McGRAW-HILL PROFESSIONAL ENGINEERING

ELECTRICAL ENGINEERING

Power System Load Flow Analysis

- Formulate nonlinear load flow equations
- Develop solution methods and algorithms
- Master a range of load flow analysis techniques
- Model networks and equipment mathematically

Lynn Powell

Power System Load Flow Analysis

Lynn Powell

McGraw-Hill
New York Chicago San Francisco Lisbon London
Madrid Mexico City Milan New Delhi San Juan
Seoul Singapore Sydney Toronto

The McGraw-Hill Companies

Library of Congress Cataloging-in-Publication Data

Powell, Lynn.
Power system load flow analysis / Lynn Powell.
p. cm.
Includes bibliographical references and index.
ISBN 0-07-144779-2
1. Electric power systems—Load dispatching. I. Title.

TK1005.P7156 2004
621.31′7—dc22 2004054871

1 2 3 4 5 6 7 8 9 0 DOC/DOC 0 1 0 9 8 7 6 5 4

ISBN 0-07-144779-2

The sponsoring editor for this book was Stephen S. Chapman and the production supervisor was Sherri Souffrance. It was set in Century Schoolbook by International Typesetting and Composition. The art director for the cover was Margaret Webster-Shapiro.

Printed and bound by RR Donnelley.

To Sandra, for her support,
her encouragement, and, above all, her patience

Preface

The inspiration for this book comes from many sources. For me, power system engineering has been a consuming passion for the whole of my career, and the mathematical solution of power system problems an integral part of that passion.

Being introduced to the subject as an apprentice in the electricity supply industry was the key. The requirement was obvious, and planning the means of providing the requirement I found exciting. The seed was planted, and there was no hope for me.

Not until I was well through my first degree course did I encounter the individual who—through his enthusiasm alone—nurtured the seed and caused it to grow. Derek German was at that time a senior lecturer at The Welsh College of Advanced Technology, and I shall be grateful to him always.

Years later, during my power systems engineering MSc program at the University of Manchester Institute of Science and Technology, I was fortunate enough to meet Alfred Brameller, world authority in the field of power systems and numerical analysis—and I could easily go on, but the page is too short. My eternal thanks go to him, too.

These two men are responsible for my addiction.

A further stimulus for me to sit down and put pen to paper was the way in which the subject was imparted to the student; I cannot use the word "taught," for that is not the role of the lecturer at University level. With a wide curriculum of topics to cover in a short timescale, it is possible only to present the subject; it is the role of the student to follow up intelligently with any necessary detail. However, I did notice that a consequence of this strategy was that many students could not grasp the detailed fundamentals from such a cursory coverage, nor did they have time within such a busy programme to delve further.

In the same way, specialist books on power system engineering contain one chapter on the subject of load flow and therefore the coverage can only be limited. Nevertheless, the publications listed at the back of this book have provided invaluable input, and in certain cases marvellous detail.

I felt that it was time to prepare a work on load-flow analysis alone, and to deal with the subject in some detail, particularly in the numerical examples. This I have tried to do, and I hope that it will provide students of the subject with helpful insight.

Lynn Powell
Wiltshire, 2004

Introduction

During the past 50 years, electrical engineers have seen a transformation in the field of power system engineering. The advent of the semiconductor device has seen widespread and often novel application, notably in system protection, power conversion, system control, and system operation.

Initially there was an apparently cautious step from appearance of the technology to its application. This was not due to the novelty of the untried, though that was undoubtedly a concern. Rather, the caution was nothing more than a manifestation of the time scales involved.

In the United Kingdom, the building of the nationwide electrical power transmission and distribution systems after the war saw a huge investment in capital plant and the establishment of high-quality maintenance programmes. As a result, the operating life of that equipment saw its end only in the 1970s. Accordingly, replacement of the popular oil-immersed switchgear, for example, with gas or vacuum devices, did not coincide with the ability of industry to provide that technology. Similarly, solid-state protective devices for systems were available long before their electromechanical antecedents had reached the end of their useful life.

Only in one field was the technology too slow for the need, and that was in the area of predictive analysis of power networks. Electricity authorities had for years used physical simulators: analogue devices on which it was possible to model networks, but they were limited in application to medium-sized systems (without recourse to simplification) and notoriously laborious to set up and balance.

These analogue devices—network analysers—took various forms, including a direct representation device operating at 50 Hz in which the circuit elements (R, X, and C) were bulky: the analyser filled a floor of a multistory block. One used by the author was less bulky, thanks to a novel choice of operating frequency f of 1592 Hz, which gave a convenient value of 10,000 to $\omega\ (= 2\pi f)$. This meant that the circuit elements were compact, and the analyser sat in a small room.

The procedure for obtaining load-flow results from these devices was protracted, and a single study on a small system could take a couple of hours. In addition, because of the size limitation, for larger systems a simplification procedure was necessary to produce a system that would fit onto the machine. It was all too slow.

The advent of commercial computers was greeted by the engineering community like a refreshing draught, none more so than power systems engineers, who saw—at last—a means of modeling the systems that were simply too large to be accommodated on even the largest analogue devices. As detailed in this book, the mathematical concepts were well understood, and the appearance of high-level languages gave the engineers themselves the tools to build their programs. The big problem was the computers themselves. Having minuscule central processing unit (CPU) memory (by today's standards vanishingly small) and being expensive to run in their air-conditioned temples, they gave little to their potential users and demanded much of their ingenuity. Faced with these problems, engineers were forced to optimize their programming with every problem, both to squeeze each program segment into memory and to ensure each program would run as quickly as possible. The search for better techniques began almost as soon as computer applications became available.

The load-flow problem was first solved using the simplest techniques, soon to be replaced by more sophisticated methods. The limitations in memory were avoided (to a degree) by clever use of optimization techniques such as sparsity programming, where only the elements describing each system were used in the analysis. The fact that power system admittance matrices are highly sparse means that only nonzero elements need be stored. This in itself demands sophisticated indexing techniques, another level of complication in the search for maximising computer utilization.

Many alternative solution techniques have been investigated during the intervening decades, and many modifications have been attempted to speed them up. The great debate on whether nodal or mesh analysis is better gradually died when the simpler programming involved in the nodal approach, coupled with the use of high-efficiency techniques such as Newton methods, saw the end of the practical use of mesh analysis. That is why the techniques described in this book are exclusively nodal methods.

The development of computer methods for load-flow analysis actually benefited from the availability of only slow-speed, limited-memory computers. It was necessary always to seek techniques that would optimize their performance, and as a result today's commercial load-flow programs offer the advantage of high-speed operation, along with the availability of cheap, high-speed computers. It must be remembered that a main-frame machine such as the IBM System 360 of the mid-1960s is far superseded in performance by the average desk top PC of today.

The importance of high-speed operation of load-flow programs can best be understood by considering their application in terms of system control, rather than system planning. Whereas the planning engineer requires fast response to examine differing network configurations and loadings, the control engineer requires an ideally instantaneous response, so that the effects of possible network changes are available to him in real time. In practical terms, modern load-flow programs can provide him with a high-speed response not far removed from the real-time ideal.

In addition to providing a prediction of the loading conditions on power systems, a load-flow solution is required as a necessary precursor to

1. A system fault study
2. A transient stability study

System short circuit studies are important in deciding upon the type and sensitivity of protection to be provided on the system. It is vital to understand the magnitude of the currents that will flow on a faulted system, so that equipment of the correct type and rating can be selected to clear fault(s) as quickly as possible, thereby minimizing damage to system elements and widespread supply interruption. Computer analysis of faulted systems requires an initial load flow of the healthy system to establish prefault starting conditions.

Transient stability studies examine the effect of network disturbances on the postdisturbance ability of synchronous machines connected within the network to remain in synchronism with one another. Loss of synchronism can lead to undesirable power swings, again causing widespread disruption to supplies. Computer programs that examine the effects of such disturbances are step-by-step methods that solve the differential equations describing all elements connected to the network. When it is considered that a single generator might be described by multiple differential equations (perhaps 10), then the complexity of analyzing a multimachine system can be appreciated. Once more, an initial load flow is required to establish predisturbance starting conditions.

This book presents the major nodal analysis techniques that have been developed through the last five decades. Each technique is developed mathematically, and the algorithm used by the author to develop the appropriate computer program is presented as a flow chart. The application of the technique is illustrated for a small reference power system, which allows the relative performance of each method to be assessed.

Detail of the numerical analysis for the first iteration of each technique is presented as an aid to understanding of the relevant technique.

ABOUT THE AUTHOR

Lynn Powell is an electrical engineer who is now retired after spending 40 years with various U.K. organizations: the South Wales Electricity Board, the Central Electricity Generating Board, the British Steel Corporation, and the Ministry of Defence. For virtually all that time he was engaged in power system design for land-based and ship-based systems. He holds a master's degree in power systems engineering from the University of Manchester Institute of Science and Technology and is a Fellow of the Institution of Electrical Engineers. A Welshman by birth, he lives in Wiltshire, England.

Contents

Preface xi
Introduction xiii

Chapter 1. System Representation 1

1.1 Introduction 3
1.2 The Per-Unit System 4
1.3 Per-Unit Transformer Representation 6
1.4 Per-Unit Power System Representation 8

Chapter 2. The Load-Flow Problem 13

2.1 Physical Statement of the Problem 15
2.2 Mathematical Statement of the Problem 16
2.3 Representation of System Elements 19
2.3.1 Lines and cables 19
2.3.2 Generators 20
2.3.3 Transformers 20
2.3.4 Loads 20
2.3.5 Shunt elements 20

Chapter 3. Reference System 21

3.1 Introduction 23
3.2 System Configuration 23
3.3 Formulation of System Admittance Matrix 23

Chapter 4. Jacobi Method 29

4.1 Introduction 29
4.2 Development of the Algorithm 29
4.3 Jacobi Method Solution for Reference System 32

Chapter 5. Gauss-Seidel Method 37

5.1 Introduction 39
5.2 Development of the Algorithm 39

5.3 Gauss-Seidel Solution for Reference System 41
5.4 Acceleration 43

Chapter 6. Z-Matrix Methods 47

6.1 Introduction 49
6.2 Development of the Method 49
6.3 Z-Matrix Method: Algorithm for Block Substitution 51
6.4 Z-Matrix (Block Substitution) Solution for Reference System 53
6.5 Z-Matrix Method: Algorithm for Forward Substitution 56
6.6 Z-Matrix (Forward Substitution) Solution for Reference System 56

Chapter 7. Newton-Raphson Methods 63

7.1 Solution of Equation $y = f(x)$ 65
7.2 Solution of Multivariable Nonlinear Equations 66
7.3 Newton-Raphson and the Load-Flow Problem 68

Chapter 8. Newton-Raphson Method Using Cartesian Coordinates 71

8.1 Development of the Algorithm 73
8.2 Newton-Raphson (Cartesian Coordinates) Solution for Reference System 76

Chapter 9. Newton-Raphson Method Using Polar Coordinates 83

9.1 Development of the Algorithm 85
9.2 Newton-Raphson (Polar Coordinates) Solution for Reference System 92

Chapter 10. Fast Decoupled Method 97

10.1 Introduction 99
10.2 Decoupled Newton-Raphson Method 101
10.3 Development of the Fast Decoupled Method 101
10.4 Development of the Algorithm 104
10.5 Fast Decoupled Solution for Reference System 104

Chapter 11. DC Load Flow 111

11.1 Introduction 113
11.2 Development of the Method 114
11.3 Development of the Algorithm 116
11.4 DC Load-Flow Solution for Reference System 116

Chapter 12. Voltage Control (1): Generators 119

12.1 Introduction 121
12.2 Performance of a Synchronous Machine 121
12.3 Generator Representation in the Load-Flow Problem 125
12.4 Solution for Reference System Including a Generator Busbar 127

Chapter 13. Voltage Control (2): On-Load Tap-Changing (OLTC) Transformers 133

13.1 Introduction 135
13.2 Development of Transformer Equivalent Circuit for Tap Changing 136
13.3 Transformer Tap Changing: Illustrative Example 139
13.4 Changes in Admittance Matrix Resulting from Tap Changing 140
13.5 Tap Changing within the Load-Flow Process 143
13.6 Reference System Including OLTC Transformers 145
13.7 Gauss-Seidel Solution for Modified Reference System 147

Chapter 14. Results Output 151

14.1 Introduction 153
14.2 Original Reference System 153
14.2.1 Busbar conditions 153
14.2.2 Line flows 155
14.3 Reference System Including Generator 157
14.3.1 Busbar conditions 158
14.3.2 Line flows 158
14.4 Reference System Including OLTC Transformers 159
14.4.1 Busbar conditions 159
14.4.2 Line flows 159

Chapter 15. Solution Difficulties 161

15.1 Introduction 163
15.2 Common Considerations 163
15.2.1 Conditioning 163
15.2.2 Sparsity 164
15.2.3 Storage of matrix elements 165
15.2.4 Use of implicit functions 165
15.2.5 Ordered elimination and matrix inversion 166
15.3 Starting Conditions 166

Appendix 169
References 173
Further Reading 175
Index 177

Chapter

1

System Representation

1.1 Introduction

It is usual to represent a balanced three-phase network as a single-phase equivalent, and to use per-unit quantities within that scenario. This is not merely convenient; it would be unnecessary to embark upon the more complex three-phase analysis for balanced three-phase conditions—there, the single-phase analysis would be accurate.

Of course, if the loading of *individual* phases was known—in terms of connected consumers and their (single-phase and three-phase) predicted demands—then it might seem logical to embark upon the more complete analysis afforded by the three-phase case. However, this would require high accuracy in demand prediction, supported by a well-detailed engineering database of consumer connections, and would need rigorous justification.

Each power system is composed of a number of busbars interconnected by a network of impedances. At each busbar one or more of the following may be connected:

1. A generator supplying real power into the network, and operating at either a lagging power factor (supplying reactive power) or a leading power factor (absorbing reactive power)
2. A load absorbing real power from the network, and operating at either a lagging power factor (absorbing reactive power) or a leading power factor (supplying reactive power)
3. Inductive or capacitive compensation devices, equivalent to static reactances employed for voltage control purposes
4. Rotating machinery capable of supplying or absorbing real and reactive power in any of a number of operating quadrant combinations

1.2 The Per-Unit System

In the per-unit system, voltages, currents, impedances, and powers are expressed in a normalized fashion as percentages (or *per-unit*) of predefined base quantities. The advantages of this method of description include ease of system representation, elimination of transformer turns ratios, and simplicity of number manipulation.

A per-unit (p.u.) quantity is one that is expressed as a decimal fraction of a predefined base quantity. For example, if a base voltage were selected as 1000 V, then an actual voltage of 920 V would be expressed as 0.92 per-unit (of that base quantity).

To maintain consistency in the per-unit system, two base quantities are chosen: voltage and voltamperes. The base voltage is normally the nominal system voltage and the voltampere base may be selected as some multiple of an equipment rating. Typically, the largest rotating machine rating is used or, alternatively, a convenient round number approximating that value.

If the selected voltage base is V_{base} and the selected voltampere base is VA_{base}, then the remaining base quantities may be derived as:

$$I_{\text{base}} = \frac{VA_{\text{base}}}{V_{\text{base}}}$$

and

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \quad \text{or} \quad \frac{V_{\text{base}}^2}{VA_{\text{base}}}$$

As mentioned in the beginning of this chapter, three-phase systems are generally represented as single-phase equivalents, in which case the base quantities are all *phase* values.

If V_{base} above is the *line* voltage of the system and VA_{base} is the three-phase voltampere quantity, then per phase:

$$\text{Base voltage} = \frac{V_{\text{base}}}{\sqrt{3}}$$

and

$$\text{Base voltamperes} = \frac{VA_{\text{base}}}{3}$$

from which

$$I_{\text{base}} = \frac{VA_{\text{base}}}{\sqrt{3}V_{\text{base}}}$$