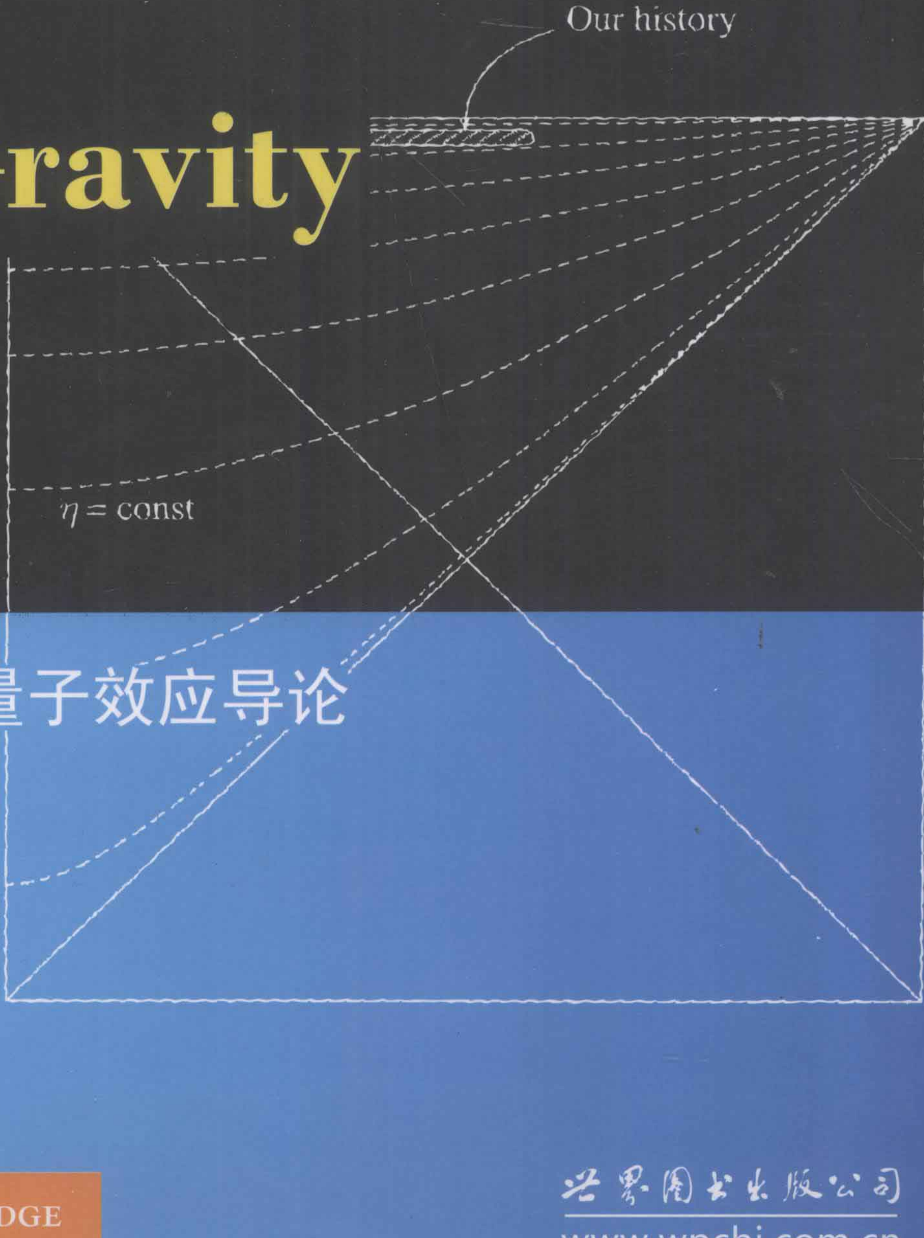
V. F. Mukhanov，S. Winitzki

Introduction to Quantum Effects in Gravity



引力的量子效应导论

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INTRODUCTION TO QUANTUM EFFECTS IN GRAVITY

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INTRODUCTION TO QUANTUM EFFECTS IN GRAVITY

This is the first introductory textbook on quantum field theory in gravitational backgrounds intended for undergraduate and beginning graduate students in the fields of theoretical astrophysics, cosmology, particle physics, and string theory. The book covers the basic (but essential) material of quantization of fields in expanding universe and quantum fluctuations in inflationary spacetime. It also contains a detailed explanation of the Casimir, Unruh, and Hawking effects, and introduces the method of effective action used for calculating the backreaction of quantum systems on a classical external gravitational field.

The broad scope of the material covered will provide the reader with a thorough perspective of the subject. Complicated calculations are avoided in favor of simpler ones, which still contain the relevant physical concepts. Every major result is derived from first principles and thoroughly explained. The book is self-contained and assumes only a basic knowledge of general relativity. Exercises with detailed solutions are provided throughout the book.

VIATCHESLAV F. MUKHANOV is Professor of Physics at Ludwig-Maximilians University, Munich. His main result is the theory of inflationary cosmological perturbations. Professor Mukhanov is author of *Physical Foundations of Cosmology* (Cambridge University Press, 2005). He also serves on the editorial boards of leading research journals and is Scientific Director of the *Journal of Cosmology and Astroparticle Physics* (*JCAP*).

SERGEI WINITZKI is Research Associate in the Department of Physics at Ludwig-Maximilians University, Munich. His main areas of research include quantum cosmology, the theory of dark energy, the global structure of spacetime, and quantum gravity.

Preface

This book is an expanded and reorganized version of the lecture notes for a course taught at the Ludwig-Maximilians University, Munich, in the spring semester of 2003. The course is an elementary introduction to the basic concepts of quantum field theory in classical backgrounds. A certain level of familiarity with general relativity and quantum mechanics is required, although many of the necessary concepts are introduced in the text.

The audience consisted of advanced undergraduates and beginning graduate students. There were 11 three-hour lectures. Each lecture was accompanied by exercises that were an integral part of the exposition and encapsulated longer but straightforward calculations or illustrative numerical results. Detailed solutions were given for all the exercises. Exercises marked by an asterisk (*) are more difficult or cumbersome.

The book covers limited but essential material: quantization of free scalar fields; driven and time-dependent harmonic oscillators; mode expansions and Bogolyubov transformations; particle creation by classical backgrounds; quantum scalar fields in de Sitter spacetime and the growth of fluctuations; the Unruh effect; Hawking radiation; the Casimir effect; quantization by path integrals; the energy-momentum tensor for fields; effective action and backreaction; regularization of functional determinants using zeta functions and heat kernels. Topics such as quantization of higher-spin fields or interacting fields in curved spacetime, direct renormalization of the energy-momentum tensor, and the theory of cosmological perturbations are left out.

The emphasis of this course is primarily on concepts rather than on computational results. Most of the required calculations have been simplified to the barest possible minimum that still contains all relevant physics. For instance, only free scalar fields are considered for quantization; background spacetimes are always chosen to be conformally flat; the Casimir effect, the Unruh effect, and the Hawking radiation are computed for massless scalar fields in suitable

$1+1$-dimensional spacetimes. Thus a fairly modest computational effort suffices to explain important conceptual issues such as the nature of vacuum and particles in curved spacetimes, thermal effects of gravitation, and backreaction. This should prepare students for more advanced and technically demanding treatments suggested below.

The authors are grateful to Josef Gaßner and Matthew Parry for discussions and valuable comments on the manuscript. Special thanks are due to Alex Vikman who worked through the text and prompted important revisions, and to Andrei Barvinsky for his assistance in improving the presentation in the last chapter.

The entire book was typeset with the excellent LyX and TeX document preparation system on computers running Debian GNU/Linux. We wish to express our gratitude to the creators and maintainers of this outstanding free software.

Suggested literature

The following books offer a more extensive coverage of the subject and can be studied as a continuation of this introductory course.

N. D. BIRRELL and P. C. W. DAVIES, *Quantum Fields in Curved Space* (Cambridge University Press, 1982).

S. A. FULLING, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, 1989).

A. A. GRIB, S. G. MAMAEV, and V. M. MOSTEPANENKO, *Vacuum Quantum Effects in Strong Fields* (Friedmann Laboratory Publishing, St. Petersburg, 1994).

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Summary Quantum fields as a set of harmonic oscillators. Vacuum state. Particle interpretation of field theory. Examples of particle production by external fields.

We begin with a few elementary observations concerning the vacuum in quantum field theory.

1.1 Classical field

A classical field is described by a function $\phi(\mathbf{x}, t)$, where $\mathbf{x}$ is a three-dimensional coordinate in space and t is the time. At every point the function $\phi(\mathbf{x}, t)$ takes values in some finite-dimensional "configuration space" and can be a scalar, vector, or tensor.

The simplest example is a real scalar field $\phi(\mathbf{x}, t)$ whose strength is characterized by real numbers. A free massive scalar field satisfies the Klein–Gordon equation

$$\frac{\partial^2 \phi}{\partial t^2} - \sum_{j=1}^{3} \frac{\partial^2 \phi}{\partial x_j^2} + m^2 \phi \equiv \ddot{\phi} - \Delta\phi + m^2 \phi = 0, \tag{1.1}$$

which has a unique solution $\phi(\mathbf{x}, t)$ for $t > t_0$ provided that the initial conditions $\phi(\mathbf{x}, t_0)$ and $\dot{\phi}(\mathbf{x}, t_0)$ are specified.

Formally one can describe a free scalar field as a set of decoupled "harmonic oscillators." To explain why this is so it is convenient to begin by considering a field $\phi(\mathbf{x}, t)$ not in infinite space but in a box of finite volume V, with some boundary conditions imposed on the field ϕ. The volume V should be large enough to avoid artifacts induced by the finite size of the box or by physically irrelevant boundary conditions. For example, one might choose the box as a cube with sides of length L and volume $V = L^3$, and impose the *periodic* boundary conditions,

$$\phi(x=0,y,z,t) = \phi(x=L,y,z,t)$$

and similarly for y and z. The Fourier decomposition is then

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{1.2}$$

where the sum goes over three-dimensional wavenumbers $\mathbf{k}$ with components

$$k_x = \frac{2\pi n_x}{L}, \quad n_x = 0, \pm1, \pm2, \ldots$$

and similarly for k_y and k_z. The normalization factor $\sqrt{V}$ in equation (1.2) is chosen to simplify formulae (in principle, one could rescale the modes $\phi_{\mathbf{k}}$ by any constant). Substituting (1.2) into equation (1.1), we find that this equation is replaced by an infinite set of decoupled ordinary differential equations:

$$\ddot{\phi}_{\mathbf{k}} + \left(k^2 + m^2\right)\phi_{\mathbf{k}} = 0,$$

with one equation for each $\mathbf{k}$. In other words, each complex function $\phi_{\mathbf{k}}(t)$ satisfies the harmonic oscillator equation with the frequency

$$\omega_k \equiv \sqrt{k^2 + m^2},$$

where $k \equiv |\mathbf{k}|$. The "oscillators" with complex coordinates $\phi_{\mathbf{k}}$ "move" not in real three-dimensional space but in the *configuration space* and characterize the strength of the field ϕ. The total energy of the field ϕ in the box is simply equal to the sum of energies of all oscillators $\phi_{\mathbf{k}}$,

$$E = \sum_{\mathbf{k}} \left[\frac{1}{2}\left|\dot{\phi}_{\mathbf{k}}\right|^2 + \frac{1}{2}\omega_k^2 \left|\phi_{\mathbf{k}}\right|^2\right].$$

In the limit of infinite space when $V \to \infty$ the sum in (1.2) is replaced by the integral over all wavenumbers $\mathbf{k}$,

$$\phi(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}}\phi_{\mathbf{k}}(t). \tag{1.3}$$

1.2 Quantum field and its vacuum state

The quantization of a free scalar field is mathematically equivalent to quantizing an infinite set of decoupled harmonic oscillators.

Harmonic oscillator A classical harmonic oscillator is described by a coordinate $q(t)$ satisfying

$$\ddot{q} + \omega^2 q = 0. \tag{1.4}$$

The solution of this equation is unique if we specify initial conditions $q(t_0)$ and $\dot{q}(t_0)$. We may identify the "ground state" of an oscillator as the state without motion, i.e. $q(t) \equiv 0$. This lowest-energy state is the solution of the classical equation (1.4) with the initial conditions $q(0) = \dot{q}(0) = 0$.

When the oscillator is quantized, the classical coordinate q and the momentum $p = \dot{q}$ (for simplicity, we assume that the oscillator has a unit mass) are replaced by operators $\hat{q}(t)$ and $\hat{p}(t)$ satisfying the Heisenberg commutation relation

$$[\hat{q}(t), \hat{p}(t)] = [\hat{q}(t), \hat{\dot{q}}(t)] = i\hbar. \tag{1.5}$$

The solution $\hat{q}(t) \equiv 0$ does not satisfy the commutation relation. In fact, the oscillator's coordinate always fluctuates. The ground state with the lowest energy is described by the normalized wave function

$$\psi(q) = \left[\frac{\omega}{\pi\hbar}\right]^{\frac{1}{4}} \exp\left(-\frac{\omega q^2}{2\hbar}\right).$$

The energy of this minimal excitation state, called the *zero-point energy*, is $E_0 = \frac{1}{2}\hbar\omega$. The typical amplitude of fluctuations in the ground state is $\delta q \sim \sqrt{\hbar/\omega}$ and the measured trajectories $q(t)$ resemble a random walk around $q = 0$.

Field quantization In the case of a field, each mode $\phi_{\mathbf{k}}(t)$ is quantized as a separate harmonic oscillator. The classical "coordinates" $\phi_{\mathbf{k}}$ and the corresponding conjugated momenta $\pi_{\mathbf{k}} \equiv \dot{\phi}^*_{\mathbf{k}}$ are replaced by operators $\hat{\phi}_{\mathbf{k}}$, $\hat{\pi}_{\mathbf{k}}$. In a finite box they satisfy the following equal-time commutation relations:

$$\left[\hat{\phi}_{\mathbf{k}}(t), \hat{\pi}_{\mathbf{k}'}(t)\right] = i\delta_{\mathbf{k},-\mathbf{k}'},$$

where $\delta_{\mathbf{k},-\mathbf{k}'}$ is the Kronecker symbol equal to unity when $\mathbf{k} = -\mathbf{k}'$ and zero otherwise. In the limit of infinite volume the commutation relations become

$$\left[\hat{\phi}_{\mathbf{k}}(t), \hat{\pi}_{\mathbf{k}'}(t)\right] = i\delta\left(\mathbf{k} + \mathbf{k}'\right), \tag{1.6}$$

where $\delta(\mathbf{k} + \mathbf{k}')$ is the Dirac δ function. To simplify the formulae, we shall almost always use the units in which $\hbar = c = 1$.

Vacuum state The *vacuum* is a state corresponding to the intuitive notions of "the absence of anything" or "an empty space." Generally, the vacuum is defined as the state with the lowest possible energy. In the case of a classical field the vacuum is a state where the field is absent, that is, $\phi(\mathbf{x}, t) = 0$. This is a solution of the classical equations of motion. When the field is quantized it becomes impossible to satisfy simultaneously the equations of motion for the operator $\hat{\phi}$ and the commutation relations by $\hat{\phi}(\mathbf{x}, t) = 0$. Therefore, the field always fluctuates and has a nonvanishing value even in a state with the minimal possible energy.

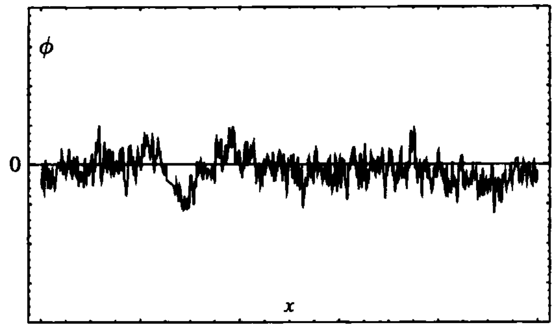

Fig. 1.1 A field configuration $\phi(x)$ that could be measured in the vacuum state.

Since all modes $\phi_{\mathbf{k}}$ are decoupled, the ground state of the field can be characterized by a *wave functional* which is the product of an infinite number of wave functions, each describing the ground state of a harmonic oscillator with the corresponding wavenumber $\mathbf{k}$:

$$\Psi[\phi] \propto \prod_{\mathbf{k}} \exp\left(-\frac{\omega_k |\phi_{\mathbf{k}}|^2}{2}\right) = \exp\left[-\frac{1}{2}\sum_{\mathbf{k}} \omega_k |\phi_{\mathbf{k}}|^2\right]. \tag{1.7}$$

The ground state of the field has the minimum energy and is called the vacuum state. Strictly speaking, equation (1.7) is valid only for a field quantized in a box. Note that if we had normalized the Fourier components $\phi_{\mathbf{k}}$ in equation (1.2) differently, then there would be a volume factor in front of ω_k.

The square of the wave function (1.7) gives us the probability density for measuring a certain field configuration $\phi(\mathbf{x})$. This probability is independent of time t. The field fluctuates in the vacuum state and the field configurations can be visualized as small random deviations from zero (see Fig. 1.1).

When the volume of the box becomes very large, we have to replace sums by integrals,

$$\sum_{\mathbf{k}} \to \frac{V}{(2\pi)^3}\int d^3\mathbf{k}, \quad \phi_{\mathbf{k}} \to \sqrt{\frac{(2\pi)^3}{V}}\phi_{\mathbf{k}}, \tag{1.8}$$

and the wave functional (1.7) becomes

$$\Psi[\phi] \propto \exp\left[-\frac{1}{2}\int d^3\mathbf{k}\, |\phi_{\mathbf{k}}|^2\, \omega_k\right]. \tag{1.9}$$

Exercise 1.1
The vacuum wave functional (1.9) contains the integral

$$I \equiv \int d^3\mathbf{k}\, |\phi_{\mathbf{k}}|^2 \sqrt{k^2+m^2}, \tag{1.10}$$