

**CONTRIBUTIONS
TO MANAGEMENT SCIENCE**

Elio Canestrelli
Editor

Current Topics in Quantitative Finance



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Elio Canestrelli (Ed.)

Current Topics in Quantitative Finance

With 14 Figures
and 23 Tables



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Preface

The present volume collects a selection of revised papers which were presented at the 21st Euro Working Group on Financial Modelling Meeting, held in Venice (Italy), on October 29-31, 1997.

The Working Group was founded in September 1986 in Lisbon with the objective of providing an international forum for the exchange of information and experience; encouraging research and interaction between financial economic theory and practice of financial decision making, as well as circulating information among universities and financial institutions throughout Europe.

The attendance to the Meeting was large and highly qualified. More than 80 participants, coming from 20 different Countries debated on 5 invited lectures and 40 communications in regular sessions.

The sessions were located at the Island of San Servolo, on the Venetian lagoon, just in front of the Doges Palace. San Servolo Island is a natural oasis, in the midst of a unique urban setting, offering great relaxation in a peaceful park and a panoramic view of Venice. The friendly atmosphere added great benefit to the formal and informal discussions among the participants, -which is typical of E.W.G.F.M. Meetings.

It is interesting to consider the story of the Meeting. The previous locations were held at Cyprus, Crete and Dubrovnik - former milestones of the Venetian Republic influence on the Mediterranean Sea. Therefore, that this Meeting should be harboured in the heart of the Republic itself (namely, the Saint Mark basin), was only a matter of consequence.

Going back to the scientific activity of this Meeting, the main discussed topics were the following: corporate finance; asset price analysis; fixed income securities; portfolio management; decision theory; artificial intelligence for finance; foreign exchange markets; financial derivatives and insurance.

The papers presented in this book provide a representative, though not complete sample of the fields to which the members of the working group devote their scientific activity. Such activity is not only theoretical but also practical because it tries to combine theoretic analyses with empirical evidence. In every-day reality, as well as in the world of

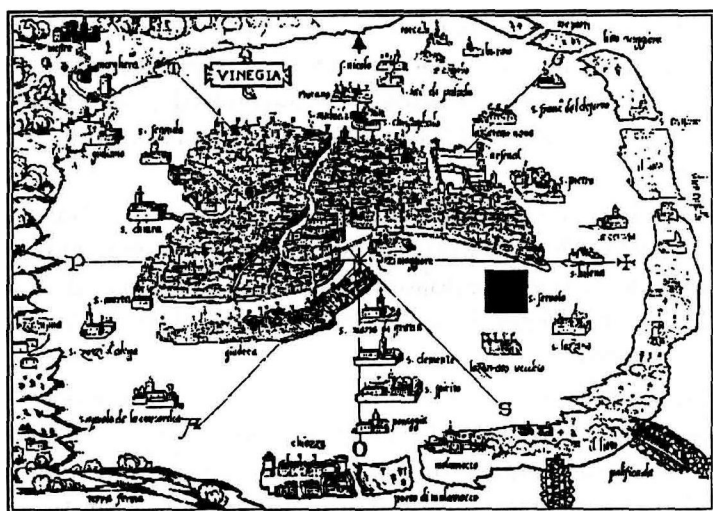
finance, no model is perfect or definite, but only more or less suitable to explain and forecast the taking place of some phenomena.

The E.W.Group wishes to express its deepest thanks and appreciation to the Dpt. of Applied Mathematics and Computer Science of Venice University, which were responsible for the Meeting organization and to the Italian National Research Council (C.N.R.) which offered financial support for the printing of the present book.

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Venezia, March 1999

Elio Canestrelli



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Performance Evaluation of Algorithms for Black-Derman-Toy Lattice

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Abstract. Within the framework of sensitivity of the optimal value of the portfolio management problem described in Dupačová and Bertocchi (1996), Dupačová and Bertocchi (1997) with respect to lattice calibration, we compare Bjerk Sund and Stensland approximation algorithm, Kang Pan-Zenios algorithm and a modified Kang Pan-Zenios algorithm to generate short-rate interest rates tree according to Black-Derman-Toy model. Numerical testing of the behaviour of the three algorithms are given. The necessary inputs for Black-Derman-Toy model are yield curve and log-yield volatilities: we provide an evidence on the relatively large sensitivity of the parameters of the fitted lattice on the chosen volatility curve. The reported numerical experience is based on data from the Italian bond market.

Keywords: Yield curve, volatility curve, BDT model, approximation and Newton-Raphson algorithms.

1 Introduction

In the sensitivity analysis of the bond portfolio management problem, that has been formulated as a stochastic program based on interest

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rate scenarios, see for instance Dupačová and Bertocchi (1996), Golub and al. (1995), the main source of uncertainty comes from the evolution of interest rates.

There are various models of evolution of interest rates; we consider interest rate scenarios sampled from the binomial lattice obtained according to Black-Derman-Toy (BDT) model (1990). The sensitivity of the optimal function value of the portfolio problem with respect to the methodology used for implementing BDT is one of open questions that we want to study. In the literature there are various references to methodologies for implementing BDT model: Kang Pan and Zenios (1992) proposed to use Newton-Raphson steps in an iterative way, Jamshidian (1991) and Rebonato (1996) refer to the use of forward induction methodology and Bjerk Sund and Stensland (1996) suggest new formulas to approximate the short term interest rate tree.

To fit the binomial lattice one needs the initial *term structure* which consists of the *yield curve* and the *volatility curve*, i. e., of the yields and standard deviations of logarithms of yields of zero-coupon government bonds of all maturities covered by the horizon N of the designed bond portfolio management model. To this purpose two main approaches may be used, parametric and nonparametric one, see Dupačová and al. (1997) for detailed discussion. In this paper we shall use linear and nonlinear parametric regression techniques. The BDT model and the inputs for it are discussed in Section 2. Section 3 gives a short description of the considered algorithms. The numerical discussion of the considered techniques is done in Section 4 based on real life data from the Italian bond market.

2 Black-Derman-Toy model

The Black-Derman-Toy model (1990) is a one-factor model which assumes that the short rate is locally lognormal, i.e. small change dr in r during the interval dt is proportional to r , which guarantees the short rate never becomes negative. See also Rebonato (1996) for detailed comments on the model.

The discretized form of the model leads to a lattice that can be fitted by matching the current market information. One important feature of the model is that the path independent property in the lattice is satisfied implying a recombining short interest rate tree with up and down movements equally likely. The term structure of interest

rates, required as an input for the Black-Derman-Toy model, consists of the yields and of the log-yield volatilities valid for the zero-coupon government bonds of all maturities.

The calibration of the binomial lattice in agreement with the (estimated) today's market term structure, provides 2^{N-1} interest rate scenarios \mathbf{r}^s whose common first component equals r_0 and the subsequent components r_n^s (valid for interval $(n, n+1]$, where $n = 1, \dots, N-1$), depend on scenario s .

One can express r_n^s as the product of r_{n0} , the lowest short rate that may occur at time n and $k_n^{l(s)}$ (where $l(s), l < n$, is the number of up movements till time n) the volatility between two adjacent short rates at time n . The lattice is completely defined by the vectors $\mathbf{r}_0 = (r_{01}, \dots, r_{0N-1})$ and $\mathbf{k} = (k_1, \dots, k_{N-1})$.

2.1 Inputs for Black-Derman-Toy model

The uncertainty concerning the interest rate scenarios, prices and the resulting optimal value of portfolio management problem stems mostly from the input information used for calibration and fitting the binomial lattice, namely, on the the initial *term structure* obtained from the existing market data. The term structure consists of the *yield curve* and the *volatility curve*, i. e., of the yields and standard deviations of logarithms of yields of zero-coupon government bonds of all maturities $n = 1, \dots, N$.

To get the yield curve, one uses the observed yields of fixed coupon government bonds traded on a given day and applies parametric or nonparametric regression techniques.

Let the market information at the chosen date consist of the yields y_i , $i = 1, \dots, m$ of various fixed coupon government bonds (without option) characterized by their maturities t_i . The postulated theoretical model

$$y_i = g(t_i; \theta) + e_i, \quad i = 1, \dots, m \quad (1)$$

includes the yield curve $g(t; \theta)$ of a prespecified parametric form where t is usually expressed in years, y is the annualized yield to maturity and $\theta \in \Theta$ is a p -dimensional vector of parameters to be estimated.

Given the market data and the theoretical model of yields, the parameters θ are estimated by the least squares method. It means

that the estimate $\hat{\theta}$ of the true parameter vector θ^* is obtained as a solution of

$$\min_{\theta \in \Theta} S(\theta) := \sum_{i=1}^m (y_i - g(t_i; \theta))^2 \quad (2)$$

The common assumption is that the residuals e_i in (1) are independent, with zero mean values and an equal unknown variance σ^2 which is estimated by

$$s^2 = S(\hat{\theta})/(m - p) \approx S(\hat{\theta})/m$$

for large m .

Provided that the matrix $\mathbf{G}(\theta)$ of gradients $\nabla_{\theta} g(t_i; \theta)$, $i = 1, \dots, m$ is of full rank, the estimates $\hat{\theta}$ from least square approximation are approximately normal, with the mean value equal to θ^* and the covariance matrix $\sigma^2 \Sigma^{-1}$, $\Sigma = \mathbf{G}(\hat{\theta})^{\top} \mathbf{G}(\hat{\theta})$ where σ^2 is estimated by s^2 ; see, e. g., Seber and Wild (1988) for details. This allows to construct approximate confidence intervals for components of the true θ^* and an approximate distribution for $g(t; \hat{\theta})$. This distribution is again approximately normal with the mean value $g(t; \theta^*)$ and variance $\sigma^2 Q^2(t)$, where

$$Q^2(t) = \nabla_{\theta} g(t; \hat{\theta})^{\top} \Sigma^{-1} \nabla_{\theta} g(t; \hat{\theta}) \quad (3)$$

As we mentioned above, in BDT we have to use as input values the yields of zero coupon bonds of all required maturities which are not directly observable. Hence, for each \tilde{t} we replace these yields by their estimates based on the estimated yield curve $g(\tilde{t}; \hat{\theta})$. These estimates are subject to error.

For the yield model we assume that the yield \tilde{y} of a zero coupon government bond with maturity \tilde{t} equals

$$\tilde{y} = g(\tilde{t}; \theta^*) + \tilde{e}$$

with $\tilde{e} \sim \mathcal{N}(0, \sigma^2)$ independent of e_i , $i = 1, \dots, m$. Then the differences of yields corresponding to the estimated and to the true parameter values are approximately normal

$$\tilde{y} - g(\tilde{t}; \hat{\theta}) \sim \mathcal{N}(0, \sigma^2(1 + Q^2(\tilde{t}))) \quad (4)$$

where $Q^2(\tilde{t})$ comes from (3).

Having tried different parametric nonlinear models, as reported in Dupačová, Bertocchi and Abaffy (1996), we chose to use a simple form of the yield curve applied already in Bradley and Crane (1972)

$$y(t; \theta) = \alpha t^\beta e^{\gamma t} \quad (5)$$

We also applied the linearized version of Bradley and Crane's model using logarithms of the already computed yields to maturity as the input and estimating the parameters $\lg \alpha, \beta, \gamma$ by the least squares method.

The techniques for obtaining volatilities of the yields are less obvious and most of the authors work with implied volatility or with an ad hoc fixed constant volatility, say $V(t) = V$ (see e.g. Hull and White (1990), Heath et al. (1992)). In case of a constant volatility, however, the model does not display any mean reversion, see Rebonato (1996). We propose therefore to use the approximate standard deviations of $\lg \tilde{y}$, see Dupačová, Bertocchi and Abaffy (1996).

One can use also volatility curve built from historical data or from implied volatilities; we refer to Kahn (1991), Kuberek (1992), Litterman et al. (1991), Dupačová et al. (1997), Risk Metrics Technical Document (1995) for discussions of various aspects of these different techniques.

3 The three algorithms

The next step is calibration of the binomial lattice in agreement with the (estimated) today's market term structure. The algorithms that we take in considerations for testing refer to Kang Pan and Zenios (1992), Bjerksund and Stensland (1996) and Jamshidian (1991) and our proposal for a modification of Kang Pan-Zenios procedure.

The Bjerksund-Stensland's Backward Algorithm is characterized by two closed formulas that generate an approximate short interest rate tree. The idea behind relies on approximation of expected future short rates (using risk-adjusted probabilities) by their corresponding implicit forward rates and using a risk-neutral valuation for a contingent claim to be evaluated at time step n along the tree.

As concerns Kang Pan and Zenios' technique, our implementation is slightly different because the nominal rate used to discount is not compounded two times per year. Moreover, we suggest to use a new strategy (Modified Kang Pan Zenios algorithm) that allows to compute

Table 1.

Date	n	α	β	γ	means	s^2
Jun 24 '92	28	.123	-.004	-.0053	4.e-08	2.e-06
Jun 03 '93	34	.102	.011	.0038	6.e-09	2.e-06
Jun 13 '94	47	.077	.135	-.0099	1.e-06	2.e-06
Jun 26 '95	24	.102	.044	-.0019	-7.e-08	4.e-06
Jun 24 '96	57	.073	-.027	.0126	4.e-06	2.e-05
Apr 17 '97	60	.057	-.017	.0108	3.e-06	2.e-05

the components of vectors \mathbf{r}_0 and \mathbf{k} all together (that is to solve the system of $2N - 2$ non linear equations in $2N - 2$ unknowns) instead of getting a pair of components by repeated solution of a system of 2 non linear equations in 2 unknowns (see end of Step 2). This allows us to compare convergence and precision of Kang Pan-Zenios procedure and to validate it. For details on these algorithms see Abaffy et al. (1997).

4 Numerical testing

All the numerical testing has been done on DEC 5000/240 workstation under ULTRIX v.4.3 using C and Fortran 77 language. Routines for solving nonlinear equations, nonlinear regression and systems of nonlinear equations come from IMSL and MINPACK library. Accuracy for stopping rules has been set to 10^{-6} . Table 1 reports selected results related to the yield curve obtained by nonlinear regression model (2) and (5) applied for different dates in 1992-1997 using net yields from the Italian treasury bonds (BTP) market to estimate the parameters of the yield curve. The mean values of residuals can be found under heading "means".

The condition number of Σ is of order 2-4, meaning that the matrix is well-conditioned.

The results for the linearized version are reported in Table 2; the estimated values of α are obtained from estimates of their logarithms.

The obtained estimates of parameters reported in Tables 1 and 2 are comparable and the plots of estimated yields /logarithms of yields versus squares of estimated residuals do not indicate any linear trend in the plot neither for the nonlinear nor for the linearized regression for the considered dates that include a sufficiently large number of observations. Both models seem to repeat the same pattern in the plots and the same outliers can be identified. However, goodness of

Table 2.

Date	n	α	β	γ	R^2	s^2
Jun 24 '92	28	.123	-.004	-.0055	.738	1.e-04
Jun 03 '93	34	.102	.011	.0040	.589	2.e-04
Jun 13 '94	47	.076	.137	-.0101	.918	3.e-04
Jun 26 '95	24	.102	.043	-.0019	.792	4.e-04
Jun 24 '96	57	.073	-.029	.0144	.369	3.e-03
Apr 17 '97	60	.057	-.017	.0116	.351	4.e-03

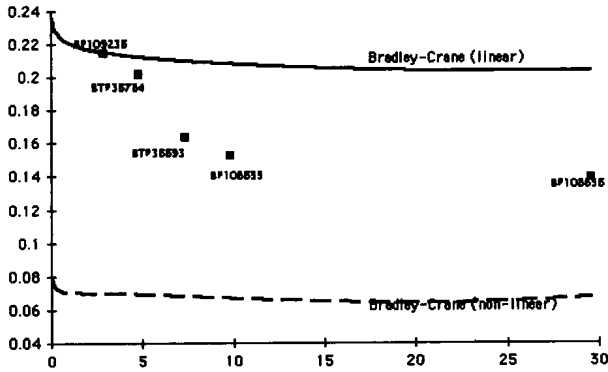


Fig. 1. Volatility structure on April 17, 1997

fit test reported in Dupačová et al. (1997) supports application of the linearized version.

The estimated approximate volatility curves are of similar character both for the nonlinear and the linearized model, see Figure 1. Out of the two models the nonlinear one gives rather low volatilities (yearly) in the range between 1% and 2% in the period 1992-95 and 5% and 7% in the period 1996-97; these volatilities are comparable with the overall standard deviation of the log-yields which come from the market prices of the traded bonds at the given day. The magnitude of volatilities obtained for the linearized Bradley -Crane model is comparable (ranging from 3% to 8% in 1992-95 and 13% to 20% in 1996-97) with the magnitude of the implied volatilities. An approximation of the volatility curve by an exponential smoothing of the implied volatilities has also been considered.

We report in Tables 3,4 complete results of the lattice, i.e. r_{n0} and k_n , using the mentioned algorithms (forward case) for June 24, 1996. For Kang Pan-Zenios' algorithm and for the modified one results are identical. Time steps on the lattice correspond to multiples of six

Table 3. Parameter values for different algorithms - June 24, 1996

Bjersund and Stensland Kang Pan-Zenios and Modified					
time	r_{n0}	k_n	r_{n0}	k_n	volatility
0	0.036643	1.000000	0.036643	1.000000	0.0
1	0.032038	1.230354	0.031974	1.234822	0.149148
2	0.028822	1.226743	0.028786	1.228161	0.148065
3	0.026112	1.224415	0.026088	1.225117	0.147217
4	0.023753	1.222556	0.023736	1.222984	0.146486
5	0.021670	1.220944	0.021656	1.221241	0.145816
6	0.019812	1.219535	0.019801	1.219761	0.145192
7	0.018147	1.218270	0.018137	1.218453	0.144598
8	0.016645	1.217148	0.016636	1.217304	0.144027
9	0.015286	1.216167	0.015278	1.216305	0.143475
10	0.014051	1.215316	0.014043	1.215442	0.142939
11	0.012926	1.214576	0.012919	1.214694	0.142416
12	0.011895	1.214007	0.011888	1.214119	0.141907
13	0.010952	1.213259	0.010946	1.213637	0.141409
14	0.010086	1.213170	0.010080	1.213276	0.140921
15	0.009285	1.212999	0.009279	1.213104	0.140445
16	0.008547	1.212925	0.008541	1.213030	0.139979
17	0.007866	1.212965	0.007860	1.213071	0.139522
18	0.007231	1.213202	0.007225	1.213310	0.139076
19	0.006643	1.213561	0.006637	1.213673	0.138640

months and we cover till 10 years. The inputs are the yields obtained by linearized Bradley and Crane model and volatilities obtained by approximate standard deviation of $\lg y$. Results for the backward cases are identical.

Table 4 shows evidence that there is a large sensitivity of the parameters of fitted lattice on the chosen volatility curve. Increasing input volatility implies an increase in parameter k_n and a decrease in base rate r_{n0} . The strong influence appears in all the experiments we did.

Since now, we shall analyze the methods for date of April 17th, 1997; in this date we were able to collect some of implied volatilities. Among the government bonds (with fixed coupons and without options), BTPs, traded on that day, we have excluded BTP36606 maturing in two weeks horizon. In that date two bonds with very long maturity (around 30 years) were quoted, see Dupačová et al. (1997) for detailed comments on this day. The yield curves estimated according to Bradley-Crane model and according to its linearized version are plotted in Figure 2.

Table 4. Bjerk Sund and Stensland algorithm - June 24, 1996

time	volatility= 0.15		volatility= 0.16		volatility= 0.20	
	r_{n0}	k_n	r_{n0}	k_n	r_{n0}	k_n
0	0.036643	1.000000	0.036643	1.000000	0.036643	1.000000
1	0.032017	1.231812	0.031773	1.249055	0.030802	1.320478
2	0.028690	1.231906	0.028255	1.249169	0.026559	1.320711
3	0.025848	1.232071	0.025263	1.249376	0.023025	1.321152
4	0.023346	1.232323	0.022645	1.249690	0.020009	1.321829
5	0.021116	1.232669	0.020326	1.250123	0.017406	1.322762
6	0.019114	1.233116	0.018257	1.250681	0.015145	1.323967
7	0.017308	1.233670	0.016403	1.251372	0.013173	1.325464
8	0.015674	1.234335	0.014736	1.252203	0.011447	1.327273
9	0.014191	1.235117	0.013233	1.253181	0.009932	1.329414
10	0.012842	1.236023	0.011876	1.254315	0.008600	1.331915
11	0.011614	1.237059	0.010647	1.255613	0.007428	1.334802
12	0.010493	1.238232	0.009535	1.257086	0.006396	1.338108
13	0.009470	1.239550	0.008526	1.258744	0.005486	1.341869
14	0.008536	1.241022	0.007610	1.260598	0.004684	1.346126
15	0.007681	1.242656	0.006778	1.262662	0.003979	1.350926
16	0.006899	1.244462	0.006023	1.264950	0.003360	1.356322
17	0.006184	1.246452	0.005338	1.267477	0.002817	1.362379
18	0.005530	1.248637	0.004715	1.270260	0.002343	1.369167
19	0.004931	1.251031	0.004151	1.273319	0.001930	1.376770

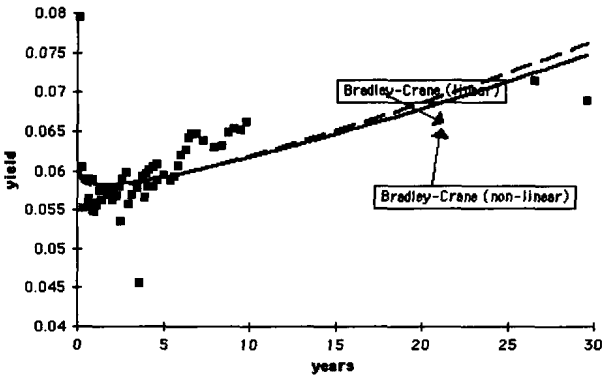


Fig. 2. Term structure on April 17, 1997

Table 5. Average CPU times (in seconds) - April 17, 1997

Case	2x2	$N \times N$ approximation		volatility
backward	0.46	2.94	0.19	constant 0.10
	0.46	2.85	0.27	constant 0.15
	0.50	2.51	0.40	constant 0.20
	1.24	4.13	0.98	s.d. of log yields
forward	0.12	1.32	0.04	constant 0.10
	0.11	1.94	0.06	constant 0.15
	0.12	0.68	0.08	constant 0.20
	0.92	2.01	0.82	s.d. of log yields

In Table 5 we report for linearized yield curve average computational times of the three algorithms in cases of constant volatility or standard deviation of log-yields as input. The modified Kang Pan-Zenios' algorithm is definitely the worst, while Bjerksund and Stensland's approach is the best. CPU time for modified Kang Pan-Zenios' algorithm strongly depends from the chosen initial starting point. It is evident that the forward approach is more than two times faster than the backward one. Moreover, it is less demanding in term of memory occupation. As to the accuracy of results for the linearized input, the Kang Pan-Zenios and the Modified Kang Pan-Zenios algorithms give identical results, but the computing time is much worst for the latter one. The accuracy of Bjerksund and Stensland's algorithm is comparable with that of Kang Pan-Zenios, i.e., it is identical till 3rd decimal digit in k_n and 5th decimal digit in r_{n0} .

5 Conclusions

Algorithms to calibrate Black-Derman-Toy lattice, i.e. Bjerksund and Stensland (1996), Kang Pan-Zenios (1992) and Modified Kang Pan-Zenios, have been compared both from accuracy and CPU times point of view. For purposes of sensitivity analysis with respect to inputs in the dynamic stochastic portfolio management as described in Dupačová et al. (1997), linearized regression model for the yield curve together with Kang Pan-Zenios' algorithm show up to be suitable.

As concerns volatility, evidence is given that BDT lattice parameters are rather sensitive to volatility curve data. However, it appears essential to search deeply into volatility aspects.

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