Review of ELEMENTARY MATHEMATICS

•

by

BARNETT RICH, Ph.D.

Review of ELEMENTARY MATHEMATICS

by

BARNETT RICH, Ph.D.
Former Head of the Mathematics Department
Brooklyn Technical High School

SCHAUM'S OUTLINE SERIES

McGRAW-HILL BOOK COMPANY

New York St. Louis San Francisco Auckland Bogotá Düsseldorf Johannesburg London Madrid Mexico Montreal New Delhi Panama Paris São Paulo Singapore Sydney Tokyo Toronto

0-07-052260-X

2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 SH SH 7 8 3 2 1 0 9 8 7

Library of Congress Cataloging in Publication Data

Rich, Barnett, date
Review of elementary mathematics.

(Schaum's outline series)
Includes index.
1. Mathematics—1961- 2. Mathematics—Problems, exercises, etc. I. Title.
QA39.2.R527 512'.1 77-9937
ISBN 0-07-052260-X

Preface

This survey of pre-college mathematics is designed for those who want intensive help in learning arithmetic, geometry, and algebra, or who need a concentrated review of these crucial subjects. It provides maximum assistance, far beyond the traditional book in elementary mathematics, for the following reasons:

(1) Each important rule, formula, and principle is stated in simple language, and is immediately applied to one or more sets of solved problems.

(2) Each procedure is developed step-by-step, with each step applied to problems alongside the procedure.

(3) Each set of solved problems is used to clarify and illustrate a rule or principle. The particular character of each such set is indicated by a title. *There are 2500 carefully selected and fully solved problems.*

(4) Each set of supplementary problems provides further application of a rule or principle. A guide number with each such set refers a student needing help to the related set of solved problems. *There are 3200 supplementary problems.* Each of these contains its required answer and, where needed, further aids to solution.

The author wishes to acknowledge the cooperation of Mr. Thomas J. Dembofsky and his staff.

BARNETT RICH

Contents

Chapter 1

Fundamentals of Arithmetic: Number

1. NUMBER

Numbers and Sets of Numbers

Number is a fundamental idea in mathematics. A number may be expressed by a symbol called a *numeral.* A number may be named by a word.

Thus, the number 5 may be written as a numeral "5" or named by the word "five."

A *set of numbers* is a collection of identifiable numbers. Each number in a set is a member or element of the set. In mathematics, the study of sets is fundamental to the study of other branches of the subject.

To specify a set of numbers by *roster*, list the numbers inside braces. A capital letter may be used to refer to a set. Thus, use $S = \{1, 2, 3, 4, 5\}$ to specify the set of the first five counting numbers.

Finite and Infinite Sets

A *finite set* is one having a limited number of members. The members of a finite set can be counted. Thus, $S = \{1, 2, 3, 4, 5\}$ is a finite set. Also, the set of the first million numbers is a finite set.

An *infinite set* is one which is not a finite set. Since there is no limit to the number of counting numbers, the set of counting numbers is an infinite set. Three dots (. . .) are needed to list an infinite set. Read the three dots as "and so on" or "and so on in the same pattern."

Thus, the infinite set of counting numbers may be listed as $C = \{1, 2, 3, 4, 5, \ldots\}$.

The symbol of three dots (. . .) may also be used to list a finite set having a great number of members. Thus, the set of the first hundred counting numbers may be listed as $S = \{1, 2, 3, \ldots, 98, 99, 100\}$.

Natural Numbers

Any counting number is called a natural number since counting can be done using one's natural fingers. In fact, the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are called *digits*, from a Latin word meaning finger. Any natural number of our decimal system may be expressed using only these ten digits.

Thus, the digits 1 and 7 are used to express the natural or counting numbers 17 and 71.

The *successor* of a natural number is the next greater natural number. Thus, the successor of 99 is 100.

Whole Numbers

A *whole number* is either 0 or a natural number. Zero (0) is not a counting number or a natural number.

Odd and Even Numbers

The *set of even numbers* consists of 0, 2, 4, 6, 8, and all numbers whose last digit is one of these. In Chapter 3, we will treat even numbers as numbers divisible by 2.

The *set of odd numbers* consists of 1, 3, 5, 7, 9, and all numbers whose last digit is one of these. A whole number is either an odd number or an even number.

Thus, 1352 is an even number and 2461 is an odd number.

Specifying Infinite Sets of Numbers

The infinite sets of natural numbers, whole numbers, even numbers, and odd numbers can be specified as follows:

Set of natural numbers: $N = \{1, 2, 3, 4, 5, \ldots\}$
Set of whole numbers: $W = \{0, 1, 2, 3, 4, 5, \ldots\}$
Set of even numbers: $E = \{0, 2, 4, 6, 8, 10, \ldots\}$
Set of odd numbers: $Q = \{1, 3, 5, 7, 9, 11, \ldots\}$

Two-Digit and Three-Digit Numbers

Many of our problems involve numbers having two or three digits. A number having two digits is a *two-digit number* as long as the first digit is not a zero digit. A number having three digits is a *three-digit number* as long as the first digit is a nonzero digit.

Thus, 19 is a two-digit number while 09 is not. Also, 139 is a three-digit number while 039 and 009 are not.

1.1 NAMING MISSING NUMBERS

Name the missing numbers in each set.

(*a*) {5, 6, 7, . . . , 11, 12, 13}
(*b*) {3, 5, 7, . . . , 15, 17, 19}
(*c*) {20, 22, 24, . . . , 34, 36, 38}
(*d*) {10, 20, 30, . . . , 60, 70, 80}
(*e*) {22, 33, 44, . . . , 77, 88, 99}
(*f*) {151, 252, 353, . . . , 757, 858, 959}

Illustrative Solution (*c*) The numbers in the set beginning with 20 and ending with 38 are even numbers. Hence, the missing numbers are 26, 28, 30, and 32. *Ans.*

Ans. (*a*) 8, 9, 10 (*b*) 9, 11, 13 (*d*) 40, 50 (*e*) 55, 66 (*f*) 454, 555, 656

1.2 DETERMINING WHETHER A NUMBER IS ODD OR EVEN

Determine whether each number is odd or even.

(*a*) 25 (*b*) 52 (*c*) 136 (*d*) 163 (*e*) 135792 (*f*) 246801

Solutions

The odd numbers are those whose last digit is odd. Hence the odd numbers are 25 in (*a*), 163 in (*d*), and 246801 in (*f*).

The even numbers are those whose last digit is even. Hence the even numbers are 52 in (*b*), 136 in (*c*), and 135792 in (*e*).

1.3 LISTING FINITE SETS

List each set: (*a*) the set of whole numbers less than 5, (*b*) the set of natural numbers less than 5, (*c*) the set of odd numbers less than 13, (*d*) the set of even numbers greater than 7 and less than 15, (*e*) the set of two-digit numbers less than 50 with both digits the same, (*f*) the set of two-digit numbers the sum of whose digits is 4.

Illustrative Solution (*f*) The two-digit numbers whose digits have a sum of 4 are 13, 22, 31, and 40. Do not include 04 since the first digit is 0. Think of 04 as 4, a single-digit number.

Ans. (*a*) {0, 1, 2, 3, 4} (*c*) {1, 3, 5, 7, 9, 11} (*e*) {11, 22, 33, 44}
(*b*) {1, 2, 3, 4} (*d*) {8, 10, 12, 14}

1.4 NAMING NUMBERS

Name each number: (*a*) the least natural number, (*b*) the greatest natural number, (*c*) the least two-digit odd number, (*d*) the greatest two-digit even number, (*e*) the least three-digit odd number all of whose digits are the same, (*f*) the greatest three-digit even number all of whose digits are different.

Illustrative Solution (*b*) There is no greatest natural number. The set of natural numbers is an infinite set without limit to the number of members.

Ans. (*a*) 1 (*c*) 11 (*d*) 98 (*e*) 111 (*f*) 986

1.5 LISTING FINITE AND INFINITE SETS USING THREE DOTS

Using three dots, list each set: (*a*) the set of natural numbers greater than 10, (*b*) the set of whole numbers between 25 and 75, (*c*) the set of odd numbers greater than 10, (*d*) the set of even numbers less than 100, (*e*) the set of whole numbers less than 100 whose last digit is 5, (*f*) the set of three-digit whole numbers.

Illustrative Solution (*b*) List a large finite set by naming the first three and the last three members of the set; thus, {26, 27, 28, . . . , 72, 73, 74}. Note the three dots representing all the numbers that are not listed.

Ans. (*a*) {11, 12, 13, . . .} (*d*) {0, 2, 4, . . . , 94, 96, 98} (*f*) {100, 101, 102, . . . , 997, 998, 999}
(*c*) {11, 13, 15, . . .} (*e*) {5, 15, 25, . . . , 75, 85, 95}

1.6 NAMING SUCCESSORS OF NATURAL OR WHOLE NUMBERS

Name the successor of each.

(*a*) 99 (*b*) 909 (*c*) 990 (*d*) 9009 (*e*) 9090 (*f*) 9999

Illustrative Solution (*e*) To obtain the successor of a number, add 1 to the number. Hence, the successor of 9090 is 9090 + 1, or 9091 *Ans.*

Ans. (*a*) 100 (*b*) 910 (*c*) 991 (*d*) 9010 (*f*) 10,000

2. DECIMAL SYSTEM OF NUMERATION

Examine the numbers in Table 1-1 and note, as we go from ones to billions, how the place value of a digit becomes ten times as great from any place to the place immediately to the left.

Table 1-1. Place Values

Number	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units or Ones
(*a*)										3
(*b*)									4	0
(*c*)								5	0	0
(*d*)								5	4	3
(*e*)				5	0	0	4	0	3	0
(*f*)	5	0	0	4	0	0	0	3	0	0

Discussion of Numbers in Table 1-1

(*a*) The number 3 is a one-digit number. The only digit of a one-digit number is a *units digit.* The units digit 3 has a *face value* of 3.

(*b*) The number 40 is a two-digit number. In a two-digit number, the first digit is a *tens digit* and the last digit is a units digit. The tens digit 4 has a value of 40. The units digit 0 enables us to place a nonzero digit such as 4 in tens place.

(*c*) The number 500 is a three-digit number. In a three-digit number, the first digit is a *hundreds digit*, the second digit is a tens digit, and the third digit is a units digit. The hundreds digit 5 has a value of 500. The zero digits in tens and units places enable us to place 5 in hundreds place. *Think of a zero digit as the absence of any value.*

(*d*) The number 543 is a three-digit number. Since the hundreds digit is 5, the tens digit is 4, and the units digit is 3, the value of 543 is 5 hundreds + 4 tens + 3 units; that is 500 + 40 + 3. The expression 500 + 40 + 3 is the *expanded form* of 543.

(*e*) The number 5,004,030 has three nonzero digits, 5, 4, and 3. Since the millions digit is 5, the thousands digit is 4, and the tens digit is 3, the value of 5,004,030 is 5 millions + 4 thousands + 3 tens; that is, 5,000,000 + 4,000 + 30. The expanded form, 5,000,000 + 4,000 + 30, shows that the number should be read as "five million, four thousand, thirty." "Thirty" is a modification of "three tens." Note, as shown here, that *commas may be used to mark off three-digit groups from right to left.* Marking off three-digit groups in this way simplifies the reading and also the writing of large numbers.

(*f*) The number 5,004,000,300 has three nonzero digits, 5, 4, and 3. Since the billions digit is 5, the millions digit is 4, and the hundreds digit is 3, the value of 5,004,000,300 is 5 billions + 4 millions + 3 hundreds. The expanded form, 5,000,000,000 + 4,000,000 + 300, shows that the number should be read as "five billion, four million, three hundred." Note the use of commas to mark off three-digit groups from right to left.

Expressing Numbers in Expanded Form

A number is expressed in expanded form as follows: (1) express the value of each nonzero digit separately, and (2) place plus signs between these values.

Thus, as above, express 543 in expanded form as 500 + 40 + 3. Also, express 5,004,030 in expanded form as 5,000,000 + 4,000 + 30.

Naming the Digits of Two-Digit and Three-Digit Numbers

A two-digit number consists of a tens digit followed by a units digit.

Thus, the digits of 45 are the tens digit 4 and the units digit 5. Read 45 as "forty five. "Forty" is a modification of "four tens."

A three-digit number consists of a hundreds digit, a tens digit, and lastly, a units digit.

Thus, the digits of 135 are the hundreds digit 1, the tens digit 3, and lastly, the units digit 5. Read 135 as "one hundred, thirty five."

Reading and Marking Off Numbers Having More Than Three Digits

Note the way in which numbers (*e*) and (*f*) are marked off in groups of three digits from right to left. Separating large numbers in groups of three digits in this way simplifies both the reading and the writing of these numbers.

Thus, 123,000 is read as "one hundred twenty three *thousand*" and 123,000,000 is read as "one hundred twenty three *million*."

Important Notes:

(1) Do not use "and" when reading whole numbers.
(2) Note the use of the singular. Read 500 as "five hundred," not "five hundreds."

2.1 NAMING NONZERO DIGITS ACCORDING TO PLACE

Name each nonzero digit according to its place in the number.

(*a*) 57 (*b*) 570 (*c*) 5,700 (*d*) 489 (*e*) 48,090 (*f*) 4,080,900

Illustrative Solution (*b*) Since 570 = 500 + 70, 5 is hundreds digit and 7 is tens digit.

Ans. (*a*) 5 is tens digit, 7 is units digit; (*c*) 5 is thousands digit, 7 is hundreds digit; (*d*) 4 is hundreds digit, 8 is tens digit, 9 is units digit; (*e*) 4 is ten thousands digit, 8 is thousands digit, 9 is tens digit; (*f*) 4 is millions digit, 8 is ten thousands digit, 9 is hundreds digit

2.2 READING NUMBERS

Read each: (*a*) 67, (*b*) 76, (*c*) 607, (*d*) 7,600, (*e*) 60,070, (*f*) 3,004,050.

Illustrative Solution (*c*) Since 607 = 600 + 7, read this as "six hundred seven."

Ans. (*a*) sixty seven (*b*) seventy six (*d*) seven thousand, six hundred (*e*) sixty thousand, seventy (*f*) three million, four thousand, fifty

2.3 WRITING NUMBERS IN EXPANDED FORM

Write each in expanded form:

(*a*) 38 (*b*) 83 (*c*) 308 (*d*) 8,003 (*e*) 459 (*f*) 40,509

Illustrative Solution (*c*) Expand using only the nonzero digits, 3 and 8. Hence, 308 = 300 + 8 *Ans.*

Ans. (*a*) 30 + 8 (*b*) 80 + 3 (*d*) 8,000 + 3 (*e*) 400 + 50 + 9 (*f*) 40,000 + 500 + 9

2.4 CONVERTING NUMBERS FROM VERBAL TO DIGIT FORM

Write each in digit form:

(*a*) twenty
(*b*) two hundred four
(*c*) four thousand, two hundred
(*d*) two hundred forty thousand
(*e*) twenty million four hundred thousand
(*f*) four hundred billion, two hundred million, forty two

Illustrative Solution (*d*) Write 240 for two hundred forty, then add three zeros for thousand. *Ans.* 240,000.

Ans. (*a*) 20 (*b*) 204 (*c*) 4,200 (*e*) 20,400,000 (*f*) 400,200,000,042

3. NUMBER LINE

Constructing a Number Line

A *number line is constructed* by dividing a line into equal segments, as in Fig. 1-1. The arrowhead in Fig. 1-1 indicates that the number line extends to the right without end.

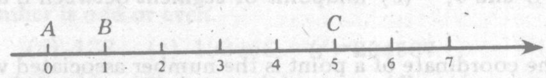


Fig. 1-1

Pairing Points and Numbers on a Number Line

As in Fig. 1-1, the points of division of a number line may be paired with successive whole numbers and identified by capital letters. For each pair of points and numbers, the number is the *coordinate* of the point and the point is the *graph* of the number.

Thus, in Fig. 1-1, 0 is the coordinate of point *A*, and *A* is the graph of 0. Also, 5 is the coordinate of point *C*, and *C* is the graph of 5.

Models of a Number Line

A good *model of a number line* may be the edge of a ruler, a yardstick, or a tape measure. In Fig. 1-2, the edge of a ruler in centimeters serves as a model of a number line.

Number Line on a Ruler

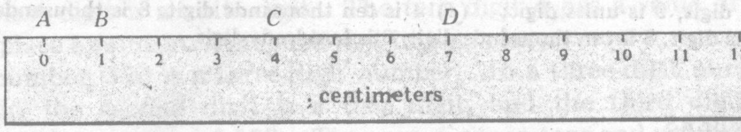


Fig. 1-2

The unit of the number line, Fig. 1-2, is one centimeter, the length of one of the equal segments. Note in Fig. 1-2 that 4 is the coordinate of point *C*, and point *D* is the graph of 7.

Graphing a Set of Numbers on a Number Line

As in Fig. 1-3, a set of numbers is graphed on a number line by making heavy the points that are the graphs of the numbers in the set.

To Graph a Set of Numbers on a Number Line

Graph the set of even numbers between 1 and 9.

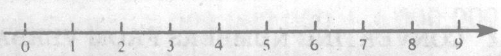
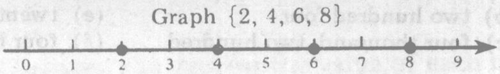
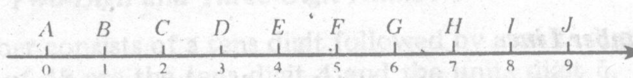
PROCEDURE	SOLUTION
1. Construct a number line having a convenient unit:	1. 
2. Find the numbers to be graphed:	2. The even numbers between 1 and 9 are 2, 4, 6, and 8.
3. Make heavy the points to be graphed:	3. Graph {2, 4, 6, 8} 

Fig. 1-3

3.1 NAMING THE COORDINATES OF GIVEN POINTS

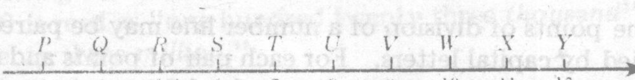


Name the coordinate of: (*a*) point *B*, (*b*) point *F*, (*c*) numbered points between *B* and *F*, (*d*) point halfway between *D* and *J*, (*e*) midpoint of segment between *E* and *G*, (*f*) point 3 units to the right of *F*.

Illustrative Solution (*e*) The coordinate of a point is the number associated with it. The required coordinate is 5, the number associated with point *F*, the midpoint of the segment between *E* and *G*.

Ans. (*a*) 1 (*b*) 5 (*c*) 2, 3, 4 (*d*) 6 (*f*) 8

3.2 NAMING THE GRAPHS OF GIVEN COORDINATES



Name the graph of: (*a*) 6, (*b*) 10, (*c*) the odd coordinates, (*d*) the last two even coordinates, (*e*) coordinates greater than 4 and smaller than 8, (*f*) the successor of 9.

Illustrative Solution **(*d*) The graph of a coordinate is the point associated with the coordinate. The last two even coordinates are 10 and 12. The graphs or points associated with 10 and 12 are *W* and *Y* respectively.**

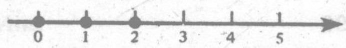
Ans. **(*a*) *S* (*b*) *W* (*c*) *P, R, T, V, X* (*e*) *R, S, T* (*f*) *W***

3.3 GRAPHING SETS OF NUMBERS

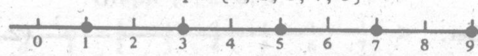
Graph: (*a*) the set of the first three whole numbers; (*b*) the set of the last digits of odd numbers; (*c*) the set of even numbers between 3 and 9; (*d*) the set of odd numbers greater than 0 and less than 6; (*e*) the set of the successors of 1, 4, and 7; (*f*) the set of the numbers whose successors are 2, 7, and 9.

Solutions

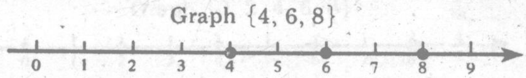
(*a*) Graph {0, 1, 2}



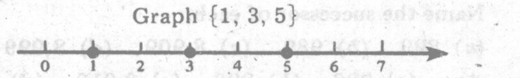
(*b*) Graph {1, 3, 5, 7, 9}



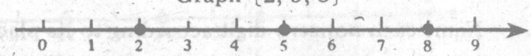
(*c*) Graph {4, 6, 8}



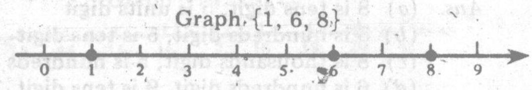
(*d*) Graph {1, 3, 5}



(*e*) Graph {2, 5, 8}



(*f*) Graph {1, 6, 8}



Supplementary Problems

The numbers in parentheses at the right of each set of problems indicate where to find the same type of example in this chapter. For example, (1.1) indicates that Set 1.1 in this chapter involves problems of the same type. If help is needed, a student should review the set to which reference is made.

1. **Name the missing numbers in each set.** **(1.1)**

(*a*) {9, 10, 11, . . . , 15, 16, 17} **(*d*) {15, 25, 35, . . . , 65, 75, 85}**
(*b*) {4, 6, 8, . . . , 16, 18, 20} **(*e*) {25, 30, 35, . . . , 70, 75, 80}**
(*c*) {99, 101, 103, . . . , 113, 115, 117} **(*f*) {939, 838, 737, . . . 333, 232, 131}**

Ans. **(*a*) 12, 13, 14** **(*c*) 105, 107, 109, 111** **(*e*) 40, 45, 50, 55, 60, 65**
(*b*) 10, 12, 14 **(*d*) 45, 55** **(*f*) 636, 535, 434**

2. **Determine whether each number is odd or even.** **(1.2)**

(*a*) 32 (*b*) 67 (*c*) 245 (*d*) 138 (*e*) 123456 (*f*) 234567

Ans. **(*a*) even (*b*) odd (*c*) odd (*d*) even (*e*) even (*f*) odd**

3. **List each set: (*a*) the set of whole numbers between 10 and 15; (*b*) the set of odd numbers less than 30 and greater than 20; (*c*) the set of two-digit even numbers less than 20; (*d*) the set of two-digit odd numbers both of whose digits are the same; (*e*) the set of two-digit numbers less than 50, the sum of whose digits is 5; (*f*) the set of three-digit numbers, each of which has the digits 1, 2, and 3.** **(1.3)**

Ans. **(*a*) {11, 12, 13, 14}** **(*c*) {10, 12, 14, 16, 18}** **(*e*) {14, 23, 32, 41}**
(*b*) {21, 23, 25, 27, 29} **(*d*) {11, 33, 55, 77, 99}** **(*f*) {123, 132, 213, 231, 312, 321}**

4. **Name each number: (*a*) the least whole number, (*b*) the greatest whole number, (*c*) the least two-digit odd number each of whose digits is greater than 3, (*d*) the greatest two-digit** **(1.4)**

even number both of whose digits are the same, (*e*) the least three-digit odd number all of whose digits are different, (*f*) the greatest three-digit even number having the digits 1, 2, and 3.

Ans. (*a*) 0 (*b*) Such a number does not exist. (*c*) 45 (*d*) 88 (*e*) 103 (*f*) 312

5. Using three dots, list: (*a*) the set of whole numbers greater than 25, (*b*) the set of natural numbers between 20 and 50, (*c*) the set of odd numbers less than 100, (*d*) the set of even numbers greater than 500, (*e*) the set of natural numbers between 100 and 200 whose last digit is 9, (*f*) the set of three-digit odd numbers. (1.5)

Ans. (*a*) {26, 27, 28, . . .} (*d*) {502, 504, 506, . . .}
(*b*) {21, 22, 23, . . . , 47, 48, 49} (*e*) {109, 119, 129, . . . , 179, 189, 199}
(*c*) {1, 3, 5, . . . , 95, 97, 99} (*f*) {101, 103, 105, . . . , 995, 997, 999}

6. Name the successor of each. (1.6)

(*a*) 899 (*b*) 989 (*c*) 8,909 (*d*) 8,999 (*e*) 9,899 (*f*) 908,999

Ans. (*a*) 900 (*b*) 990 (*c*) 8,910 (*d*) 9,000 (*e*) 9,900 (*f*) 909,000

7. Name each nonzero digit according to its place in the number. (2.1)

(*a*) 85 (*b*) 850 (*c*) 8,500 (*d*) 691 (*e*) 90,016 (*f*) 90,100,060

Ans. (*a*) 8 is tens digit, 5 is units digit
(*b*) 8 is hundreds digit, 5 is tens digit
(*c*) 8 is thousands digit, 5 is hundreds digit
(*d*) 6 is hundreds digit, 9 is tens digit, 1 is units digit
(*e*) 9 is ten thousands digit, 1 is tens digit, 6 is units digit
(*f*) 9 is ten millions digit, 1 is hundred thousands digit, 6 is tens digit

8. Read each. (2.2)

(*a*) 92 (*b*) 209 (*c*) 2,900 (*d*) 678 (*e*) 600,807 (*f*) 60,800,700

Ans. (*a*) ninety two (*d*) six hundred seventy eight
(*b*) two hundred nine (*e*) six hundred thousand, eight hundred seven
(*c*) two thousand, nine hundred (*f*) sixty million, eight hundred thousand, seven hundred

9. Write each in expanded form. (2.3)

(*a*) 82 (*b*) 208 (*c*) 2,800 (*d*) 359 (*e*) 300,509 (*f*) 30,500,900

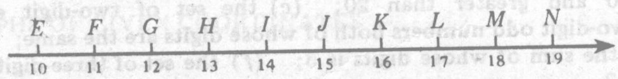
Ans. (*a*) 80 + 2 (*c*) 2,000 + 800 (*e*) 300,000 + 500 + 9
(*b*) 200 + 8 (*d*) 300 + 50 + 9 (*f*) 30,000,000 + 500,000 + 900

10. Write each in digit form. (2.4)

(*a*) eighty (*d*) three hundred forty two
(*b*) eight hundred five (*e*) four thousand, two hundred three
(*c*) five thousand eight (*f*) forty million, twenty thousand, thirty

Ans. (*a*) 80 (*b*) 805 (*c*) 5,008 (*d*) 342 (*e*) 4,203 (*f*) 40,020,030

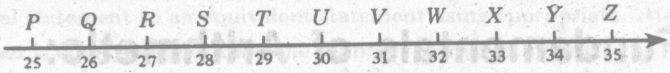
11. Name the coordinate of each point specified below. (3.1)



(*a*) point *H*, (*b*) point *M*, (*c*) numbered points between *I* and *L*, (*d*) point halfway between *E* and *M*, (*e*) point 4 units to the left of *N*, (*f*) point 5 units to the right of point *F*.

Ans. (*a*) 13 (*b*) 18 (*c*) 15, 16 (*d*) 14 (*e*) 15 (*f*) 16

12. Name the graph of each coordinate specified below. (3.2)



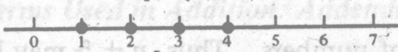
(*a*) 29, (*b*) 35, (*c*) the first three even coordinates, (*d*) the last two odd coordinates, (*e*) the coordinates greater than 26 and less than 30, (*f*) the successor of the first coordinate having both odd digits.

Ans. (*a*) *T* (*b*) *Z* (*c*) *Q, S, U* (*d*) *X, Z* (*e*) *R, S, T* (*f*) *W*

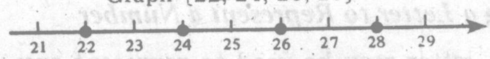
13. Graph: (*a*) the set of the first four natural numbers; (*b*) the set of the last digits of even numbers; (*c*) the set of odd numbers between 4 and 14; (*d*) the set of even numbers greater than 21 and less than 29; (*e*) the set of successors of 13, 18, and 19; (*f*) the set of numbers whose successors are 24, 29, and 32. (3.3)

Ans.

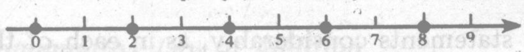
(*a*) Graph {1, 2, 3, 4}



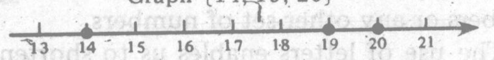
(*d*) Graph {22, 24, 26, 28}



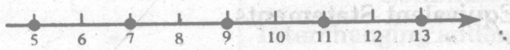
(*b*) Graph {0, 2, 4, 6, 8}



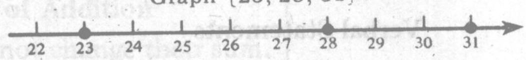
(*e*) Graph {14, 19, 20}



(*c*) Graph {5, 7, 9, 11, 13}



(*f*) Graph {23, 28, 31}



Chapter 2

Fundamentals of Arithmetic: Addition and Multiplication

1. FUNDAMENTAL OPERATIONS OF ARITHMETIC

The four fundamental operations of arithmetic are

1. ADDITION (sum)
2. SUBTRACTION (difference)
3. MULTIPLICATION (product)
4. DIVISION (quotient)

The words enclosed in parentheses name the results of the operations.

This chapter involves the operations of addition and multiplication of whole numbers. Chapter 3 will involve the operations of subtraction and division.

Using a Letter to Represent a Number

A letter may be used to represent any number in a set of numbers. Thus, $n + 5$ may be used to represent "any number increased by 5." The set of numbers may be the set of whole numbers or any other set of numbers.

The use of letters enables us to shorten verbal statements considerably, as in each of the following:

Verbal Statements	Equivalent Statements USING ×	Equivalent Statements OMITTING ×
1. The sum of three times a number and twice the same number equals five times the number.	$3 \times n + 2 \times n = 5 \times n$	$3n + 2n = 5n$
2. The product of one and any number equals the number.	$1 \times n = n$	$1n = n$
3. The product of any two numbers is the same as the product obtained by interchanging the numbers.	$a \times b = b \times a$	$ab = ba$

Omitting the Multiplication Sign

The multiplication sign may be omitted between a number and a letter, or between two letters.

Thus, in the first equivalent statement to the right above, $3n$ was used for "three times a number." Also, ab in the last equivalent statement represents "the product of any two numbers." The multiplication sign cannot be omitted between numbers being multiplied. We cannot write 3×4 as 34!

1.1 STATING PRODUCTS WITHOUT MULTIPLICATION SIGNS

State each product without multiplication signs.

(*a*) $10 \times n$ (*c*) $b \times c$ (*e*) $4 \times 2 \times z$
(*b*) $1 \times n$ (*d*) $7 \times a \times b$ (*f*) $2 \times 3 \times a \times g$

Illustrative Solution (*f*) Use 6, the product of 2 and 3. Omit the multiplication sign between 6 and a, and between a and g. *Ans.* $6ag$.

Ans. (*a*) $10n$ (*b*) $1n$ or n (*c*) bc (*d*) $7ab$ (*e*) $8z$

1.2 CHANGING VERBAL STATEMENTS TO EQUIVALENT STATEMENTS

Change each verbal statement to an equivalent statement using appropriate letters.

(*a*) Ten times a number is equal to eight times the number, increased by twice the number.
(*b*) The sum of a number and twice the number equals three times the number.
(*c*) Five times a number less twice the number equals three times the number.
(*d*) The perimeter of a square is four times the length of one of its sides.
(*e*) The selling price of an article is the sum of the cost and the profit.
(*f*) The difference between ten times a number and the number is nine times the number.

Illustrative Solution (*d*) Use p for "perimeter of a square" and s for "length of one of its sides."
Ans. $p = 4s$.

Ans. (*a*) $10n = 8n + 2n$ (*c*) $5n - 2n = 3n$ (*f*) $10n - n = 9n$
(*b*) $n + 2n = 3n$ (*e*) $s = c + p$

2. FUNDAMENTAL PROPERTIES OF ADDITION

Terms Used in Addition: Addends, Sum

Addends are numbers being added. Their *sum* is the answer obtained. Thus, in $2 + 3 = 5$, the addends are 2 and 3. Their sum is 5.

Commutative Law of Addition

Commutative Law of Addition

Interchanging addends does not change their sum.

Thus, $2 + 3 = 3 + 2$. In general, for any two numbers a and b, $a + b = b + a$.

Using a cash register, if first \$5.37 is rung up, and then \$4.25, the sum displayed must be the same as that which would be displayed if \$4.25 were the first to be rung up followed by \$5.37. Here, \$5.37 + \$4.25 = \$4.25 + \$5.37.

Additive Identity Property: Additive Identity

Additive Identity Property

Adding zero to any number results in the same number.

Thus, $543 + 0 = 543$. In general, for any number n, $n + 0 = n$.

Zero (0) is the *additive identity*, since adding zero to any number results in identically the same number.

Using a Number Line to Perform Addition

To Add Two Numbers Using a Number Line

Using a number line, add:	(*a*) $3 + 4$	(*b*) $4 + 3$
PROCEDURE	**SOLUTIONS**	
1. Begin at point whose coordinate is the first number:	Begin at 3, Fig. 2-1(*a*).	Begin at 4, Fig. 2-1(*b*).
2. Go to the right a number of units equal to the second number:	Go right 4 units.	Go right 3 units.
3. The sum is the coordinate of the point reached:	Reach 7, the sum. *Ans.* 7	Reach 7, the sum. *Ans.* 7