Ferdinand P. Beer ▪ E. Russell Johnston, Jr. ▪ John T. DeWolf

MECHANICS of MATERIALS

Fourth Edition

Fourth Edition

MECHANICS OF MATERIALS

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About the Authors

As publishers of the books written by Ferd Beer and Russ Johnston, we are often asked how did they happen to write the books together, with one of them at Lehigh and the other at the University of Connecticut.

The answer to this question is simple. Russ Johnston's first teaching appointment was in the Department of Civil Engineering and Mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in mechanics. Born in France and educated in France and Switzerland (he held an M.S. degree from the Sorbonne and an Sc.D. degree in the field of theoretical mechanics from the University of Geneva), Ferd had come to the United States after serving in the French army during the early part of World War II and had taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Born in Philadelphia, Russ had obtained a B.S. degree in civil engineering from the University of Delaware and an Sc.D. degree in the field of structural engineering from MIT.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the mechanics courses. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented to them in a graphic way. Together they wrote lecture notes in statics and dynamics, to which they later added problems they felt would appeal to future engineers, and soon they produced the manuscript of the first edition of *Mechanics for Engineers*. The second edition of *Mechanics for Engineers* and the first edition of *Vector Mechanics for Engineers* found Russ Johnston at Worcester Polytechnic Institute and the next editions at the University of Connecticut. In the meantime, both Ferd and Russ had assumed administrative responsibilities in their departments, and both were involved in research, consulting, and supervising graduate students—Ferd in the area of stochastic processes and random vibrations, and Russ in the area of elastic stability and structural analysis and design. However, their interest in improving the teaching of the basic mechanics courses had not subsided, and they both taught sections of these courses as they kept revising their texts and began writing together the manuscript of the first edition of *Mechanics of Materials*.

Ferd and Russ's contributions to engineering education earned them a number of honors and awards. They were presented with the Western Electric Fund Award for excellence in the instruction of engineering students by their respective regional sections of the American Society for Engineering Education, and they both received the Distinguished Educator Award from the Mechanics Division of the same society. In 1991 Russ received the Outstanding Civil Engineer Award from the Connecticut Section of the American Society of Civil Engineers, and in 1995 Ferd was awarded an honorary Doctor of Engineering degree by Lehigh University.

John T. DeWolf, Professor of Civil Engineering at the University of Connecticut, joined the Beer and Johnston team as an author on the second edition of *Mechanics of Materials*. John holds a B.S. degree in civil engineering from the University of Hawaii and M.E. and Ph.D. degrees in structural engineering from Cornell University. His research interests are in the area of elastic stability, bridge monitoring, and structural analysis and design. He is a member of the Connecticut Board of Examiners for Professional Engineers.

PREFACE

OBJECTIVES

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. The authors hope that it will help instructors achieve this goal in that particular course in the same way that their other texts may have helped them in statics and dynamics.

GENERAL APPROACH

In this text the study of the mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to clearly indicate the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

Free-body Diagrams Are Used Extensively. Throughout the text free-body diagrams are used to determine external or internal forces. The use of "picture equations" will also help the students understand the superposition of loadings and the resulting stresses and deformations.

Design Concepts Are Discussed Throughout the Text Whenever Appropriate. A discussion of the application of the factor of safety to design can be found in Chap. 1, where the concepts of both allowable stress design and load and resistance factor design are presented.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the examples, sample problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Since a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find most desirable for their class.

Optional Sections Offer Advanced or Specialty Topics. Topics such as residual stresses, torsion of noncircular and thin-walled members, bending of curved beams, shearing stresses in non-symmetrical members, and failure criteria, have been included in optional sections for use in courses of varying emphases. To preserve the integrity of the subject, these topics are presented in the proper sequence, wherever they logically belong. Thus, even when not covered in the course, they are highly visible and can be easily referred to by the students if needed in a later course or in engineering practice. For convenience all optional sections have been indicated by asterisks.

CHAPTER ORGANIZATION

It is expected that students using this text will have completed a course in statics. However, Chap. 1 is designed to provide them with an opportunity to review the concepts learned in that course, while shear and bending-moment diagrams are covered in detail in Secs. 5.2 and 5.3. The properties of moments and centroids of areas are described in Appendix A; this material can be used to reinforce the discussion of the determination of normal and shearing stresses in beams (Chaps. 4, 5, and 6).

The first four chapters of the text are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. Each analysis is based on a few basic concepts, namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study of each type of loading is complemented by a large number of examples, sample problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The concept of stress at a point is introduced in Chap. 1, where it is shown that an axial load can produce shearing stresses as well as normal stresses, depending upon the section considered. The fact that stresses depend upon the orientation of the surface on which they are computed is emphasized again in Chaps. 3 and 4 in the cases of torsion and pure bending. However, the discussion of computational techniques—such as Mohr's circle—used for the transformation of stress at a point is delayed until Chap. 7, after students have had the opportunity to solve problems involving a combination of the basic loadings and have discovered for themselves the need for such techniques.

The discussion in Chap. 2 of the relation between stress and strain in various materials includes fiber-reinforced composite materials. Also, the study of beams under transverse loads is covered in two separate chapters. Chapter 5 is devoted to the determination of the normal stresses in a beam and to the design of beams based on the allowable normal stress in the material used (Sec. 5.4). The chapter begins with a discussion of the shear and bending-moment diagrams (Secs. 5.2 and 5.3) and includes an optional section on the use of singularity functions for the determination of the shear and bending moment in a beam (Sec. 5.5). The chapter ends with an optional section on nonprismatic beams (Sec. 5.6).

Chapter 6 is devoted to the determination of shearing stresses in beams and thin-walled members under transverse loadings. The formula for the shear flow, $q = VQ/I$, is derived in the traditional way. More advanced aspects of the design of beams, such as the determination of the principal stresses at the junction of the flange and web of a W-beam, are in Chap. 8, an optional chapter that may be covered after the transformations of stresses have been discussed in Chap. 7. The design of transmission shafts is in that chapter for the same reason, as well as the determination of stresses under combined loadings that can now include the determination of the principal stresses, principal planes, and maximum shearing stress at a given point.

Statically indeterminate problems are first discussed in Chap. 2 and considered throughout the text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics. In this way, they will have become thoroughly familiar with this fundamental method by the end of the course. In addition, this approach helps the students realize that stresses themselves are statically indeterminate and can be computed only by considering the corresponding distribution of strains.

The concept of plastic deformation is introduced in Chap. 2, where it is applied to the analysis of members under axial loading. Problems involving the plastic deformation of circular shafts and of prismatic beams are also considered in optional sections of Chaps. 3, 4, and 6. While some of this material can be omitted at the choice of the instructor, its inclusion in the body of the text will help students realize the limitations of the assumption of a linear stress-strain relation and serve to caution them against the inappropriate use of the elastic torsion and flexure formulas.

The determination of the deflection of beams is discussed in Chap. 9. The first part of the chapter is devoted to the integration method and to the method of superposition, with an optional section (Sec. 9.6) based on the use of singularity functions. (This section should be used only if Sec. 5.5 was covered earlier.) The second part of Chap. 9 is optional. It presents the moment-area method in two lessons.

Chapter 10 is devoted to columns and contains material on the design of steel, aluminum, and wood columns. Chapter 11 covers energy methods, including Castigliano's theorem.

PEDAGOGICAL FEATURES

Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems.

Chapter Lessons. The body of the text has been divided into units, each consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally can be covered in one lesson.

Examples and Sample Problems. The theory sections include many examples designed to illustrate the material being presented and facilitate its understanding. The sample problems are intended to show some of the applications of the theory to the solution of engineering problems. Since they have been set up in much the same form that students will use in solving the assigned problems, the sample problems serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and help the students understand the basic principles used in mechanics of materials. The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty. Problems requiring special attention have been indicated by asterisks. Answers to problems are given at the end of the book, except for those with a number set in italics.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in the chapter. Notes in the margin have been included to help the students organize their review work, and cross references provided to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Computer Problems. The availability of personal computers makes it possible for engineering students to solve a great number of challenging problems. A group of six or more problems designed to be solved with a computer can be found at the end of each chapter. Developing the algorithm required to solve a given problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem.

Fundamentals of Engineering Examination. Engineers who seek to be licensed as *Professional Engineers* must take two exams. The first exam, the *Fundamentals of Engineering Examination,* includes subject material from *Mechanics of Materials.* Appendix E lists the topics in *Mechanics of Materials* that are covered in this exam along with problems that can be solved to review this material.

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We also gratefully acknowledge the help, comments and suggestions offered by the many users of previous editions of *Mechanics of Materials.*

E. Russell Johnston, Jr.
John T. DeWolf

List of Symbols

a Constant; distance
A, B, C, . . . Forces; reactions
$A, B, C, \ldots$ Points
A, a Area
b Distance; width
c Constant; distance; radius
C Centroid
$C_1, C_2, \ldots$ Constants of integration
C_P Column stability factor
d Distance; diameter; depth
D Diameter
e Distance; eccentricity; dilatation
E Modulus of elasticity
f Frequency; function
F Force
$F.S.$ Factor of safety
G Modulus of rigidity; shear modulus
h Distance; height
H Force
H, J, K Points
$I, I_x, \ldots$ Moment of inertia
$I_{xy}, \ldots$ Product of inertia
J Polar moment of inertia
k Spring constant; shape factor; bulk modulus; constant
K Stress concentration factor; torsional spring constant
l Length; span
L Length; span
L_e Effective length
m Mass
M Couple
$M, M_x, \ldots$ Bending moment
M_D Bending moment, dead load (LRFD)
M_L Bending moment, live load (LRFD)
M_U Bending moment, ultimate load (LRFD)
n Number; ratio of moduli of elasticity; normal direction
p Pressure
P Force; concentrated load
P_D Dead load (LRFD)
P_L Live load (LRFD)
P_U Ultimate load (LRFD)
q Shearing force per unit length; shear flow
Q Force
Q First moment of area
r Radius; radius of gyration
R Force; reaction
R Radius; modulus of rupture
s Length
S Elastic section modulus
t Thickness; distance; tangential deviation
T Torque
T Temperature
u, v Rectangular coordinates
u Strain-energy density
U Strain energy; work
v Velocity
V Shearing force
V Volume; shear
w Width; distance; load per unit length
W, W Weight, load
x, y, z Rectangular coordinates; distance; displacements; deflections
$\bar{x}, \bar{y}, \bar{z}$ Coordinates of centroid
Z Plastic section modulus
α, β, γ Angles
α Coefficient of thermal expansion; influence coefficient
γ Shearing strain; specific weight
γ_D Load factor, dead load (LRFD)
γ_L Load factor, live load (LRFD)
δ Deformation; displacement
ϵ Normal strain
θ Angle; slope
λ Direction cosine
ν Poisson's ratio
ρ Radius of curvature; distance; density
σ Normal stress
τ Shearing stress
ϕ Angle; angle of twist; resistance factor
ω Angular velocity

MECHANICS OF MATERIALS

Contents

Preface xiii
List of Symbols xix

1
INTRODUCTION—CONCEPT OF STRESS
2

1.1	Introduction	2
1.2	A Short Review of the Methods of Statics	2
1.3	Stresses in the Members of a Structure	5
1.4	Analysis and Design	6
1.5	Axial Loading; Normal Stress	7
1.6	Shearing Stress	9
1.7	Bearing Stress in Connections	11
1.8	Application to the Analysis and Design of Simple Structures	12
1.9	Method of Problem Solution	14
1.10	Numerical Accuracy	15
1.11	Stress on an Oblique Plane under Axial Loading	23
1.12	Stress under General Loading Conditions; Components of Stress	24
1.13	Design Considerations	27
	Review and Summary for Chapter 1	38

2
STRESS AND STRAIN—AXIAL LOADING
47

2.1	Introduction	47
2.2	Normal Strain under Axial Loading	48
2.3	Stress-Strain Diagram	50
*2.4	True Stress and True Strain	55
2.5	Hooke's Law; Modulus of Elasticity	56
2.6	Elastic versus Plastic Behavior of a Material	57

2.7 Repeated Loadings; Fatigue 59
2.8 Deformations of Members under Axial Loading 61
2.9 Statically Indeterminate Problems 70
2.10 Problems Involving Temperature Changes 74
2.11 Poisson's Ratio 84
2.12 Multiaxial Loading; Generalized Hooke's Law 85
*2.13 Dilatation; Bulk Modulus 87
2.14 Shearing Strain 89
2.15 Further Discussion of Deformations under Axial Loading; Relation among E, ν, and G 92
*2.16 Stress-Strain Relationships for Fiber-Reinforced Composite Materials 95
2.17 Stress and Strain Distribution under Axial Loading; Saint-Venant's Principle 104
2.18 Stress Concentrations 107
2.19 Plastic Deformations 109
*2.20 Residual Stresses 113

Review and Summary for Chapter 2 121

3 TORSION 132

3.1 Introduction 132
3.2 Preliminary Discussion of the Stresses in a Shaft 134
3.3 Deformations in a Circular Shaft 136
3.4 Stresses in the Elastic Range 139
3.5 Angle of Twist in the Elastic Range 150
3.6 Statically Indeterminate Shafts 153
3.7 Design of Transmission Shafts 165
3.8 Stress Concentrations in Circular Shafts 167
*3.9 Plastic Deformations in Circular Shafts 172
*3.10 Circular Shafts Made of an Elastoplastic Material 174
*3.11 Residual Stresses in Circular Shafts 177
*3.12 Torsion of Noncircular Members 186
*3.13 Thin-Walled Hollow Shafts 189

Review and Summary for Chapter 3 198

4 PURE BENDING 209

4.1 Introduction 209
4.2 Symmetric Member in Pure Bending 211
4.3 Deformations in a Symmetric Member in Pure Bending 213
4.4 Stresses and Deformations in the Elastic Range 216

4.5 Deformations in a Transverse Cross Section 220
4.6 Bending of Members Made of Several Materials 230
4.7 Stress Concentrations 234
*4.8 Plastic Deformations 243
*4.9 Members Made of an Elastoplastic Material 246
*4.10 Plastic Deformations of Members with a Single Plane of Symmetry 250
*4.11 Residual Stresses 250
4.12 Eccentric Axial Loading in a Plane of Symmetry 260
4.13 Unsymmetric Bending 270
4.14 General Case of Eccentric Axial Loading 276
*4.15 Bending of Curved Members 285
Review and Summary for Chapter 4 298

5
ANALYSIS AND DESIGN OF BEAMS FOR BENDING
308

5.1 Introduction 308
5.2 Shear and Bending-Moment Diagrams 311
5.3 Relations among Load, Shear, and Bending Moment 322
5.4 Design of Prismatic Beams for Bending 332
*5.5 Using Singularity Functions to Determine Shear and Bending Moment in a Beam 343
*5.6 Nonprismatic Beams 354
Review and Summary for Chapter 5 363

6
SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS
372

6.1 Introduction 372
6.2 Shear on the Horizontal Face of a Beam Element 374
6.3 Determination of the Shearing Stresses in a Beam 376
6.4 Shearing Stresses τ_{xy} in Common Types of Beams 377
*6.5 Further Discussion of the Distribution of Stresses in a Narrow Rectangular Beam 380
6.6 Longitudinal Shear on a Beam Element of Arbitrary Shape 388
6.7 Shearing Stresses in Thin-Walled Members 390
*6.8 Plastic Deformations 392
*6.9 Unsymmetric Loading of Thin-Walled Members; Shear Center 402
Review and Summary for Chapter 6 414

7
TRANSFORMATIONS OF STRESS AND STRAIN
423

7.1 Introduction 423
7.2 Transformation of Plane Stress 425
7.3 Principal Stresses: Maximum Shearing Stress 428
7.4 Mohr's Circle for Plane Stress 436
7.5 General State of Stress 446
7.6 Application of Mohr's Circle to the Three-Dimensional Analysis of Stress 448
*7.7 Yield Criteria for Ductile Materials under Plane Stress 451
*7.8 Fracture Criteria for Brittle Materials under Plane Stress 453
7.9 Stresses in Thin-Walled Pressure Vessels 462
*7.10 Transformation of Plane Strain 470
*7.11 Mohr's Circle for Plane Strain 473
*7.12 Three-Dimensional Analysis of Strain 475
*7.13 Measurements of Strain; Strain Rosette 478
Review and Summary for Chapter 7 486

8
PRINCIPAL STRESSES UNDER A GIVEN LOADING
496

*8.1 Introduction 496
*8.2 Principal Stresses in a Beam 497
*8.3 Design of Transmission Shafts 500
*8.4 Stresses under Combined Loadings 508
Review and Summary for Chapter 8 521

9
DEFLECTION OF BEAMS
530

9.1 Introduction 530
9.2 Deformation of a Beam under Transverse Loading 532
9.3 Equation of the Elastic Curve 533
*9.4 Direct Determination of the Elastic Curve from the Load Distribution 538
9.5 Statically Indeterminate Beams 540
*9.6 Using Singularity Functions to Determine the Slope and Deflection of a Beam 549
9.7 Method of Superposition 558

9.8 Application of Superposition to Statically Indeterminate Beams 560
*9.9 Moment-Area Theorems 569
*9.10 Application to Cantilever Beams and Beams with Symmetric Loading 571
*9.11 Bending-Moment Diagrams by Parts 573
*9.12 Application of Moment-Area Theorems to Beams with Unsymmetric Loadings 582
*9.13 Maximum Deflection 584
*9.14 Use of Moment-Area Theorems with Statically Indeterminate Beams 586
Review and Summary for Chapter 9 594

10
COLUMNS
607

10.1 Introduction 607
10.2 Stability of Structures 608
10.3 Euler's Formula for Pin-Ended Columns 610
10.4 Extension of Euler's Formula to Columns with Other End Conditions 614
*10.5 Eccentric Loading; the Secant Formula 625
10.6 Design of Columns under a Centric Load 636
10.7 Design of Columns under an Eccentric Load 652
Review and Summary for Chapter 10 662

11
ENERGY METHODS
670

11.1 Introduction 670
11.2 Strain Energy 670
11.3 Strain-Energy Density 672
11.4 Elastic Strain Energy for Normal Stresses 674
11.5 Elastic Strain Energy for Shearing Stresses 677
*11.6 Strain Energy for a General State of Stress 680
11.7 Impact Loading 693
11.8 Design for Impact Loads 695
11.9 Work and Energy under a Single Load 696
11.10 Deflection under a Single Load by the Work-Energy Method 698
*11.11 Work and Energy under Several Loads 709
*11.12 Castigliano's Theorem 711
*11.13 Deflections by Castigliano's Theorem 712
*11.14 Statically Indeterminate Structures 716
Review and Summary for Chapter 11 726