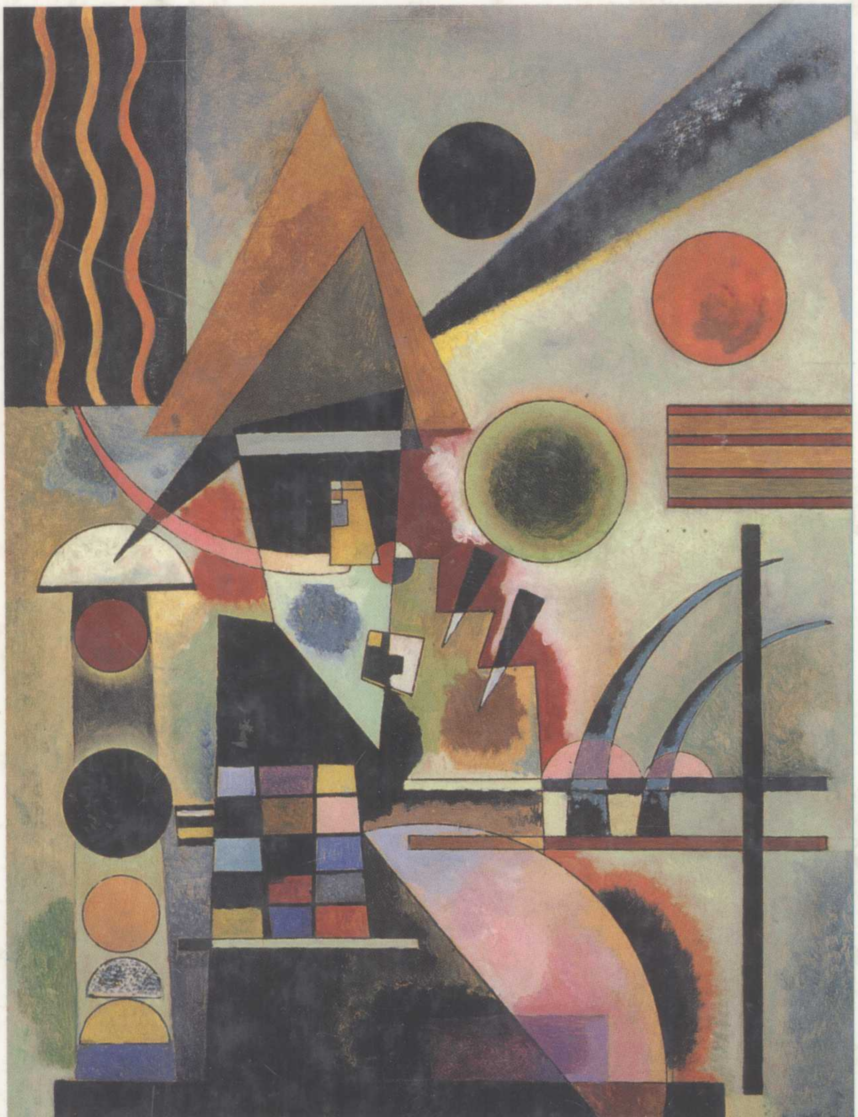
For All Practical Purposes

Introduction to Contemporary Mathematics



Third Edition

FOR ALL PRACTICAL PURPOSES

INTRODUCTION TO CONTEMPORARY MATHEMATICS

THIRD EDITION

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Preface

We began the preface to the first edition of *For All Practical Purposes* with a basic statement of our philosophy:

> Every mathematician at some time has been called upon to answer the innocent question, "Just what is mathematics used for?" With understandable frequency, usually at social gatherings, the question is raised in similar ways: "What do mathematicians do, practice, or believe in?" At a time when success in our society depends heavily on satisfying the need for developing quantitative skills and reasoning ability, the mystique surrounding mathematics persists. *For All Practical Purposes: Introduction to Contemporary Mathematics* is our response to these questions and our attempt to fill this need.
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> *For All Practical Purposes* represents our effort to bring the excitement of contemporary mathematical thinking to the nonspecialist, as well as help him or her develop the capacity to engage in logical thinking and to read critically the technical information with which our contemporary society abounds. We attempt to implement for the study of mathematics Thomas Jefferson's notion of an "enlightened citizenry," in which individuals having acquired a broad knowledge of topics exercise sound judgment in making personal and political decisions. Environmental and economic issues dominate modern life, and behind these issues are complex matters of science, technology, and mathematics that call for an awareness of fundamental principles.
>
> To encourage achievement of these goals, *For All Practical Purposes* stresses the connections between contemporary mathematics and modern society. Since the technological explosion that followed World War II, mathematics has become a cluster of mathematical sciences encompassing statistics, computer science, operations research, and decision science, as well as the more traditional areas. In science and industry mathematical models are the tools par excellence for solving complex problems. In this book our goal is to convey the power of mathematics as illustrated by the great variety of problems that can be modeled and solved by quantitative means.

This philosophy has guided our efforts through the second edition and now into our third. Moreover, the success of previous editions of *For All Practical Purposes* and the emergence of new texts that adopt much of our content yield proof that this philosophy has truly changed the nature of Mathematics for Liberal Arts courses across the country. We are justifiably proud of these results and grateful to those faculty who worked with us to make this revolution possible. We have only one concern. It was our expressed desire, through the publication of FAPP, to demonstrate to students and faculty the broad range of modern mathematics and its applications. We did not intend to substitute one orthodoxy for another — to set in stone a more modern table of contents.

As a consequence, in this edition we have added a good deal of new material, including some results (on envy-free fair division) discovered as recently as 1992. We were fortunate indeed to be able to secure the efforts of Professor Joseph A. Gallian to write two new chapters on Coding Theory. This is exciting and accessible material. The point is a simple one—mathematics is dynamic; new mathematics is being invented and applied in new ways every day. No one set syllabus can capture this continuous sense of invention. Our goal, and our text, must be flexible enough to demonstrate to students the contemporary nature of our subject and its myriad of applications to our daily lives.

To that end, we have clarified, reorganized, and revised topics in the third edition of *For All Practical Purposes* in the following ways:

- Part I, "Management Science," includes an improved version of the edge walker algorithm, further explication of the sorted edges algorithm, and an expanded discussion on linear programming.
- Part II, "Statistics, The Science of Data," includes a discussion on calculating the standard deviation (Chapter 6), more on counting (Chapter 7), and further explication of confidence intervals (Chapter 8). Setting off key principles and definitions within text columns has improved text readability.
- Part III, "Coding Information," features two chapters that explore the role of mathematics in coding information in the modern world, from bar codes on milk cartons to identification numbers on your driver's license to television services. Unavailable in any other textbook, this is accessible material to which students can easily relate.
- Part IV, "Social Choice and Decision Making," now includes separate chapters on Fair Division (Chapter 13) and on Apportionment (Chapter 14) to more fully develop these topics. A discussion of envy-free fair division makes Chapter 13 the most current and unique treatment of fair division available. Weighted Voting (Chapter 12) has been rewritten and includes a discussion on the Shapley-Shubik power index. Other improvements include further explication of topics such as the Hill-Huntington method (Chapter 14), mixed strategies in Game Theory (Chapter 15), Arrow's theorem, and the Balinski-Young theorem (Chapter 11).
- Part V, "On Size and Shape," now devotes individual, shorter chapters to telescopes, Euclidean and non-Euclidean geometry, symmetry and patterns, and tilings.
- Two special 8-page color inserts are featured in this edition. The first, a photo essay on fractals by Richard F. Voss, presents a stunningly beautiful and coherent view of fractal geometry, with captions in ordinary language that students can understand. The second, a collection of tilings and Escher sketches and drawings, complements discussions in Chapter 21 (Symmetry and Patterns) and Chapter 22 (Tilings).
- Writing projects for every chapter encourage students to explore mathematical concepts in a nontraditional way. Students are encouraged to extend the techniques and ideas introduced in text chapters to the discussion of current social and political events, controversies, and other real life situations.
- All chapters include new examples, exercises, and spotlights.

SUPPLEMENTS

We are pleased to provide the following outstanding supplements to accompany the third edition of *For All Practical Purposes:*

- An *Instructor's Guide,* prepared by Chris Leary, St. Bonaventure University, offers detailed chapter outlines and summaries, skill objectives, teaching tips, work sheets for in-class or homework assignments, answers to the even-numbered exercises in the text, as well as an additional exercise section (includes answers). Also featured is a helpful digest of the videotape programs, including running times of each segment and suggestions for coordinating the Annenberg videotape programs with lectures.

- A set of 100 black-and-white *Transparency Masters* with enlarged illustrations and tables from the text, designed to produce excellent overhead transparencies for use in the classroom or lecture hall. This set includes 40 new images from the text.
- A *Telecourse Guide* to accompany the Annenberg video series of 26 half-hour programs. This is a valuable study guide for students enrolled in the telecourse, as well as for students who are enrolled in courses that use the videos in the classroom. Updated to reflect changes within chapters, it includes a cross-reference chart between the videos and the second and third edition texts, program overviews, skill objectives, self-tests, and sample problems. Answers to both self-test questions and the sample problems are included.
- A one-hour video, *Geometry in New Technologies,* with five 12-minute presentations that illustrate some of the exciting modern uses of geometry. It covers vertex coloring to resolve conflict situations; motion planning for robots; creating error-correcting codes of photo transmission from outer space; developing soft-tissue x-rays for medical diagnosis, and the use of Euler circuits to solve cost problems. A printed viewer's guide with exercises and solutions accompanies the video.
- *Election Theory Software,* with two hands-on exercises that will enhance student understanding of the concepts introduced in "Social Choice and Decision Making." Available in Macintosh and IBM formats.

For more information and to request copies of these supplements, please contact Sales Support, W. H. Freeman and Company, 41 Madison Avenue, New York, NY 10010.

For more information on Telecourse preview, purchase, or rental, please call 1-800-LEARNER, or write The Annenberg CPB Project, P.O. Box 2345, South Burlington, VT 05407-2345.

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Alan D. Taylor, Union College

Part V On Size and Shape

Paul J. Campbell, Beloit College

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Solomon Garfunkel
COMAP

To The Student

Have fun. Enjoy. That is what this book is all about. You may at one time or another have wondered why people go into mathematics. Do they have different genes, that they like this stuff? This text is one attempt to show some of the payoffs for working with mathematics. The areas we discuss and the problems we tackle are real. Most of the people you will meet in these pages are alive and much of their work has been done in the last twenty or thirty years. We have tried to show you mathematics as we see it—solving problems we care about and have fun doing.

We do not expect you to become a mathematician. We understand that you won't remember much of the techniques presented here in a year or two. That is not the point. What we hope is that you will do some mathematics, solve some problems, and gain an appreciation for what we do and why. To a large extent mathematics is not about formulas and equations; it is about CDs and CAT-scans, and parking meters, and presidential polls, and computer graphics. Mathematics is about looking at our world and creating representations we can work with to solve problems that count, and we hope it is in this spirit that you use this book. Welcome to the world of contemporary mathematics.

Contents

Those who were watching live television one night in July 1969 will never forget the spectacle of seeing the first person walk on the moon. The element of danger and uncertainty, heightened by a history of trial and error, added to the suspense of the lunar landing. Before the 1960s, no one really knew if rockets would ever be able to carry humans into space.

Neil Armstrong's first step onto the moon's surface was a triumph for American science and technology and the culmination of a national quest that had begun in the office of President John F. Kennedy. It was Kennedy's goal to put a man on the moon before the decade was out, a goal realized in the Nixon administration.

In the eight years from 1961 to 1969, we moved from a president's vision to the reality of a lunar landing. Most of us think of this achievement in terms of the tremendous scientific advances it represented—in physics, engineering, chemistry, and associated technologies. But there was another side to this far-reaching project. Someone had to set the objectives, commission the work, suffer the setbacks, overcome unforeseen obstacles, and tie together a project with thousands of disparate components. The kind of science responsible for such details is a branch of mathematics called **management science.**

NASA administrators faced many new problems in the Apollo moon project: they had to choose the best design for the

The first steps on the moon represented a great leap for management science. Here, Buzz Aldrin poses for Neil Armstrong. [NASA.]

SPOTLIGHT *Apollo 11* Launch Owes Success to Management Science

Captain Robert F. Freitag, who later became director of Policy and Plans for NASA's space station, in 1969 headed the team responsible for landing the *Apollo 11* safely on the moon. The success of the lunar mission can be traced to management-science techniques that ensured that thousands of small tasks would come together to meet a single giant objective. Freitag shares his observations about that historic event:

> I think the feeling most of us in NASA shared was, "My gosh, now we really have to do it." When you think that the enterprise we were about to undertake was ten times larger than any that had ever been undertaken, including the Manhattan Project, it was a pretty awesome event. But we knew it was the kind of thing that could be broken down into manageable pieces and that if we could get the right people and the right arrangement of these people, it would be possible.
>
> In the case of the Apollo program, it was very important that we take a comprehensive system-engineering approach. We had to analyze in a very strict sense exactly what the mission was going to be, what each piece of equipment needed was and how it would perform, and all the elements of the system from the concept on through to the execution of the mission, to its recovery back on earth.

Captain Robert F. Freitag, NASA.

spacecraft, design realistic ground simulations, and weigh the priorities of conducting experiments with immediate returns against carrying out tests that would serve long-term goals. When NASA commissioned the Apollo module, it was asking several hundred companies to design, build, test, and deliver components and systems that had never been built before.

Supporting these space age goals, however, were the nuts-and-bolts issues that make up the major concerns of management science, namely, finding ways to make the operations as productive and economical as possible; details of this support are in the Spotlight on the *Apollo 11* launch.

The *Apollo 11* launch is certainly not the only project in which efficiency and timeliness are im-

We started out, in a very logical way, by having a space station in earth orbit. We would then take the lunar spacecraft and build it in orbit, and then send it off to the moon and bring it back. It turned out that this approach was probably a little more risky and took a lot longer, so with the analyses we made, we shifted our whole operation to building a rocket that would go all the way to the moon after it took off from Cape Canaveral. It would then go into orbit around the moon rather than landing on the moon, and from orbit around the moon would descend to the surface of the moon and perform its exploration. Then it would return to its orbit around the moon and come back home.

Well, that was a very comprehensive analysis job. It was probably more deep-seated than the kind of job one would do for building an airplane or a dam because there were so many variables involved. What you do is break it down into pieces: the launch site, the launch vehicles, the spacecraft, the lunar module, and worldwide tracking networks, for example. Then, once these pieces are broken down, you assign them to one organization or another. They, in turn, take those small pieces, like the rocket, and break it down into engines or structures or guidance equipment. And this breakdown, or "tree," is the really tough part about managing.

In the Apollo program, it was decided that three NASA centers would do the work. One was Huntsville, where Dr. von Braun and his team built the rocket. The other was Houston, where Dr. Gerous and his team built the spacecraft and controlled the flight operations. The third was Cape Canaveral, were Dr. Debries and his team did the launching and the preparation of the rocket.

Those three centers were pieces, and they could break their pieces down into about 10 or 20 major industrial contractors who would build pieces of the rocket. And then each of those industrial contractors would break them down into maybe 20 to 30 or 50 subcontractors — and they, in turn, would break them down into perhaps 300,000 or 400,000 pieces, each of which would end up being the job of one person. But you need to be sure that the pieces come together at the right time, and that they work when put together. Management science helps with that. The total number of people who worked on the *Apollo* was about 400,000 to 500,000, all working toward a single objective. But that objective was clear when President Kennedy said, "I want to land a man on the moon and have him safely returned to the earth, and to do so within the decade." Of course, Congress set aside $20 billion. So you had cost, performance, and schedule, and you knew what the job was in one simple sentence. It took a lot of effort to make that happen.

portant. In a variety of modern projects, ranging from the making of a Hollywood movie to the building of the "Chunnel" (a tunnel under the English Channel), the difference between success and failure depends on whether the project is on time and on budget. Chapters 2 and 3 present some project planning and scheduling techniques that were pioneered in the Apollo program.

But projects are not the only area where organization and efficiency are valuable. Day-to-day operations, such as the delivery of city services discussed in Chapter 1, and the production problems faced by manufacturing firms discussed in Chapter 4, offer big opportunities for cost savings using management science.

Naturally, efficiency is not a modern inven-