ADVANCED MECHANICS OF SOLIDS

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PREFACE

A good foundation in the mechanics of deformable solids is essential for most engineers. To meet this requirement many engineering educational programmes include, after an introductory course on strength of materials, an advanced course on the subject. The conventional method of treating the advanced subject as an extension of the introductory course leaves many things incomplete. At the same time, to treat the advanced subject completely from the continuum mechanics or elasticity theory approach is to make it unnecessarily complicated. A compromise therefore, is needed between these two approaches. The present book is expected to meet this requirement.

The contents of the book can logically be divided into two parts—the first part dealing with principles and the second part with applications or specific problems. The first two chapters discuss the analysis of stress and the analysis of strain. These two chapters follow the continuum mechanics approach without much mathematical complexity. This approach lays a good foundation to the subject matter. These topics are analysed in the language of 'strength of materials' rather than in the language of the 'elasticity theory', without losing rigour. The third chapter dealing with stress-strain relations for linearly elastic solids makes use of physical interpretations to arrive at the results rather than depending too heavily on a formal approach. Students have found this approach more appealing than the conventional formal one.

In most textbooks, the theories of failure or yield criteria are discussed towards the end of the book. However, its logical place is immediately after the stress-strain relations since it establishes the condition or conditions when yielding begins. Consequently, Chapter 4 deals with theories of failure or yield criteria. This chapter also contains a brief introduction to ideally plastic solid. Chapter 5 dealing with energy methods is quite exhaustive and covers many important topics like the reciprocal theorem, theorems of Castigliano, the theorem of virtual work, Engesser's theory and Maxwell-Mohr integrals.

The last five chapters deal with applications or specific problems. Bending of beams is discussed in Chapter 6. Asymmetrical bending, shear centre, curved beams and deflections of thick curved bars are treated in this chapter. Torsion of solid cylindrical rods and of thin-walled multiple-cell closed sections are discussed in Chapter 7. Timoshenko's book on the advanced strength of materials had influenced many authors to include a chapter on plates and shells in their books. However, the trend is changing. The theory of plates and shells is taught separately in most institutions and not much justice can be done in including a brief discussion on these two topics. Instead, a chapter dealing with axisymmetric problems and another on thermal stresses are included in this book. Since many engineering problems deal with these, it is believed that their inclusion will be more useful. The last chapter contains three sections. These sections deal respectively with beam columns, treatment of stability problem as an eigenvalue problem, and energy methods to solve buckling problems.

All chapters have worked examples. Problems for solution are given at the end of each chapter. Answers are given to most of the problems. In dealing with numerical examples and problems, quantities are given in MKS as well as SI units. I shall be grateful if my attention is drawn to errors that might have crept in.

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1

ANALYSIS OF STRESS

1.1 Introduction

In this book we shall deal with the mechanics of deformable solids. The starting point for discussion can be either the analysis of stress or the analysis of strain. In books on the theory of elasticity, one usually starts with the analysis of strain which deals with the geometry of deformation without considering the forces that cause the deformation. However, one is more familiar with forces, though the measurement of force is usually done through the measurement of deformations caused by the force. Books on the strength of materials, begin with the analysis of stress. The concept of stress has already been introduced in the elementry strength of materials. When a bar of uniform cross-section, say a circular rod of diameter d, is subjected to a tensile force F along the axis of the bar, average stress σ induced across any transverse section perpendicular to the axis of the bar and away from the region of loading is given by

$$\sigma = \frac{F}{\text{Area}} = \frac{4F}{\pi d^2}$$

It is assumed that the reader is familiar with the elementary flexural stress and torsional stress concepts. In general, a structural member or a machine element will not possess uniform geometry of shape or size, and the loads acting on it will also be complex. For example, an automobile crankshaft or a piston inside an engine's cylinder or an aircraft wing are subject to loadings which are both complex as well as dynamic in nature. In such cases, one will have to introduce the concept of the state of stress at a point and

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its analysis which will be the subject of discussion in this chapter. However, we shall not deal with forces which vary with time.

It will be assumed that the matter of the body that is being considered is continuously distributed over its volume, so that if we consider a small volume element of the matter surrounding a point and shrink this volume, in the limit we shall not come across a void. In reality, however, all materials are composed of many discrete particles which are often microscopic, and when an arbitrarily selected volume element is shrunk, in the limit one may end up in a void. But in our analysis, we assume that the matter is continuously distributed. Such a body is called a continuous medium and the mechanics of such a body or bodies is called continuum mechanics.

1.2 Body Force, Surface Force and Stress Vector

Consider a body **B** occupying a region of space referred to a rectangular coordinate system *Oxyz*, as shown in Fig. 1.1. In general, the body will be subjected to two types of forces—body forces and surface forces. The body force acts on each volume element of the body. Examples of this kind of force are the gravitational force, the inertia force and the magnetic force. The surface forces act on the surface or area elements of the body. When

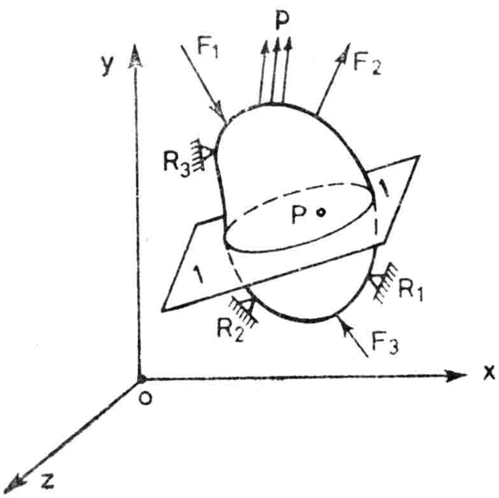

Fig. 1.1 Body subjected to forces

the area considered lies on the actual boundary of the body, the surface force distribution is often termed surface traction. In Fig. 1.1, the surface forces F_1, F_2, F_3, ... F_n are concentrated forces, while P is a distributed force.

The support reactions R_1, R_2 and R_3 are also surface forces. It is explicitly assumed that under the action of both body forces and surface forces, the body is in equilibrium.

Let P be a point inside the body with coordinates (x, y, z). Let the body be cut into two parts C and D by a plane 1-1 passing through point P, as shown in Fig. 1.2. If we consider the free-body diagrams of C and D, then each part is in equilibrium under the action of the externally applied forces and the internally distributed forces across the interface. In part D, let ΔA be a small area surrounding point P. In part C, the corresponding area at P′ is $\Delta A'$. These two areas are distinguished by their outward drawn normals $\vec{n}$ and $\vec{n}'$. The action of part C on ΔA at point P can be represented by force vector $\Delta \vec{T}$, and the action of part D on $\Delta A'$ at P′ can be represented by force vector $\Delta \vec{T}'$. We assume that as ΔA tends to zero, ratio $\Delta \vec{T}/\Delta A$ tends to a definite limit, and further, the moment of the forces acting on area ΔA about any point within the area vanishes in the limit. The limiting vector is written as

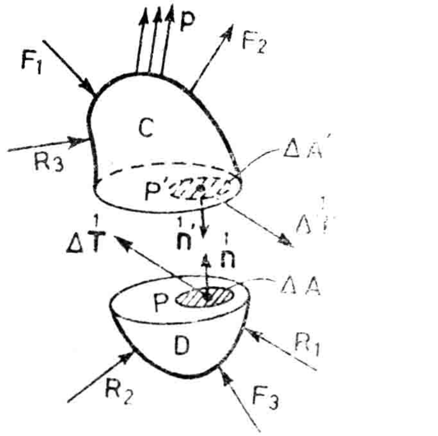

Fig. 1.2 Free body diagrams of a body cut into two parts

$$\lim_{\Delta A \to 0} \frac{\Delta \vec{T}}{\Delta A} = \frac{d\vec{T}}{dA} = \vec{T} \tag{1.1}$$

Similarly, at point P′, the action of part D on C as $\Delta A'$ tends to zero, can be represented by a vector

$$\lim_{\Delta A' \to 0} \frac{\Delta \vec{T}'}{\Delta A'} = \frac{d\vec{T}'}{dA} = \vec{T}' \tag{1.2}$$

Vectors $\vec{T}$ and $\vec{T}'$ are called the stress vectors and they represent forces per unit area acting respectively at P and P′ on planes with outward drawn normals $\vec{n}$ and $\vec{n}'$.

We further assume that stress vector $\vec{T}$ representing the action of C on D at P is equal in magnitude and opposite in direction to stress vector $\vec{T}'$ representing the action of D on C at corresponding point P′. This assumption is similar to Newton's third law which is applicable to particles. We have thus

$$\vec{T} = -\vec{T}' \tag{1.3}$$

If the body in Fig. 1.1 is cut by a different plane, 2-2 passing through the same point P, then the stress vector representing the action of C_2 on D_2 will be represented by $\overset{2}{T}$ (Fig. 1.3), i.e.

$$\overset{2}{T} = \lim_{\Delta A \to 0} \frac{\Delta T_2}{\Delta A}$$

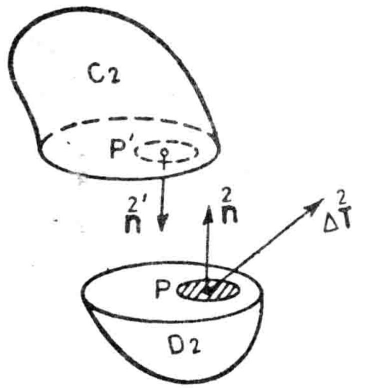

Fig. 1.3 Body cut by another plane

In general, stress vector $\overset{1}{T}$ acting at point P on a plane with outward drawn normal $\overset{1}{n}$ will be different from stress vector $\overset{2}{T}$ acting at the same point P, but on a plane with outward drawn normal $\overset{2}{n}$. Hence the stress at a point depends not only on the location of the point (identified by coordinates x, y, z) but also on the plane passing through the point (identified by direction cosines n_x, n_y, n_z of the outward drawn normal).

1.3 The State of Stress at a Point

Since an infinite number of planes can be drawn through a point, we get an infinite number of stress vectors acting at a given point, each stress vector characterized by the corresponding plane on which it is acting. The totality of all stress vectors acting on every possible plane passing through the point is defined to be the state of stress at the point. It is the knowledge of this state of stress which is of importance to a designer in determining the critical planes and the respective critical stresses. It will be shown in Sec. 1.6, that if the stress vectors acting on three mutually perpendicular planes are known, we can determine the stress vector acting on any other arbitrary plane.

1.4 Normal and Shear Stress Components

Let $\overset{n}{T}$ be the stress vector at point P acting on a plane whose outward drawn normal is $\boldsymbol{n}$, (Fig. 1.4). This can be resolved into two components, one along the normal $\boldsymbol{n}$ and the other perpendicular to $\boldsymbol{n}$. The component parallel to

$\boldsymbol{n}$ is called the normal stress and is generally denoted by σ_n. The component perpendicular to $\boldsymbol{n}$ is known as the tangential stress or shear stress component and is denoted by τ_n. We have, therefore, the relation:

$$\left|\overset{n}{T}\right|^2 = \sigma^{n2} + \tau_n^2 \qquad (1.4)$$

where $\left|\overset{n}{T}\right|$ is the magnitude of the resultant stress. Stress vector $\overset{n}{T}$ can also be resolved into three components parallel to the x, y, z axes. If these components are denoted by $\overset{n}{T}_x, \overset{n}{T}_y, \overset{n}{T}_z$, we have

$$\left|\overset{n}{T}\right|^2 = \overset{n}{T}_x{}^2 + \overset{n}{T}_y{}^2 + \overset{n}{T}_z{}^2 \qquad (1.5)$$

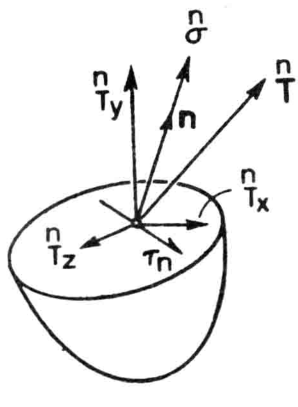

Fig. 1.4 Resultant stress vector, normal and shear stress components

1.5 Rectangular Stress Components

Let the body B, shown in Fig. 1.1, be cut by a plane parallel to the yz plane. The normal to this plane is parallel to the x axis and, hence, the plane is called the x plane. The resultant stress vector at P acting on this will be $\overset{x}{T}$. This vector can be resolved into three components parallel to the x, y, z axes. The component parallel to the x axis, being normal to the plane, will be denoted by σ_x. The components parallel to the y and z axes are shear stress components and are denoted by τ_{xy} and τ_{xz} respectively (Fig. 1.5.)

In the above designation, the first subscript x indicates the plane on which the stresses are acting and the second subscript (y or z) indicates the direction of the component. For example, τ_{xy} is the stress component on the x plane in y direction. Similarly, τ_{xz} is the stress component on the x plane in z direction. To maintain consistency, one should have denoted the normal stress component as τ_{xx}. This would be the stress component on the x plane in the x direction. However, to distinguish between a normal stress and a shear stress, the normal stress is denoted by σ and the shear stress by τ.

At any point P, one can draw three mutually perpendicular planes, the x plane, y plane and the z plane. Following the notation mentioned above, the normal and shear stress components on these planes are

σ_x, τ_{xy}, τ_{xz} on x plane
σ_y, τ_{yz}, τ_{yx} on y plane
σ_z, τ_{zx}, τ_{zy} on z plane.

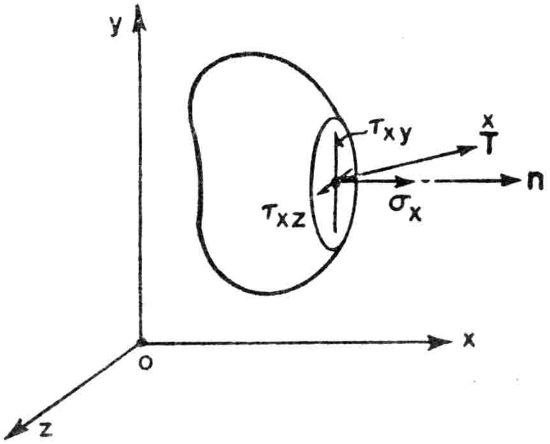

Fig. 1.5 Stress components on x plane

These components are shown acting on a small rectangular element Fig. 1.6.

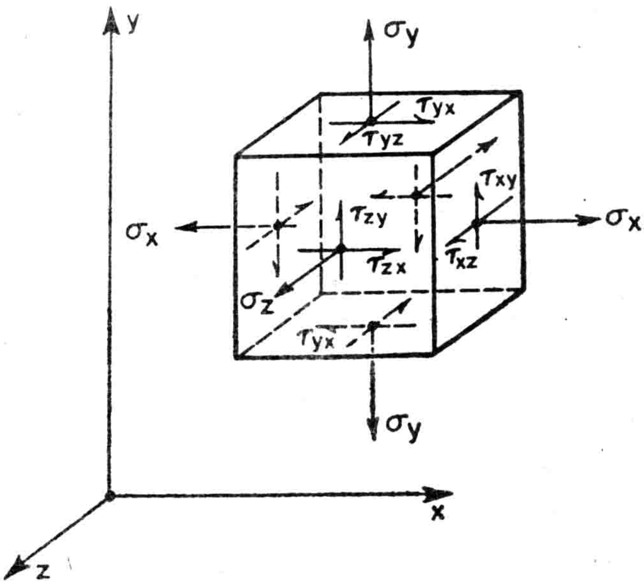

Fig. 1.6 Rectangular stress components

One should observe that the three visible faces of the rectangular element have their outward drawn normals along the positive x, y and z axes respectively. Consequently, the positive stress components on these faces will also be directed along the positive axes. The three hidden faces have their outward drawn normals in the negative x, y and z axes. The positive stress components on these faces will, therefore, be directed along the negative axes. For example, the bottom face has its outward drawn normal along the negative y axis. Hence, the positive stress components on this face, i.e., σ_y, τ_{yx} and τ_{yz}, are directed respectively along the negative y, x and z axes.

1.6 Stress Components on an Arbitrary Plane

It was stated in Sec. 1.3 that a knowledge of stress components acting on three mutually perpendicular planes passing through a point, will determine the stress components acting on any plane passing through that point. Let the three mutually perpendicular planes be the x, y and z planes and let the arbitrary plane be identified by its outward drawn normal $\boldsymbol{n}$ whose direction cosines are n_x, n_y, and n_z. Consider a small tetrahedron at P with three of its faces normal to the coordinate axes, and the inclined face having its normal parallel to $\boldsymbol{n}$. Let h be the perpendicular distance from P to the inclined face. If the tetrahedron is isolated from the body and a free-body diagram is drawn, then it will be in equilibrium under the action of the surface forces and the body forces. The free-body diagram is shown in Fig. 1.7.

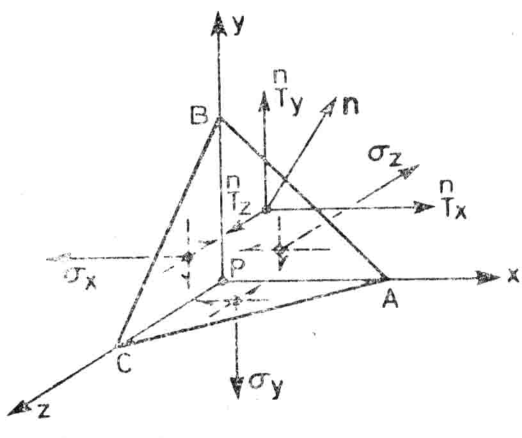

Fig. 1.7 Tetrahedron at point P

Since the size of the tetrahedron considered is very small and in the limit as we are going to make h tend to zero, we shall speak in terms of the average stresses over the faces. Let $\overset{n}{T}$ be the resultant stress vector on face