examples on important topics have been included in the book, to provide better understanding and practice to students.

We are grateful to the teachers and students for the favourable response given to the book. We welcome suggestions for the improvement of the book.

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PREFACE TO THE FIRST EDITION

The book, intended to meet the needs of B.Sc. (Pass, Honours and Subsidiary) and engineering students, covers the topics included in the syllabi of various Indian universities.

The subject-matter is divided into ten chapters. Each chapter is self-contained and is treated in a comprehensive way, using the S.I. system of units. Harmonic oscillators, linearity and superposition principle, oscillations with one degree of freedom, resonance and sharpness of resonance, quality factor, Doppler effect in sound and light, tape recording, cathode ray oscillograph, medical applications of ultrasonics, acoustic intensity and acoustic measurements are some of the important topics which have been given special attention.

While preparing the book it was assumed that the student is familiar with the basic principles of sound. However, some of the elementary discussions are included to initiate an advanced treatment of the subject. Solved numerical problems (in S.I. units), wherever necessary, are given in the text and exercises, at the end of each chapter, contain questions which are largely drawn from the university question papers of recent years. The book contains a large number of diagrams.

We hope that the book will be found useful both by students and teachers.

AUTHORS

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CHAPTER 1

HARMONIC OSCILLATORS

1.1. INTRODUCTION

In every day life we come across numerous things that move. These motions are of two types, *viz.* (*i*) the motion in which the body moves about a mean position *i.e.* a fixed point and (*ii*) the motion in which the body moves from one place to the other with respect to time. The first type of motion of a body about a mean position is called oscillatory motion. A moving train, flying aeroplane, moving ball etc correspond to the second type of motion. Examples of oscillatory motion are: an oscillating pendulum, vibrations of a stretched string, movement of water in a cup, vibration of electrons, movement of light in a laser etc.

Sometimes both the types of motion are exhibited in the same phenomenon depending on our point of view. The sea waves appear to move towards the beach but the water moves up and down about the mean position. When a stretched rope is displaced, the displacement pulse travels from one end to the other but the material of the rope vibrates about the mean position without travelling forward.

1.2. SOUND

In the case of elastic waves the disturbance travels with a velocity depending upon the elastic properties of the medium. These elastic waves are also called sound waves. In every day life sound is referred to the sensation of hearing. When an elastic wave, propagating through a solid, liquid or gas, reaches the ear, the membrane of the ear is set into vibration. The vibrations of the membrane stimulate a nervous response relating to the process of hearing. The human nervous system produces the hearing sensation for frequencies ranging from 16 hertz to 20,000 hertz. For animals the audible frequency range for hearing is different. Beyond these frequency limits, the sound is not audible. The elastic waves relating to frequencies higher than 20,000 hertz are called ultrasonic waves or ultrasonics.

A longitudinal mechanical wave of frequency less than the lower limit of audibility is called *infrasonic wave*. Waves generated by a large source *e.g.* earthquake waves are infrasonic waves in nature. The production, properties and applications of ultrasonic waves are discussed in Chapter 10. All these mechanical vibrations are simple harmonic in nature.

1.3. SIMPLE HARMONIC MOTION

Let P be a particle moving on the circumference of a circle of radius a with a uniform velocity v (Fig. 1.1). Let ω be the uniform angular velocity of the particle ($v=a\omega$). The circle along which P moves is called the **circle of reference.** As the particle P moves round the circle continuously with uniform velocity, the foot of the perpendicular M, vibrates along the diameter YY'. If the motion of P is uniform, then the motion of M is periodic *i.e.*, it takes the same time to vibrate once between the points Y and Y'. At any instant the distance of M from the centre O of the circle is called the **displacement.** If the particle moved from X to P in time t, then

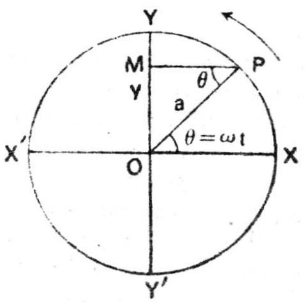


Fig. 1.1.

$$\angle POX=\angle MPO=\theta=\omega t.$$

From the $\triangle MPO$,

$$\sin\theta=\sin\omega t=\frac{OM}{a}$$

or
$$OM=y=a\sin\omega t$$

OM is called the displacement of the vibrating particle. The displacement of a vibrating particle at any instant can be defined as its distance from the mean position of rest. The maximum displacement of a vibrating particle is called its **amplitude.**

$\therefore$ Displacement$=y=a\sin\omega t$ ... (1)

The rate of change of displacement is called the **velocity** of the vibrating particle.

$\therefore$ Velocity$=\frac{dy}{dt}=+a\omega\cos\omega t$ ... (2)

The rate of change of velocity of a vibrating particle is called its acceleration.

$\therefore$ Acceleration =Rate of change of velocity

$$=\frac{d}{dt}\left(\frac{dy}{dt}\right)$$

$$=\frac{d^2y}{dt^2}=-a\omega^2\sin\omega t$$

$$=-\omega^2.a\sin\omega t=-\omega^2y$$ ... (3)

The changes in the displacement, velocity and acceleration of a vibrating particle in one complete vibration are given in the following table.

Angle ωt	Position of the vibrating particle M	Displacement $y = a \sin \omega t$	Velocity $\frac{dy}{dt} = a\omega \cos \omega t$	Acceleration $\frac{d^2y}{dt^2} = -a\omega^2 \sin \omega t$
O	O	Zero	$+a\omega$	Zero
$\frac{\pi}{2}$	Y	$+a$	Zero	$-a\omega^2$
π	O	Zero	$-a\omega$	Zero
$\frac{3\pi}{2}$	Y'	$-a$	Zero	$+a\omega^2$
2π	O	Zero	$+a\omega$	Zero

Oscillatory behaviour. At the extreme positions when y is maximum. dy/dt is zero. The acceleration d^2y/dt^2 is maximum and directed towards the mean position. This return force induces a negative velocity. When the displacement y is zero, the velocity dy/dt is maximum and is $-$ve. When the displacement is negative maximum, the velocity dy/dt is zero and the acceleration is maximum in the positive direction. This return force again induces a velocity in the positive direction which becomes positive maximum when the displacement is zero. The particle overshoots the mean position due to its velocity. The process repeats itself periodically. Thus the system oscillates. In this process, displacement y, velocity dy/dt and acceleration d^2y/dt^2 continuously change with respect to time.

Thus, the velocity of the vibrating particle is maximum (in the direction OY or OY') at the mean position of rest and zero at the maximum positions of vibration. The acceleration of the vibrating particle is zero at the mean position of rest and maximum at the maximum positions of vibration. The acceleration is always directed towards the mean position of rest and is directly proportional to the displacement of the vibrating particle. This type of motion where the acceleration is directed towards a fixed point (the mean position of rest) and is proportional to the displacement of the vibrating particle is called **simple harmonic motion.**

Further,

$$\text{Acceleration} = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$= -\omega^2 \times \text{displacement}$$

Numerically, $\omega^2 = \frac{\text{Acceleration}}{\text{Displacement}}$

or $$\omega = 2\pi n = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

or $$\frac{2\pi}{T} = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

or $$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{K}$$

Thus, in general, the time period of a particle vibrating simple harmonically is given by $T = 2\pi\sqrt{K}$ where K is the displacement per unit acceleration.

If the particle P revolves round the circle, n times per second, then the angular velocity ω is given by

$$\omega = 2\pi n = \frac{2\pi}{T} \quad \left(\because\ n = \frac{1}{T} \text{ where } T \text{ is the time period}\right)$$

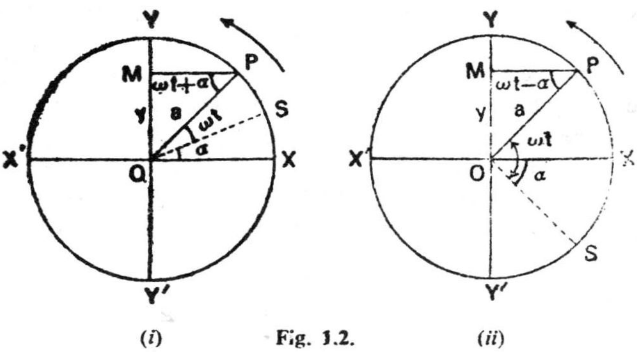
or $$y = a \sin 2\pi nt = a \sin 2\pi \frac{t}{T}.$$

On the other hand, if the time is counted [Fig. 1.2 (i)] from the instant P is at S ($\angle SOX = \alpha$) then the displacement

$$y = a \sin(\omega t + \alpha)$$
$$= a \sin\left(\frac{2\pi t}{T} + \alpha\right)$$

If the time is counted from the instant P is at S' [Fig. 1.2 (ii)], then

$$y = a \sin(\omega t - \alpha) = a \sin\left(\frac{2\pi t}{T} - \alpha\right)$$



(i) Fig. 1.2. (ii)

Phase of the vibrating particle. (i) The phase of a vibrating particle is defined as the ratio of the displacement of the vibrating particle at any instant to the amplitude of the vibrating particle (y/a) or (ii) it is also defined as the fraction of the time interval that has elapsed since the particle crossed the mean position of rest in the positive direction or (*iii*) it is also equal to the angle swept by the radius vector since the vibrating particle last crossed its mean position of rest *e.g.*, in the above equations ωt, $(\omega t+\alpha)$ or $(\omega t-\alpha)$ are called phase angles. The initial phase angle when $t=0$, is called the epoch. Thus α is called the **epoch** in the above expressions.

1.4. DIFFERENTIAL EQUATION OF SHM

For a particle vibrating simple harmonically, the general equation of displacement is,

$$y=a \sin(\omega t+\alpha) \qquad \ldots (1)$$

Here y is the displacement and a is the amplitude and α is epoch of the vibrating particle.

Differentiating equation (1) with respect to time

$$\frac{dy}{dt}=a\omega \cos(\omega t+\alpha) \qquad \ldots (2)$$

Here dy/dt represents the velocity of the vibrating particle.

Differentiating equation (2) with respect to time

$$\frac{d^2y}{dt^2}=-a\omega^2 \sin(\omega t+\alpha)$$

But $a \sin(\omega t+\alpha)=y$

$$\therefore \qquad \frac{d^2y}{dt^2}=-\omega^2 y$$

or

$$\frac{d^2y}{dt^2}+\omega^2 y=0 \qquad \ldots (3)$$

Here d^2y/dt^2 represents the acceleration of the particle. Equation (3) represents the differential equation of simple harmonic motion.

It also shows that in any phenomenon where an equation similar to equation (3) is obtained, the body executes simple harmonic motion. The general solution of equation (3) is

$$y=a \sin(\omega t+\alpha)$$

Also the time period of a vibrating particle can be calculated from equation (3).

Numerically

$$\omega=\sqrt{\frac{d^2y/dt^2}{y}}$$

or

$$\omega=\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

or

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

1.5. GRAPHICAL REPRESENTATION OF SHM

Let P be a particle moving on the circumference of a circle of radius a. The foot of the perpendicular vibrates on the diameter YY'

$$y=a \sin \omega t=a \sin 2\pi \frac{t}{T}$$

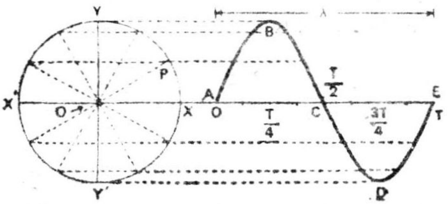


Fig. 1.3. Displacement—Time Curve.

The displacement graph is a sine curve represented by $ABCDE$ (Fig. 1.3).

The motion of the particle M is simple harmonic.

The velocity of a particle moving with simple harmonic motion is

$$v=\frac{dy}{dt}=+a\omega \cos \omega t$$

The velocity—time graph is shown in Fig. 1.4. It is a cosine curve.

The acceleration of a particle moving with simple harmonic motion is

$$\frac{d^2y}{dt^2}=-a\omega^2 \sin \omega t.$$

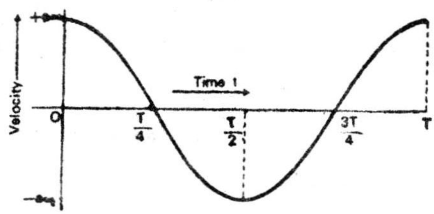


Fig. 1.4. Velocity—Time Curve.

The acceleration—time graph is shown in Fig. 1.5. It is a negative sine curve.

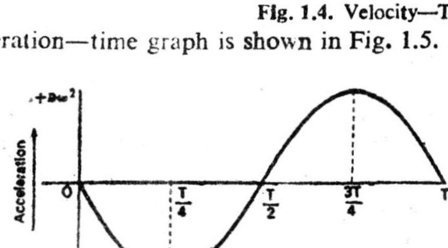


Fig. 1.5. Acceleration—Time Curve.

1.6. AVERAGE KINETIC ENERGY OF A VIBRATING PARTICLE

The displacement of a vibrating particle is given by

$$y=a \sin (\omega t+\alpha)$$

$$v=\frac{dy}{dt}=a\omega \cos (\omega t+\alpha).$$

If m is the mass of the vibrating particle, the kinetic energy at any instant

$$=\tfrac{1}{2}mv^2=\tfrac{1}{2}m.a^2\omega^2\cos^2(\omega t+\alpha).$$

The average kinetic energy of the particle in one complete vibration

$$=\frac{1}{T}\int_0^T \tfrac{1}{2}ma^2\omega^2\cos^2(\omega t+\alpha)dt$$

$$=\frac{1}{T}\cdot\frac{ma^2\omega^2}{4}\int_0^T 2\cos^2(\omega t+\alpha)dt$$

$$=\frac{ma^2\omega^2}{4T}\int_0^T [1+\cos 2(\omega t+\alpha)]dt$$

$$=\frac{ma^2\omega^2}{4T}\left[\int_0^T dt+\int_0^T \cos 2(\omega t+\alpha)dt\right]$$

But, $$\int_0^T \cos 2(\omega t+\alpha)dt=0$$

$\therefore$ Average K.E. $$=\frac{ma^2\omega^2}{4T}\cdot T+0$$

$$=\frac{ma^2\omega^2}{4}=\frac{ma^2(4\pi^2n^2)}{4}=\pi^2ma^2n^2$$

where m is the mass of the vibrating particle, a is the amplitude of vibration and n is the frequency of vibration. Also, the average kinetic energy of a vibrating particle is directly proportional to the square of the amplitude.

1.7. TOTAL ENERGY OF A VIBRATING PARTICLE

$$y=a\sin(\omega t+\alpha)$$

$$\sin(\omega t+\alpha)=\frac{y}{a}$$

$$\cos(\omega t+\alpha)=\sqrt{1-\frac{y^2}{a^2}}=\sqrt{\frac{a^2-y^2}{a^2}}$$

$$=\frac{\sqrt{a^2-y^2}}{a}$$

Velocity $v=a\omega\cos\omega t=a\omega\frac{\sqrt{a^2-y^2}}{a}=\omega\sqrt{(a^2-y^2)}$

$\therefore$ The kinetic energy of the particle at the instant the displacement is y,

$$=\tfrac{1}{2}mv^2$$
$$=\tfrac{1}{2}m.\omega^2(a^2-y^2)$$

Potential energy of the vibrating particle is the amount of work done in overcoming the force through a distance y.

Acceleration $=-\omega^2 y$

Force $=-m\omega^2 y$