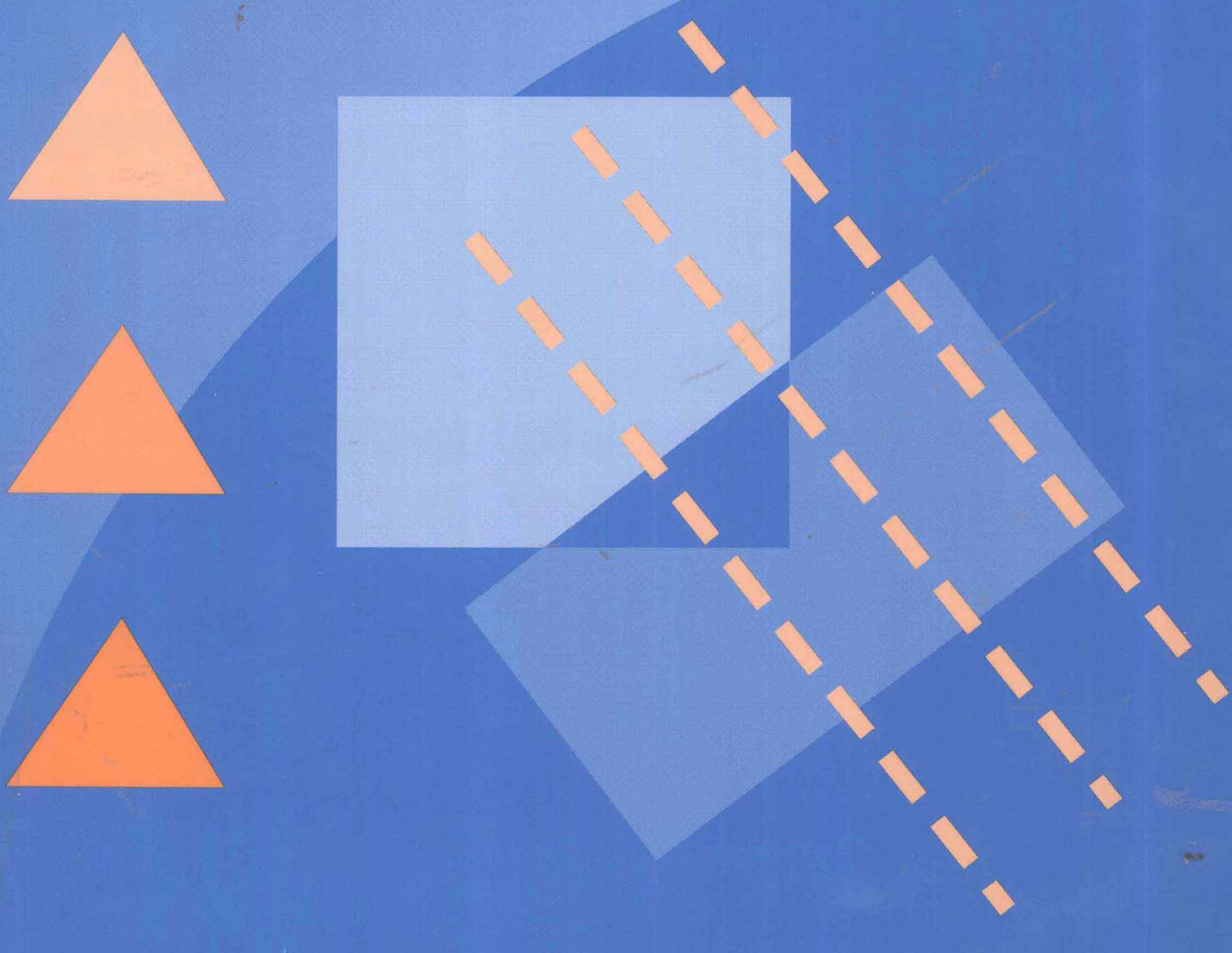
ESSENTIAL GEOMETRY

Harry L. Baldwin, Jr.



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Preface

Essential Geometry presents the basic geometric knowledge needed by educated people in the modern world. A current trend is for many universities to insist that their entering students demonstrate a level of mathematical competency roughly equivalent to that learned during the first two years of high school mathematics. The tests that evaluate such competency almost always include some geometry. A mastery of the material in this book should provide a strong geometric preparation for such a test, or for any college "entry-level" mathematics course, such as intermediate algebra, trigonometry, or college algebra.

A student can study *Essential Geometry* with the assurance that no prior exposure to geometry is assumed. Furthermore, the algebra used in the many examples and exercises is never at a level higher than what is taught in a typical beginning algebra course. This book can be used as a supplement in an algebra course (either beginning or intermediate), or as a stand-alone text for a one-unit geometry course. However, the book was written with the self-study student also in mind. Many examples have been included, all with complete explanations. The answers to all exercises appear at the end of each chapter, conveniently placed immediately following the exercise set. Most answers are accompanied by an explanation showing how the answer was obtained.

All students should be encouraged to make their geometric drawings as accurate as possible, as a means of assessing the reasonableness of their results. To set a good example in this regard, all diagrams in this book show angles with their true measures and relative lengths in correct proportion.

In the writing and production of *Essential Geometry*, much effort was spent to develop the material logically, to include accurate drawings, and to insure that the examples and the exercises (especially the answers) are error free. If any mistake has managed to sneak past our checks, it would be greatly appreciated if it is brought to my attention. I welcome all suggestions and criticisms. Please send them to

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Special gratitude goes to Jim Nulton, my office partner of many years. Not only did Jim's proofreading avert much frustration on the part of future readers, but the many hours of conversations with him helped formulate and consolidate my ideas.

All text and diagrams were computer typeset using the TEX typesetting program written by Professor Donald Knuth of Stanford University. My deepest appreciation goes to Dr. Knuth for creating this amazing program which made writing the book so much fun. The construction of the diagrams was aided by the use of two macro packages, and I thank their writers: Leslie Lamport, author of LATEX, and Michael Wichura, author of PICTEX.

Harry L. Baldwin, Jr.

Summary of Important Formulas

The symbol π, which appears in some of the formulas below, represents the irrational number "pi", which has a value slightly greater than 3.

Useful approximations to π: $\pi \approx 3.14$ $\pi \approx \frac{22}{7}$

Perimeter: The perimeter P of a closed curve (a one-dimensional figure that encloses a region) is the length of the boundary of the region. (If the boundary is a circle, then the perimeter is called the circumference C of the circle.)

Triangle:	$P = a + b + c$	where a, b, and c are the lengths of the sides of the triangle.
Square:	$P = 4s$	where s is the length of a side of the square.
Rectangle:	$P = 2L + 2W$	where L is the length, and W is the width, of the rectangle.
Circle:	$C = \pi d$	where d is the diameter of the circle.

Area: The area A of a region (a two-dimensional figure) is the amount of surface enclosed by the boundary curve.

Square:	$A = s^2$	where s is the length of a side of the square.
Rectangle:	$A = LW$	where L is the length, and W is the width, of the rectangle.
Parallelogram:	$A = bh$	where b is the length of the base (any side), and h is the corresponding height, of the parallelogram.
Triangle:	$A = \frac{1}{2}bh$	where b is the length of the base (any side), and h is the corresponding height of the triangle.
Trapezoid:	$A = \frac{1}{2}h(b_1 + b_2)$	where b_1 and b_2 are the lengths of the bases (the parallel sides), and h is the height, of the trapezoid.
Circle:	$A = \pi r^2$	where r is the radius of the circle.

Volume: The volume V of a solid (a three-dimensional figure) is the amount of space enclosed by the boundary surface.

Cube:	$V = s^3$	where s is the length of an edge of the cube.
Rectangular Solid:	$V = LWH$	where L is the length of the base (any face) of the solid, W the width of the base of the solid, and H is the height of the solid.
Cylinder:	$V = \pi r^2 H$	where r is the radius of the base, and H is the height, of the cylinder.
Pyramid:	$V = \frac{1}{3}AH$	where A is the area of the base, and H is the height, of the pyramid.
Cone:	$V = \frac{1}{3}\pi r^2 H$	where r is the radius of the base, and H is the height, of the cone.
Sphere:	$V = \frac{4}{3}\pi R^3$	where R is the radius of the sphere.

ESSENTIAL GEOMETRY

CONTENTS

Chapter 1

Length

Geometric Figures

A **plane** is a flat surface. No surface in nature, under microscopic examination, would be found to be perfectly flat, but we can imagine the existence of such an ideal surface, and we can use our paper or chalkboard as a representation of that surface. On our plane we can mark a location: a **point**. The mark we make, although small, is not really a point, for a true point has no size at all. Our mark, therefore, is merely a representation (a "picture") of a point.

The entire plane consists of an infinite number of points. Any collection or grouping of these points is called a **geometric figure**. A point is the most elementary or fundamental geometric figure. We can "draw a picture" of a point by making a small dot on our paper, or chalkboard, or whatever we are using to represent the plane that contains the point. To identify a point in a drawing, or when talking or writing about the point, it is often best if the point is given a name. Here we will follow the customary practice of using capital letters to name points.

Imagine that we take a "perfect pencil" (one whose tip is a true "point") and move it along our plane, causing it to leave a mark as it moves. Every location on that mark is a point, and this collection of points is another type of geometric figure: a **curve**. If, as we moved the pencil, we never changed the direction of motion, then the mark made would be a representation of part of a **straight line** (or just **line**). The entire straight line extends forever in opposite directions, but since our representation of the plane is limited (our paper or chalkboard is of finite size) we can draw a picture of only part of the line. In discussions and drawings, lines are often named with lowercase letters, usually near the middle of the alphabet, such as k, l, or m. Sometimes we will name, or "label", a line by using a letter together with a "subscript"—a small symbol (usually a number) written to the lower right of the letter. In such a case, the letter and its subscript (for example, j_2, which can be spoken "jay sub two") form the name of the line.

Applications of geometry involve only parts of lines rather than entire lines. A **line segment** consists of two points on a line (these points are the "endpoints" of the segment) together with all points of the line that lie between the endpoints. A line segment is named by listing the letter-names of its endpoints, in either order.

The points, lines, and line segments in Figures 1.1 through 1.4 of the examples below are referred to in statements that accompany the figures. Some of these statements have been grouped; within a grouping, the statements are "equivalent", which means that they give the same information, even though they are worded differently.

1.

Equivalent statements:

- The point named P lies on the line named m.
- P lies on m.
- m contains P.

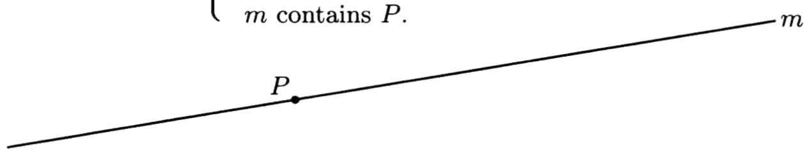


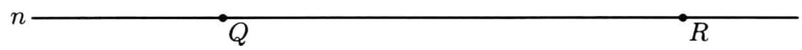
Figure 1.1

2.

Equivalent statements:

- Points Q and R lie on line n.
- Q and R lie on n.
- n contains Q and R.
- Line segment QR lies on line n.
- QR is a segment of n.
- n contains QR.
- n contains RQ.

Figure 1.2



3.

Equivalent statements:

- Point S lies on line k_1, and point S lies on line k_2.
- k_1 and k_2 each contains S.
- k_1 and k_2 intersect at S.
- S is the point of intersection of lines k_1 and k_2.
- Point S is common to lines k_1 and k_2.

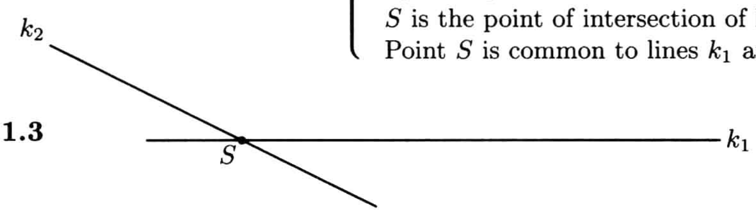


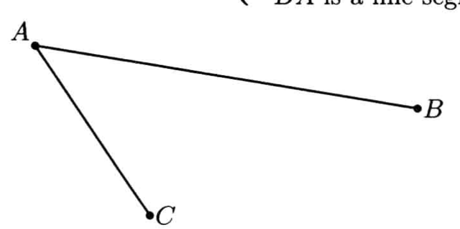
Figure 1.3

4.

Equivalent statements:

- Points A and B are the endpoints of line segment AB.
- A and B are the endpoints of AB.
- AB is a line segment.
- BA is a line segment.

Figure 1.4



Equivalent statements:

- AB and AC are line segments that intersect at A.
- A is an endpoint of line segments AB and AC.

Two points "determine" (or "define") a line. This means that there is only one line that contains a given pair of points. In the same way, two points determine a single line segment. Sometimes a drawing may show a pair of points, but not show the line (or the line segment) that is defined by that pair of points. Often it is helpful to draw a picture of the line (or segment) determined by the points. As an example, consider points X, Y, and Z which are shown in Figure 1.5a. We may alter the drawing to show the line that contains X and Y, or to show the segments XZ and YZ. The result of such alterations is shown in Figure 1.5b. (Of course, there are many other ways that the drawing could be altered.)

Figure 1.5a

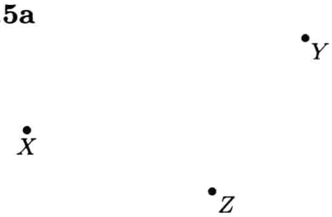
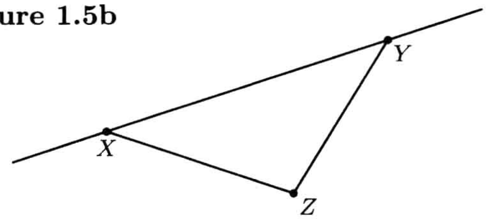


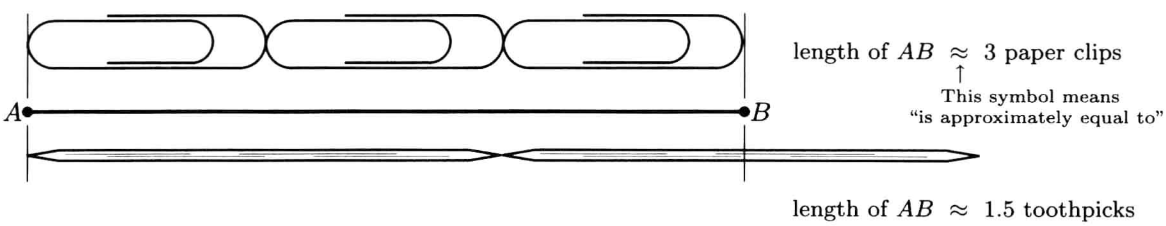
Figure 1.5b



Units of Length

The **measure of a line segment** is its **length**, which is the distance between the endpoints of the segment. A length is described by giving a number together with some word that tells what "unit of length" is being used. Consider, for example, segment AB in Figure 1.6. Suppose paper clips are placed alongside AB, and it is found that three of these paper clips placed end-to-end span about the same distance as does the segment AB. We could then describe the length of AB as being "about 3 paper clips", and anyone who is familiar with those paper clips can visualize the length of the segment. In a similar way, AB could be compared with toothpicks: the length of AB is somewhere between the length of one and two toothpicks; by mentally dividing a toothpick into halves, we might estimate the length of AB as being "about 1.5 toothpicks".

Figure 1.6



A disadvantage of using paper clips and toothpicks as units of length is that these objects come in different sizes, and there has been no agreement on just which brand of paper clip or toothpick is the "standard". In the same way, a "step" (or "pace") and a "city block" are informal units that are useful when describing larger distances, but there is no universal agreement on the exact lengths described by such units. There are, however, many units of length that are widely accepted and used throughout the world; any such unit is referred to as a **standard unit** of length.

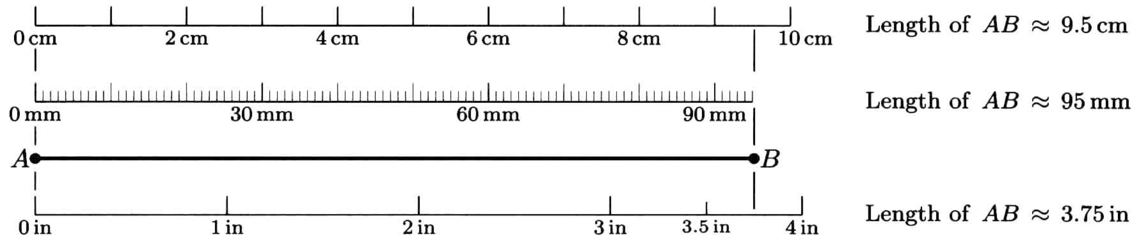
Each of the more-commonly-used standard units of length is a member of one of two "systems of measurement": the "metric system", or the "British system". Although the metric system is easier to use mathematically, the British system is firmly entrenched in the United States, and efforts to replace it with the metric system have not yet been successful. Familiarity with both systems is essential, for an application problem may involve units of length from either or both systems. Shown at the top of the next page are some units of length (and the abbreviations) that are commonly seen in everyday applications (and that are used in problems in this book).

Metric System	
units	**abbreviation**
kilometers	km
meters	m
centimeters	cm
millimeters	mm

British System	
units	**abbreviation**
miles	mi
yards	yd
feet	ft (or ′)
inches	in (or ″)

Which unit of length is most convenient to use depends on the distance that is to be measured. Large distances (such as the distances between cities) are usually given using miles or kilometers. Shorter distances, such as the length and width of a room, are best described using feet, yards, or meters. Lengths less than the width of this page are usually described using inches, centimeters, or millimeters. In Figure 1.7 we again see segment AB (first seen in Figure 1.6), where its length is now described using standard units. In Figure 1.7, the unit distances are accurate, or "at full scale". Other figures in this book may have some labeled lengths that are not at full scale, but within each drawing all lengths shown will have the proper relative size: a segment that is labeled four feet long, for example, will be twice as long as one that is labeled two feet long.

Figure 1.7



Algebraic expressions that include units (whether those units are units of length or any other type of units) can be manipulated just as if the units were variables. As an example, just as $3x + 5x = 8x$, so does $3\,\text{in} + 5\,\text{in} = 8\,\text{in}$. In contrast, the expression $4x + 7y$ cannot be replaced with a single term unless one variable can somehow be eliminated. If, say, we know that $x = 5y$, then a substitution would allow us to obtain an equivalent expression that involves only the variable y:

$$\text{Since } x = 5y, \text{ then } \quad 4x + 7y = 4(5y) + 7y = 20y + 7y = 27y.$$

In a like manner, a sum of lengths involving two different units can be expressed in terms of a single unit if we know a relationship between the units. For example, the sum $5\,\text{ft} + 2\,\text{yd}$ can be expressed in feet by using the fact that $1\,\text{yd} = 3\,\text{ft}$. Here is the conversion:

$$5\,\text{ft} + 2\,\text{yd} = 5\,\text{ft} + 2(1\,\text{yd}) = 5\,\text{ft} + 2(3\,\text{ft}) = 5\,\text{ft} + 6\,\text{ft} = 11\,\text{ft}.$$

We also could express the sum in yards, using the fact that $1\,\text{ft} = \frac{1}{3}\,\text{yd}$:

$$5\,\text{ft} + 2\,\text{yd} = 5(1\,\text{ft}) + 2\,\text{yd} = 5(\tfrac{1}{3}\,\text{yd}) + 2\,\text{yd} = \tfrac{5}{3}\,\text{yd} + 2\,\text{yd} = \tfrac{11}{3}\,\text{yd} \text{ or } 3\tfrac{2}{3}\,\text{yd}.$$

When lengths are added, it often is best if the sum is expressed using only one unit of length. With the help of one or more of the following equations, lengths can be converted between the eight units of length listed earlier on this page:

Some Length-Conversion Equations		
	Within the metric system:	$\begin{cases} 1\,\text{km} = 1000\,\text{m} \\ 1\,\text{m} = 100\,\text{cm} \\ 1\,\text{cm} = 10\,\text{mm} \end{cases}$
	Within the British system:	$\begin{cases} 1\,\text{mi} = 5280\,\text{ft} \\ 1\,\text{yd} = 3\,\text{ft} \\ 1\,\text{ft} = 12\,\text{in} \end{cases}$
	A conversion between the British and metric systems:	$1\,\text{in} = 2.54\,\text{cm}$

Any "conversion equation", such as those above, can be replaced with an equivalent equation that has one side equal to the number 1. This can always be done in two ways. As an example,

$$\text{the equation} \quad 1\,\text{ft} = 12\,\text{in} \quad \text{is equivalent to} \quad \frac{1\,\text{ft}}{12\,\text{in}} = 1 \quad \text{or} \quad \frac{12\,\text{in}}{1\,\text{ft}} = 1.$$

These two ways of writing the relationship between different units of length allow us to express a length using whatever units we desire. For example, suppose we wished to convert a length of 4 centimeters into inches. We begin by writing the equation

$$4\,\text{cm} \;=\; 4\,\text{cm}.$$

We next multiply the right side of the equation by an expression which is equivalent to 1, choosing an expression that will enable the centimeters units (cm), which we don't want, to cancel and leave inches units (in), which we do want. The fraction $\frac{1\,\text{in}}{2.54\,\text{cm}}$ is such an expression:

$$4\,\text{cm} \;=\; 4\,\text{cm}\cdot\frac{1\,\text{in}}{2.54\,\text{cm}} \;=\; \frac{4\,\cancel{\text{cm}}\cdot 1\,\text{in}}{2.54\,\cancel{\text{cm}}} \;\approx\; 1.574\,803\,15\,\text{in}.$$

The division was done with a calculator—a helpful tool for such problems. The approximation symbol (≈) was used because the division did not give an exact value, although the 9-digit number given by my calculator certainly is close to the true value. Your calculator might give more, or perhaps fewer, than 9 digits, but your value also will be very near to the exact value. Roughly speaking, the "precision" of a decimal numeral refers to the number of digits that appear to the right of the decimal point. Since the precision in the above answer is perhaps excessive (a number that shows many digits is not easy to read or visualize), we will "round" our answer to some selected precision. In this case, let's round the answer to the nearest thousandth, or three digits to the right of the decimal point. To do this rounding we will follow the same procedure that is built into many calculators: We begin by discarding all digits to the right of the location corresponding to the precision selected. (In this case our desired precision is thousandths, so we throw away all digits to the right of the 4.) If the leftmost discarded digit is 0, 1, 2, 3, or 4, then after this throwing away of digits we are through. If, however, the leftmost discarded digit is 5, 6, 7, 8, or 9 (and in this case the leftmost discarded digit is 8) then we "round up": we increase the rightmost retained digit by 1 (here, then, we increase the 4 by 1). In this example, our final approximation would be

$$1.574\,803\,15\,\text{in} \;\approx\; 1.575\,\text{in}.$$

Here are a few comments about "rounding":

- If the rightmost retained digit is a 9, and rounding is "up", then increasing the 9 by 1 would give a 0, with 1 being "carried" to the next digit to the left. For example, rounding 7.2961 to the nearest hundredth (two digits to the right of the decimal point) would give 7.30.

- If, after rounding, the rightmost digit is 0 then do not omit the digit 0 from your answer. For example, in the rounding of 7.2961 to the nearest hundredth (discussed immediately above) if we omitted the 0 and reported the answer as 7.3 rather than as 7.30, a reader would believe that the precision is to the nearest tenth rather than to the nearest hundredth. The presence of the digit 0 conveys important information about the precision of the answer.

- Any mathematics problem has some numbers that are given (these are the "input" numbers), and after certain calculations one or more answers are produced (these are the "output" numbers). The precision that should be used when reporting an answer is related to the precisions and magnitudes of the input numbers. In real-world problems, the precisions of the input numbers are usually known (or can be estimated), and procedures exist that, if followed, dictate the precision of the answer. In this book, we will assume that all input numbers to a problem are accurate. Do not round the results of any intermediate calculations in a problem, but if the final answer would have an excessive number of digits, then you will be told to what decimal place that answer should be rounded.

The techniques for converting units and rounding described above will be used in the examples that follow.

---

1. Convert a length of 7 inches to centimeters.

   We start by writing a statement that is certainly true: $7\,\text{in} = 7\,\text{in}$. We then work on the right side only, multiplying by 1 in such a way that the inches cancel and centimeters remain. The fraction that we should use is $\frac{2.54\,\text{cm}}{1\,\text{in}}$:

$$7\,\text{in} = 7\,\text{in}\cdot\frac{2.54\,\text{cm}}{1\,\text{in}} = \frac{7\,\cancel{\text{in}}\cdot 2.54\,\text{cm}}{1\,\cancel{\text{in}}} = 17.78\,\text{cm}.$$

   In this conversion problem, if the input of 7 inches is exact, then, since the conversion equation $1\,\text{in} = 2.54\,\text{cm}$ also is exact, the answer 17.78 centimeters also is exact. Since four digits in the answer is not excessive, we won't round this answer.

---

2. A desk is 118 centimeters long. What is the length of this desk in <u>feet</u>, expressed to the nearest hundredth foot?

   The conversion equations listed earlier do not include one that allows us to change a length in centimeters directly into a length in feet, but by using a couple of the given conversion equations we can make the change in two steps. We first convert from centimeters into inches, and then from inches into feet. After writing $118\,\text{cm} = 118\,\text{cm}$, we then multiply the right side by 1 <u>twice</u>, each time using "a special way of writing the number 1":

$$118\,\text{cm} = 118\,\text{cm}\cdot\frac{1\,\text{in}}{2.54\,\text{cm}}\cdot\frac{1\,\text{ft}}{12\,\text{in}} = \frac{118\,\cancel{\text{cm}}\cdot 1\,\cancel{\text{in}}\cdot 1\,\text{ft}}{2.54\,\cancel{\text{cm}}\cdot 12\,\cancel{\text{in}}} = \frac{118\,\text{ft}}{2.54\cdot 12} \approx 3.871\,391\,08\ \text{ft} \approx 3.87\,\text{ft}.$$

   Notice that the leftmost discarded digit was 1, so we made no change after discarding digits to the right of the hundredth place.

---

3. Suppose you need to convert many distances given in meters into distances given in inches. It would be helpful to have a conversion equation that tells how many inches equal one meter. Derive such an equation.

   Since $\frac{100\,\text{cm}}{1\,\text{m}} = 1$ and $\frac{1\,\text{in}}{2.54\,\text{cm}} = 1$, then

$$1\,\text{m} = 1\,\text{m}\cdot\frac{100\,\text{cm}}{1\,\text{m}}\cdot\frac{1\,\text{in}}{2.54\,\text{cm}} = \frac{1\,\cancel{\text{m}}\cdot 100\,\cancel{\text{cm}}\cdot 1\,\text{in}}{1\,\cancel{\text{m}}\cdot 2.54\,\cancel{\text{cm}}} = \frac{100\,\text{in}}{2.54} \approx 39.370\,078\,74\ \text{in}.$$

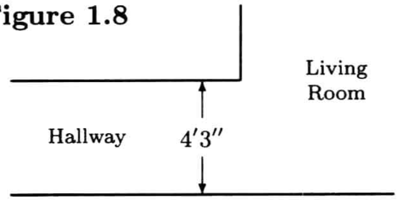
   Rounding to the nearest hundredth inch yields this "approximate equation": $1\,\text{m} \approx 39.37\,\text{in}$.

---

4. In a drawing of a house plan (part of the plan is shown in Figure 1.8 at right), a hallway is shown as having width $4'3''$. Convert this width into <u>inches</u>.

The dimension $4'3''$ is a shorter way of writing "4 ft 3 in", which means $4\,\text{ft} + 3\,\text{in}$. We will change 4 ft into inches by multiplying by the fraction $\frac{12\,\text{in}}{1\,\text{ft}}$:

Figure 1.8



$$4'3'' = 4\,\text{ft} + 3\,\text{in} = 4\,\text{ft} \cdot \frac{12\,\text{in}}{1\,\text{ft}} + 3\,\text{in} = \frac{4\,\cancel{\text{ft}} \cdot 12\,\text{in}}{1\,\cancel{\text{ft}}} + 3\,\text{in} = 48\,\text{in} + 3\,\text{in} = 51\,\text{in} \quad (\text{or } 51'').$$

Geometric Drawings

No length is described completely unless both a number <u>and</u> the units of length are given. However, in some discussions or drawings the units are omitted. In these cases, we will assume that all distances in the discussion have the <u>same</u> units of length, and that the statements in the discussion are true no matter what units would be used.

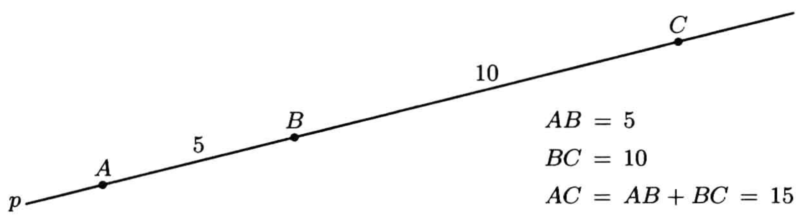
Often the length of a line segment is given using an equation, in which one side of the equation is the <u>name</u> of the segment, and the other side of the equation is the <u>length</u> of the segment. In such an equation, the name of the segment is being treated as a constant, and can be used in algebraic expressions just like any symbol that represents a constant. As an example, Figure 1.9 shows that line p contains points A, B, and C, and that the lengths of segments AB and BC are 5 and 10 (units of length), respectively. We may write

$$AB = 5 \qquad \text{and} \qquad BC = 10.$$

Furthermore, since segment AC can be formed by joining segments AB and BC, we know that

$$AC = AB + BC, \quad \text{so} \quad AC = 5 + 10 \quad \text{or} \quad AC = 15.$$

Figure 1.9



(Notice that the units of length were omitted in the discussion and drawing above. The statements are true regardless of what units of length are used, so long as all lengths have the <u>same</u> units.)

If a problem involves the lengths of line segments, then a good way to begin is by writing, on the drawing, all known lengths. As other lengths become known, they also should be written on the drawing. A number written next to a segment is giving the length of the segment whose indicated endpoints lie near, but on opposite sides of, that number. Sometimes a brace (or other symbol) must also be used, to clearly indicate what segment a length applies to. For example, Figure 1.10a shows that XZ has length 24 and XY has length 10. From this it can be deduced that YZ has length 14, and this fact is shown on the altered drawing in Figure 1.10b. (When writing the length of a line segment on a drawing, you need not redraw the figure. Merely write the length on the existing drawing.)

Figure 1.10a

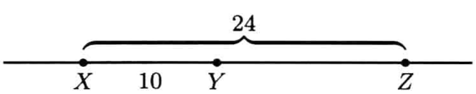
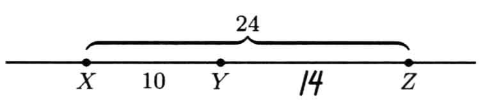


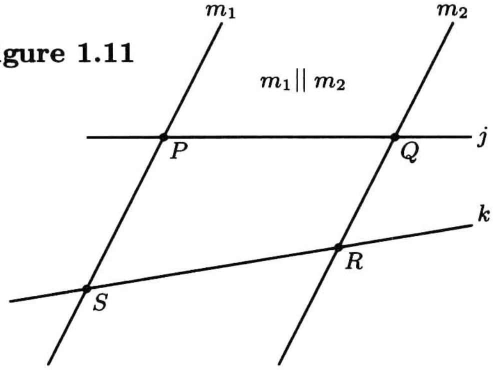
Figure 1.10b



Two lines are **parallel** if they lie in the same plane and they do not intersect. Two line segments are parallel if the lines that contain those segments are parallel. Suppose lines m_1 and m_2 are parallel, as shown in Figure 1.11. This fact can be written $m_1 \,||\, m_2$, where the symbol $||$ means "is parallel to". Lines j and k are not parallel, but intersect at some point to the right of the drawing. Because $m_1 \,||\, m_2$, and since PS and RQ are segments of lines m_1 and m_2, respectively, then each of these statements also is true:

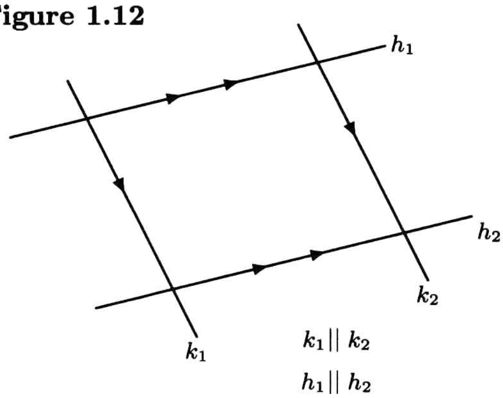
$$PS \,||\, QR \qquad PS \,||\, m_2 \qquad m_1 \,||\, QR$$

Figure 1.11



Unless information is given that two lines (or line segments) are parallel, then, even though the lines look as if they are parallel in a drawing you should not assume that they are. It is possible that lines that appear to be parallel actually intersect at some distant point. If it has been established (or if it is given) that two lines are parallel, then this fact can (and should) be indicated on the drawing by imbedding an arrow in each line of the parallel pair. Figure 1.12 shows two examples of this notation: k_1 and k_2 are parallel, as indicated by a single arrow imbedded in each line; h_1 and h_2 also are parallel, as indicated by two arrows imbedded in each line.

Figure 1.12



Some geometry problems (both in this book and in "real life") are given without any accompanying drawing. In such cases, your first goal should be to draw a picture that properly shows all of the given information. Segments with known lengths should be drawn "to scale": if, say, AB is given to be longer than CD, then your drawing should show that. Any segment having a known length should be labeled with that length, for after your drawing is complete you will then know that any segment without a labeled length has an "unknown" length. Lines or segments that are known to be parallel should be drawn so that they do not appear to intersect even if they were extended, and don't forget to draw the arrows that indicate the lines are parallel. The better your drawing, the better your chances of properly judging the reasonableness of any answers.

Suppose a drawing is to show a couple of line segments, but no information is given about the lengths of the segments, or whether two segments are parallel or not. In such a case, your drawing should be made as non-specific as possible: do not draw the segments so that they appear to have the same length, and do not draw them so that they appear to be parallel, for looking at such a drawing might cause you to think that they have equal lengths or that they are parallel, and thus you might be misled during the solving of the problem. As a problem is solved, information may become available that might make you want to alter your drawing. Sometimes an alteration will not require more than appending more segments or labels to your existing drawing. However, a change could be so major that it might be best to make a fresh start. Begin a completely new drawing that will accurately show any given facts and any facts that you have deduced from the given information.

A final bit of advice: don't become discouraged. As you practice and gain experience, making an adequate drawing in a reasonable amount of time will certainly become easier.

Chapter Summary

A **point** is a location in space.

A point has no size.

A point is named with a capital letter.

the point named A

A **line** (or **straight line**) extends forever in opposite directions.

Two points define a unique line.

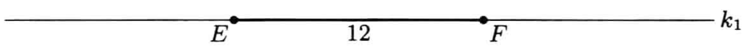
A line can be named using a lowercase letter (perhaps with a subscript).

the line named m, defined by the points B and C

A **line segment** is part of a line: two points (the endpoints) together with that part of the line between the endpoints.

A line segment has a measure: its length.

A line segment is named by naming its endpoints.



the line segment EF, of length 12 units, contained in line k_1

Some commonly-used **standard units of length** include the following:

Metric System:			**British System:**		
millimeter	⇥⇤ 1 mm		inch	⊢ 1 in ⊣	
centimeter	⊢1 cm⊣	1 cm = 10 mm	foot		1 ft = 12 in
meter		1 m = 100 cm	yard		1 yd = 3 ft
kilometer		1 km = 1000 m	mile		1 mi = 5280 ft

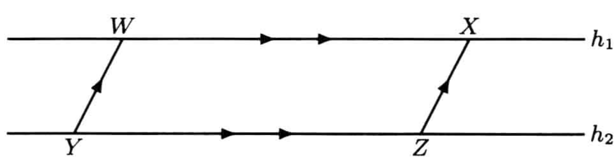
An exact conversion between the Metric and British systems: 1 in = 2.54 cm

Two lines (or line segments) are **parallel** if they do not intersect (even if extended indefinitely).

$||$ is the symbol for "is parallel to".

An arrow (or two arrows) embedded in each of a pair of lines in a drawing indicates that those lines are parallel.

$h_1 || h_2$ and $YW || ZX$



Exercises 1

The answers to each set of exercises will be found in a box immediately after the exercise set.

An exercise is identified with the symbol Ⓒ if the solving would be significantly easier with the help of a calculator.

1. In the figure, points A, B, C, and D are marked and labeled. Alter the drawing, by sketching and labeling each of the following:

a) line n, which contains A and B

b) l, which contains B and C

c) segment AC

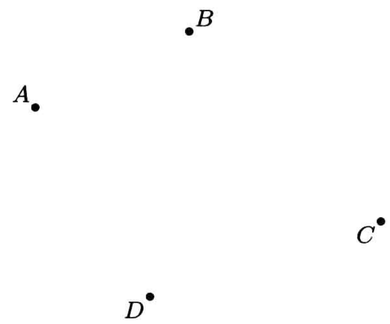
d) BD

e) E, which is the point of intersection of AC and BD

f) m, which contains C and is parallel to BD

g) F, which is the intersection of m and n

h) G, where G lies on BC, and $BG = GC$ (G can be called the "midpoint" of BC.)



2. Points E, F, G, and H are shown in the figure below. Alter the drawing, by sketching and labeling each of the following:

a) line k_1, determined by points E and G

b) segment GF

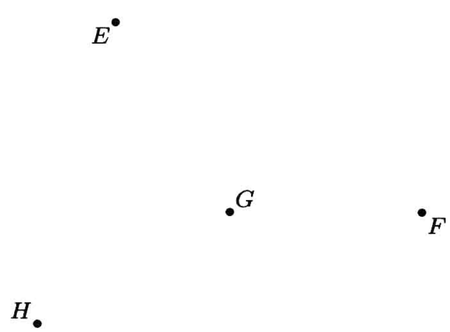
c) HF

d) X, which is the point of intersection of HF and k_1

e) k_2, which contains H and is parallel to GF

f) Y, which is the intersection of k_1 and k_2

g) Z, where Z is the midpoint of HY

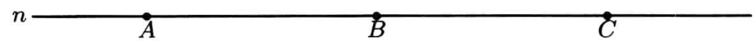


3. Indicate on the drawing the known lengths of line segments. Then determine the lengths of the other line segments, and label them on the drawing. (The labeling may require using a brace to properly identify the segment.)

a) $AB = 7$

$BC = 8$

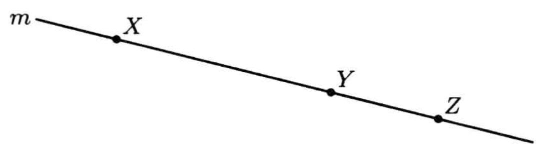
$AC =$ ______



b) $XZ = 24$

$YZ = 8$

$XY =$ ______

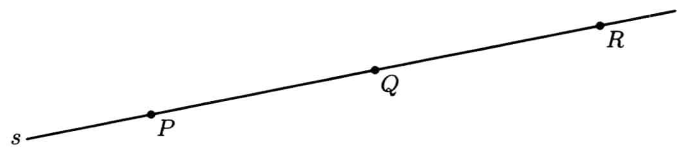


c) $PR = 18$

$PQ = QR$

$PQ =$ ______

$QR =$ ______

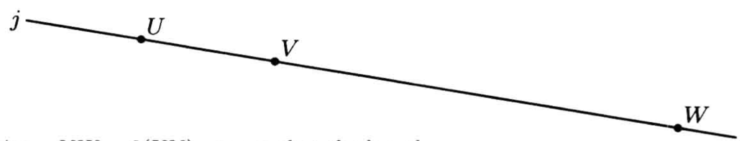


d) $UV = 7$

$VW = 3(UV)$

$VW =$ ______

$UW =$ ______



Note: $VW = 3(UV)$ means that the length of VW is three times the length of UV.

e) $EF = 20$

$HF = 18$

$EG = \frac{1}{4}(EF)$

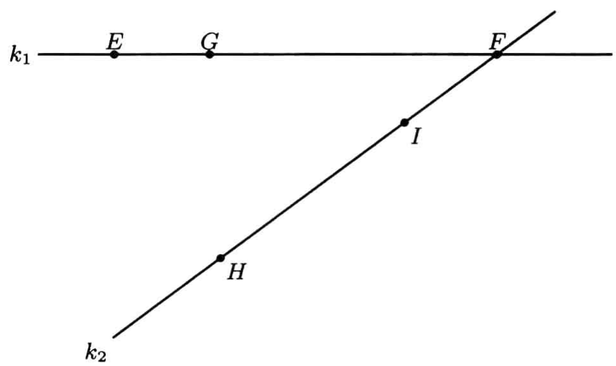
$IF = \frac{1}{3}(HF)$

$GE =$ ______

$GF =$ ______

$IF =$ ______

$HI =$ ______



f) $PR = 12$

$QR = \frac{1}{3}(PR)$

$RS = \frac{3}{4}(PQ)$

$QR =$ ______

$RS =$ ______

$PS =$ ______